COMPARISON OF GAMMA, RAYLEIGH, WEIBULL AND NRCS MODELS WITH OBSERVED RUNOFF DATA FOR CENTRAL TEXAS SMALL WATERSHEDS

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ABSTRACT

Unit hydrograph methods are applied by TxDOT designers to obtain peak discharge and hydrograph shape for hydraulic design. Unit hydrographs are applied to watersheds that either are too large for application of the rational method or are sufficiently complex that the assumptions necessary for application of the rational method do not apply. Currently, the Natural Resources Conservation Service (NRCS) dimensionless unit hydrograph method is used by TxDOT to estimate unit hydrographs for ungaged watershed in Texas. Three candidate models derived from a linear-system analysis are compared with NRCS model, along with an early empirical model. The models are Gamma model, Rayleigh model, Weibull model and the empirical model by Commons. In this research the watersheds being studied are from central Texas and are divided into location modules: Dallas, Austin, San Antonio, Fort Worth and Small Rural watersheds. The five modules contain data from over 84 stations and a combined total of 1642 storm events to run the testing models. Results show that all the models have produced acceptable prediction of runoff discharge, when supplied historical precipitation events. The Weibull model produced the best "fit" as was expected because it has the most adjustable parameters. In addition to simple model selection analysis, this research also tested the worth of constant base flow separation for this particular dataset.

TABLE OF CONTENTS

LIST OF FIGURES

FIGURES **Page**

LIST OF TABLES

CHAPTER 1

PROBLEM STATEMENT

Unit hydrographs have been widely used in hydrologic engineering for over 70 years since Sherman's introduction of the concept in 1932, and are still considered a standard-of-practice. Unfortunately, most hydraulic design is performed for watersheds without both a stream gage and one or more rain gages that together provide rainfall-runoff history. In such cases, a synthetic unit graph is estimated from statistical procedures. Synthetic unit graphs refer to unit graphs developed for a particular ungaged watershed using timing and shape parameters of the unit graphs that are statistically transferred or regionalized from nearby gaged watersheds considered to be similar to the ungaged watershed.

The purpose of this research is to determine how TxDOT should apply unit hydrograph technology for drainage analysis in Texas. The overall research program is intended to answer two questions: First, is the NRCS dimensionless unit hydrograph as currently published, representative of observed unit hydrographs for Texas watersheds? Second, if the NRCS dimensionless unit hydrograph is not representative of unit hydrographs for Texas watersheds, then can an alternative method or adjustment be developed that is representative of observed hydrographs in Texas? To answer these questions a research consortium composed of Texas Tech University, Lamar University, the United States Geologic Survey-Austin District, and the University of Houston jointly developed a relatively large database of paired rainfall-runoff measurements on small watersheds in Central Texas and independently analyzed the database using different unit hydrograph techniques. The University of Houston was assigned the task of developing instantaneous unit hydrograph (IUH) methods.

This thesis is one-component of the IUH analysis study preferred for the larger project. The specific problem addressed in this thesis is selection of an IUH function to represent the rainfall-runoff process in the Central Texas database and evaluation of the necessity of base flow separation in these data.

Five model IUH functions are proposed and used to “fit” the storms in the database. Each model function is then characterized by a set of acceptability criteria, and the model that satisfies the criteria more frequently for most of storms is selected as the preferred IUH model for future regionalization efforts.

Each model is fit to storms without and with base flow separation and the same criteria as above are applied. Base flow separation is a concern because initially the research team assumed separation would be unnecessary for these data.

The remainder of this thesis is outlined as follows. Chapter 2 is a review of relevant literature regarding unit hydrograph analysis with specific attention to instantaneous unit hydrographs. Chapter 3 explains the methods used in this component of the research. Chapter 4 is a presentation of the results, and Chapter 5 presents the conclusions for this research.

CHAPTER 2

LITERATURE REVIEW

Hydrological studies are a search for an improved physical interpretation of phenomena and for the creation of mathematical instruments for the management and control of water resources (Marco Franchini 1991).

In the measurement of water resources specific objectives of various applied problems are:

(1) Evaluation of the maximum flood discharge to be used, e.g. in the design of urban sewerage systems or reclamation systems.

(2) Evaluation of flood waves, to aid both the design of appropriate defense systems and the control of flood waves, especially by means of real-time flood forecasting;

(3) Construction, starting from the knowledge of rainfall, of daily or sub-daily runoff, for long periods of time, in order to reconstruct the runoff hydrograph itself, with particular reference to those sections without measurements;

(4) Evaluation of the influence of the type of soil and sub-soil on the runoff formation dynamic, to analyze the consequences of anthropogenic defects (inhabited areas, deforestation, etc.);

The third aspect is the main purpose for this research, and the application of the study is expected to be used for those un-gaged watersheds, especially for the central Texas area.

2.1. Runoff Prediction

Practical runoff prediction using hydrological concepts has been practiced for at least a century. The approach is to determine the runoff hydrograph from a precipitation hyetograph for a specific watershed. The procedures used prior to the 1940's were largely empirical ad-hoc models of the rainfall runoff process. It was recognized in the 1850's that runoff was related to rainfall intensity, rainfall duration (i.e. the hyetograph), and to the time required for runoff to leave a watershed. Furthermore at that time it was also recognized that the watershed's "time" characteristic was related to its slope, area, and shape. J.C.I. Dooge, who established the basis for application of linear systems theory to hydrograph analysis, and was the first to establish the theoretical basis for unit hydrographs, credits this early understanding of runoff behavior and the subsequent development of the rational method to T.J. Mulvaney in 1851 (Dooge, 1959; 1973). To date, the focus of runoff prediction has been to determine how to relate morphological and topographic characteristics to watershed response. It is as yet a largely unsolved problem (in the practical sense); yet good simple approximations are available.

One method that evolved is the "rational method" which, even though it is arguably empirical, is systematic. In all versions of the method, the drainage area is analyzed using simple hydraulics principles and topological information (typically a time-area method) to determine a time of concentration that is defined as the time that a water particle falling on the most distant location of the watershed exits at the outlet. Once this "duration" is established, a rainfall of this duration is applied at a specified intensity (intensity-duration-frequency analysis) and the peak discharge is obtained as the product of this intensity, the watershed area, and some weight that scales the rainfall intensity to the peak discharge. Equation 2.1 is a typical rational method equation.

$$Q_{\max} = C \cdot i_{t_c} \cdot A \,. \qquad (2.1)$$

Details of finding the time of concentration, the weights, the use of time-area-methods, and intensity-frequency-duration curves are found in any modern hydrology textbook (e.g. McCuen, 1998). The "method" has been explored for non-uniform rainfall and other modification by many authors and is used extensively for relatively small watersheds, less than 200 acres. TxDOT uses this method for design where contribution drainage areas are less than 200 acres.

Figure 2.1 is the hydrologic method selection chart from the TxDOT design manual. The unit hydrograph methods are indicated in three of the four suggested design-analysis techniques. In our research the focus is on unregulated watersheds in the 200 acres to 20 square miles size. Areas larger than 20 square miles are currently analyzed by regional regression equations and larger gage streams by log-Pearson analysis of the annual maxima.

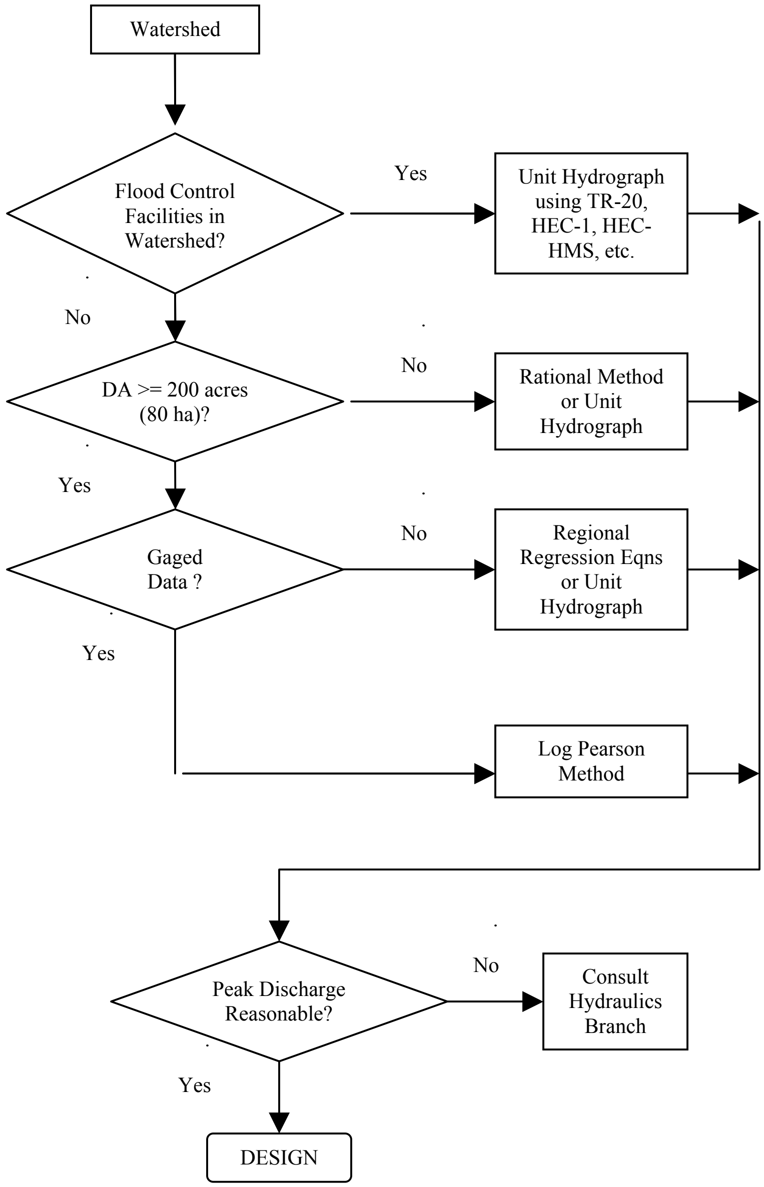

Figure 2.1. Hydrologic Method Selection Chart (adapted from TxDOT Hydraulic Design Manual, 2003)

2.2. Unit Hydrographs

A unit hydrograph (UH) is the hydrograph of the direct runoff that results from a uniformly distributed rainfall producing one unit effective depth over the basin for a specified duration.

A unit hydrograph can be determined in gaged basins by measuring the concurrent rainfall and runoff amounts for the storms. One of the fundamental principles in unit hydrograph theory is linearity; thus when a unit hydrograph is determined for a basin, and then the response to any other storm can be obtained by linear combinations of the unit hydrograph.

The unit hydrograph concept is credited to Sherman in 1932 (Sherman, 1932), although the concept was likely in use prior to that time. In his paper he illustrated a procedure to construct direct runoff hydrographs from a sequence of rainfall "units" by addition of ordinates of unit hydrographs lagged by the duration of the individual rainfall durations. Upon close examination, one concludes that Sherman's procedure is graphical convolution of responses to different input weights. Subsequent efforts by many other authors codified these ideas, and UH theory today is essentially the application of linear-systems theory to the rainfall runoff process (Dooge, 1973; Chow, *et al*, 1988).

In the 1970's, Chow and others worked on development of linear systems theory applications to hydrologic modeling. Chapter 7 in Chow, *et al* (1988) is an overview of that work. The convolution integral,

$$Q(t) = \int_0^t I(\tau)u(t-\tau)d\tau \qquad (2.2)$$

Where $Q(t)$ = output time function,

$I(\tau)$ = input time function,

$u(t-\tau)$ = impulse response function,

$(t-\tau)$ = time lag between time the impulse is applied, and

t = time.

In discrete time, the pulse response function is

$$Q_n = \sum_{m=1}^{n \le M} P_m U_{n-m+1} \tag{2.3}$$

Where U_n = unit response function (unit-graph; L^2/T), and

P_m = effective precipitation (L) for period m.

The unit-graph, then, is a linear model that has some embedded assumptions:

(1) Effective rainfall has a constant intensity within the effective duration;

(2) Effective rainfall is uniformly distributed spatially;

(3) Time base of runoff (period of time that direct runoff exceeds zero) resulting from an effective rainfall of specific duration is constant;

(4) The ordinates of direct runoff of a constant base time are directly proportional to the total amount of direct runoff represented by each hydrograph; and

(5) For a particular watershed, the size of the direct runoff hydrograph for two effective rainfall pulses is in direct proportion to the relative size of the pulses.

In fact, these assumptions are often not true, particularly for small watersheds, which have a tendency to be non-linear in response. However, the unit hydrograph approach is usually good enough to obtain engineering estimates for design purposes.

Of importance to this research is the impulse-response function in Equation 2.2. This function is the IUH, if one knows the response function (or the set of weights in the discrete model). Then one can predict the runoff hydrograph for any rainfall sequence (hyetograph) applied to the watershed (assuming the watershed behaves as a linear system).

Historically the response functions have been treated as statistical distributions although researchers have linked simplified physics to the distributions (Nash, 1958; Leinhard, 1971). Linking a series of reservoirs in a feed forward (cascade) fashion, Nash (1958) developed his IUH. The Nash model, gamma-hydrograph, and Pearson Type III hydrograph are identical distributions (under certain circumstances). Lienhard and Meyer (1967) showed that the gamma family of distributions can be explained using statistical-mechanical principles, establishing a rigorous physical basis for IUHs.

The unit hydrograph procedure should be limited to watershed drainage areas that are less than about 2,000 square miles. If storm patterns are thought to impact runoff hydrographs, then the watershed can be subdivided into smaller sub-watersheds and each of those subjected to a hydrograph analysis. The development of the procedure has been documented many times.

2.3. Synthetic Unit Hydrographs

As mentioned before, actual or observed unit hydrographs can not be determined for all the basins since there are not available rainfall and runoff data everywhere. Therefore for such basins unit hydrographs are determined synthetically, to be used in the design of hydraulic structures.

Synthetic unit hydrographs are developed using two main concepts; 1) each watershed has a unique unit hydrograph, and 2) all unit hydrographs can be represented by a single family of curves or a single equation.

Several methods have been developed for estimating synthetic unit hydrographs for locations where observations of input and response are lacking. Chow et al (1988) group synthetic unit hydrographs into three types: (1) those relating hydrograph characteristics (peak flow, time to peak, base time, etc.) to watershed characteristics (Snyder, 1938; Gray, 1961); (2) those based on conceptual models of watershed storage (Clark, 1943; Nash 1957); and (3) those based on a dimensionless unit hydrograph DUH (Soil Conservation Service 1972).Types (1) and (2) involve empirical coefficients whose validity is limited to a particular watershed or region. Type (3) is based on the expectation that, by selecting proper dimensionless ratios, all individual unit hydrographs can be transformed into one more-or-less universally applicable DUH.

A number of parameters are important in determining the shape of the unit hydrograph for a watershed. The discharge parameter which is mostly used is the peak discharge (Q_p). Lag time (t_L), time to peak (t_p), time of concentration (t_c) and base time (T_b) are often used as the time parameters. Watershed parameters of most concern, influencing the shape of the outflow hydrograph, include area (A in sq. mi.) and its shape, main stream length (L in ft), length to watershed centroid from the outlet (L_c in ft) and average slope of basin (y in %).

CHAPTER 3

METHOD OF ANALYSIS

In this research the goal is to determine an IUH from observed rainfall-runoff data. This research assumes that an IUH exists, and that it is the response function to a linear system, and the research task is to find the parameters (unknown coefficients) of the transfer function.

To accomplish this task a database must be assembled that contains appropriate rainfall and runoff values for analysis. Once the data are assembled, the runoff signal is analyzed for the presence of any base flow, and this component of the runoff signal is removed. Once the base flow is removed, the remaining hydrograph is called the direct runoff hydrograph (DRH). The total volume of discharge is determined and the rainfall input signal is analyzed for rainfall losses. The losses are removed so that the total rainfall input volume is equal to the total discharge volume. The rainfall signal after this process is called the effective precipitation. By definition, the cumulative effective precipitation is equal to the cumulative direct runoff.

If the rainfall-runoff transfer function and its coefficients are known a-priori, then the DRH signal should be obtainable by convolution of the rainfall input signal with the IUH response function. The difference between the observed DRH and the model DRH should be negligible if the data have no noise, the system is truly linear, and we have selected both the correct function and the correct coefficients.

If the analyst postulates a functional form (the procedure of this thesis) then searches for correct values of coefficients, the process is called de-convolution. In the present work by guessing at coefficient values, convolving the effective precipitation signal, and comparing the model output with the actual output, we accomplish de-convolution. A merit function is used to quantify the error between the modeled and observed output. A simple searching scheme is used to record the estimates that reduce the value of a merit function and when this scheme is completed, the parameter set is called a non-inferior (as opposed to optimal) set of coefficients of the transfer function.

3.1. Database Construction

USGS small watershed studies were conducted largely during the period spanning the early 1960's to the middle 1970's. The storms documented in the USGS studies can be used to evaluate unit hydrographs and these data are critical for unit hydrograph investigation in Texas. Candidate stations for hydrograph analysis were selected and a substantial database was assembled.

Table 3.1 is a list of the 88 stations eventually keypunched and used in this research. The first two columns in each section of the table is the watershed and sub watershed name. The urban portion of the database does not use the sub watershed naming convention, but the rural portion does. The third column is the USGS station ID number.

This number identifies the gauging station for the runoff data. The precipitation data is recorded in the same reports as the runoff data so this ID number also identifies the precipitation data. The last numeric entry is the number of rainfall-runoff records available for the unit hydrograph analysis. The details of the database construction are reported in Asquitn et. al (2004).

Table 3.1.Stations and Number of Storms used in Study

Austin			
Watershed	Sub-Shed	Station ID	#Events
BartonCreek		08155200	5
BartonCreek		08155300	8
BearCreek		08158810	8
BearCreek		08158820	2
BearCreek		08158825	2
BoggyCreek		08158050	10
BoggySouthCreek		08158880	14
BullCreek		08154700	13
LittleWalnutCreek		08158380	2
OnionCreek		08158700	6
OnionCreek		08158800	2
ShoalCreek		08156650	13
ShoalCreek		08156700	16
ShoalCreek		08156750	13
ShoalCreek		08156800	24
SlaughterCreek		08158840	9
SlaughterCreek		08158860	2
WallerCreek		08157000	40
WallerCreek		08157500	38
WalnutCreek		08158100	15
WalnutCreek		08158200	17
WalnutCreek		08158400	10
WalnutCreek		08158500	14
WalnutCreek		08158600	22
WestBouldinCreek		08155550	10
WilbargerCreek		08159150	29
WilliamsonCreek		08158920	14
WilliamsonCreek		08158930	18
WilliamsonCreek		08158970	16

Dallas			
Watershed	Sub-Shed	Station ID	#Events
AshCreek		08057320	5
BachmanBranch		08055700	41
CedarCreek		08057050	3
CoombsCreek		08057020	7
CottonWoodCreek		08057140	6
DuckCreek		08061620	8
ElamCreek		08057415	8
FiveMileCreek		08057418	7
FiveMileCreek		08057420	10
FloydBranch		08057160	8
JoesCreek		08055600	14
NewtonCreek		08057435	3
PrairieCreek		08057445	8
RushBranch		08057130	5
SouthMesquite		08061920	9
SouthMesquite		08061950	31
SpankyCreek		08057120	4
TurtleCreek		08056500	42
WoodyBranch		08057425	13

Fort Worth			
Watershed	Sub-Shed	Station ID	#Events
DryBranch		08048550	25
DryBranch		08048600	27
LittleFossil		08048820	20
LittleFossil		08048850	24
Sycamore		08048520	24
Sycamore		08048530	28
Sycamore		08048540	24
Sycamore		SSSC	21

San Antonio			
Watershed	Sub-Shed	Station ID	#Events
AlazanCreek		08178300	30
LeonCreek		08181000	10
LeonCreek		08181400	15
LeonCreek		08181450	29
OlmosCreek		08177600	12
OlmosCreek		08177700	23
OlmosCreek		08178555	10
SaladoCreek		08178600	13
SaladoCreek		08178640	10
SaladoCreek		08178645	5
SaladoCreek		08178690	39
SaladoCreek		08178736	12

SmallRuralSheds			
Watershed	Sub-Shed	Station ID	#Events
BrasosBasin	CowBayou	08096800	48
BrasosBasin	Green	08094000	28
BrasosBasin	Pond-Elm	08098300	19
BrasosBasin	Pond-Elm	08108200	21
ColoradoBasin	Deep	08139000	27
ColoradoBasin	Deep	08140000	28
ColoradoBasin	Mukewater	08136900	22
ColoradoBasin	Mukewater	08137000	38
ColoradoBasin	Mukewater	08137500	4
SanAntonioBasin	Calaveras	08182400	24
SanAntonioBasin	Escondido	08187000	31
SanAntonioBasin	Escondido	08187900	21
TrinityBasin	ElmFork	08050200	34
TrinityBasin	Honey	08057500	31
TrinityBasin	Honey	08058000	29
TrinityBasin	LittleElm	08052630	29
TrinityBasin	LittleElm	08052700	58
TrinityBasin	North	08042650	14
TrinityBasin	North	08042700	56
TrinityBasin	PinOak	08063200	33

3.2. Data Preparation

An additional processing step used in this thesis is the interpolation of the observed data into uniformly spaced, one minute intervals.

3.2.1. Base Flow Separation

Hydrograph separation is the process of separating the time distribution of base flow from the total runoff hydrograph to produce the direct runoff hydrograph (McCuen 1998). Base flow separation is a time-honored hydrologic exercise termed by hydrologists as "one of the most desperate analysis techniques in use in hydrology" (Hewlett and Hibbert 1967) and "that fascinating arena of fancy and speculation" (Appleby 1970; Nathan and McMahon 1990). Hydrograph separation is considered more of an art than a science (Black 1991). Several hydrograph separation techniques such as constant discharge, constant slope, concave method, and the master depletion curve method have been developed and used. Figure 3.1 is a sketch of a representative hydrograph that will be used in this section to explain the different base flow separation methods.

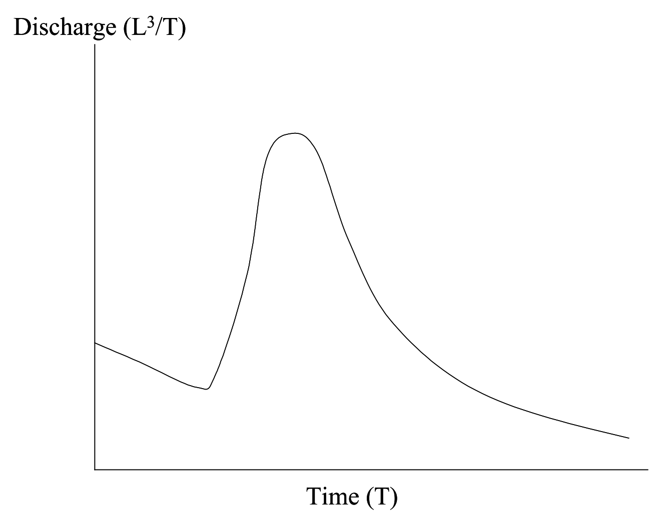

Figure 3.1 Representative Hydrograph

Constant-discharge method

The base flow is assumed to be constant regardless of stream height (discharge). Typically, the minimum value immediately prior to beginning of the storm is projected horizontally. All discharge prior to the identified minimum, as well as all discharge beneath this horizontal projection is labeled as "base flow" and removed from further analysis. Figure 3.2 is a sketch of the constant discharge method applied to the representative hydrograph. The shaded area in the sketch represents the discharge that would be removed (subtracted) from the observed runoff hydrograph to produce a direct-runoff hydrograph.

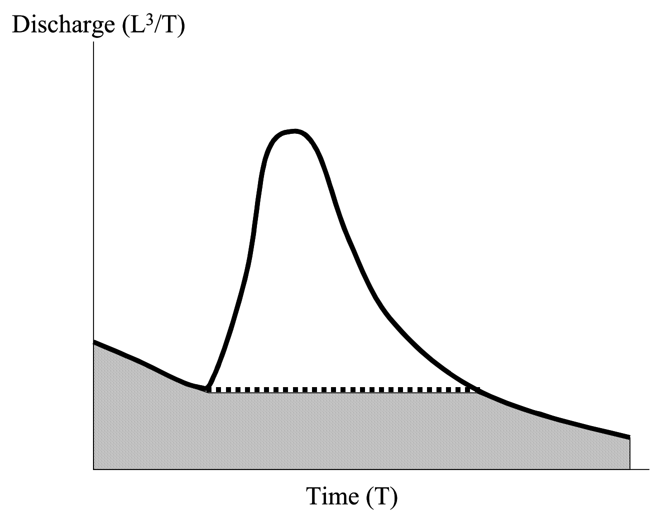

Figure 3.2. Constant-discharge base flow separation.

The principal disadvantage is that the method is thought to yield an extremely long time base for the direct runoff hydrograph, and this time base varies from storm to storm, depending on the magnitude of the discharge at the beginning of the storm

(Linsley *et, al,* 1949). The method is easy to automate, especially for multiple peak hydrographs.

<u>Constant-slope method</u>

A line is drawn from the inflection point on the receding limb of the storm hydrograph to the beginning of storm hydrograph, as depicted on Figure 3.3. This method assumes that the base flow began prior to the start of the current storm, and arbitrarily sets to the inflection point.

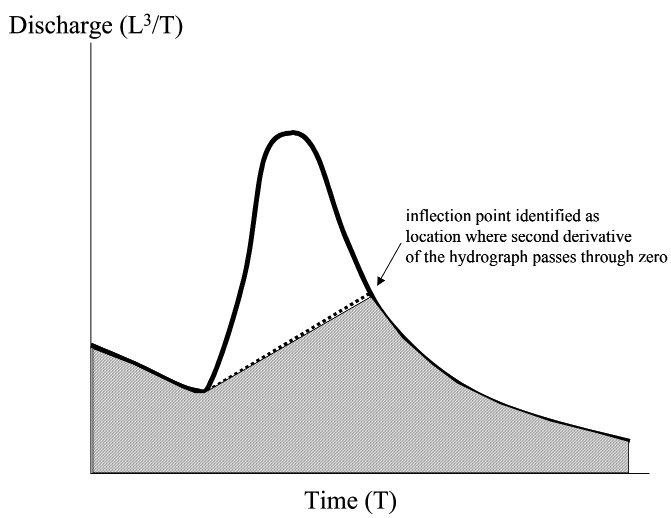

Figure 3.3. Constant-slope base flow separation.

The inflection point is located either as the location where the second derivative passes through zero (curvature changes) or is empirically related to watershed area. This method is also relatively easy to automate, except multiple peaked storms will have multiple inflection points.

Concave method

The concave method assumes that base flow continues to decrease while stream flow increases to the peak of the storm hydrograph. Then at the peak of the hydrograph, the base flow is then assumed to increase linearly until it meets the inflection point on the recession limb.

Figure 3.4 is a sketch illustrating the method applied to the representative hydrograph. This method is also relatively easy to automate except for multiple peak hydrographs which, like the constant slope, method will have multiple inflection points.

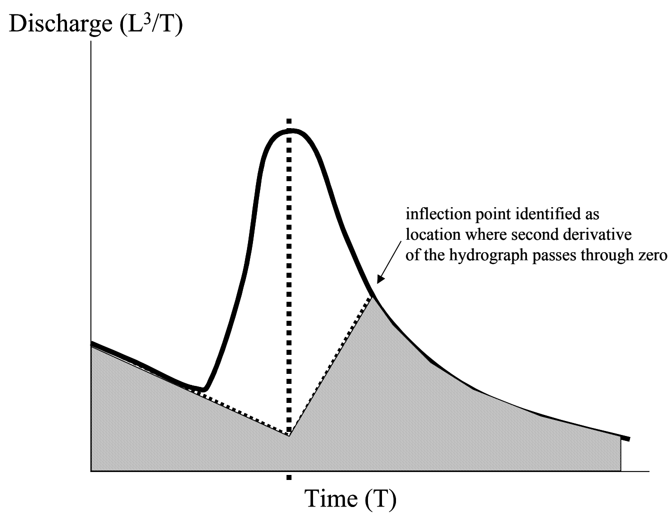

Figure 3.4 Concave-method base flow separations

Depletion curve method

This method models base flow as discharge from accumulated groundwater storage. Data from several recessions are analyzed to determine the basin recession constant. The base flow is modeled as an exponential decay term $q_b(t) = q_{b,o}\exp(-kt)$. The time constant, k, is the basin recession coefficient that is inferred from the recession portion of several storms.

Individual storms are plotted with the logarithm of discharge versus time. The storms are time shifted by trial-and-error until the recession portions all fall along a straight line. The slope of this line is proportional to the basin recession coefficient and the intercept with the discharge axis at zero time is the value for $q_{b,o}$. Figure 3.5 illustrates five storms plotted along with a test storm where the base flow separation is being determined. The storm with the largest flow at the end of the recession is plotted without any time shifting. The recession is extrapolated from this storm as if there were no further input to the groundwater store. The remaining storms are time shifted so that the straight line portion of their recession limbs come tangent to this curve. By trial-and-error the master depletion curve can be adjusted and the storms time shifted until a reasonable agreement of all storms recessions with the master curve is achieved.

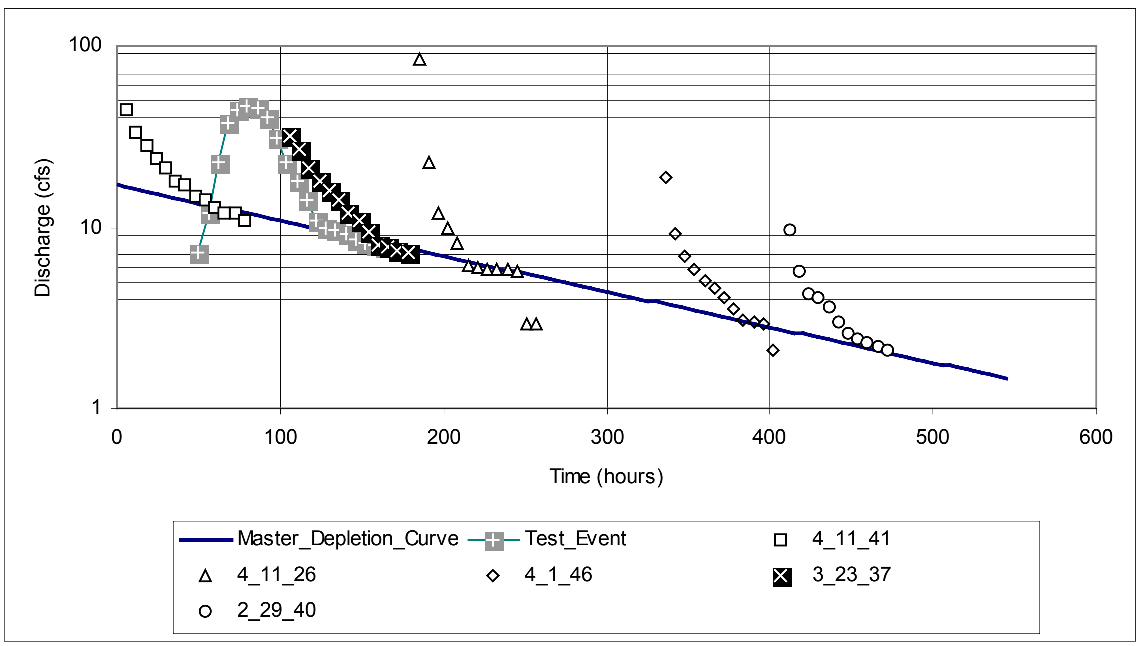

Figure 3.5 Master-Depletion Curve Method

(Data from McCuen, 1998, Table 9-2, pp 486)

Once the master curve is determined, then the test storm is plotted on the curve and shifted until its straight-line portion come tangent to the master curve, and the point of intersection is taken as the base flow value for that storm. In the example in Figure 3.5, the base flow for the test event is approximately *9.1* cfs, the basin recession constant is *0.0045/hr*, and the base flow at the beginning of the recession is *17* cfs. Once the base flow value is determined for a particular test event, then base flow separation proceeds use the constant discharge method.

The depletion curve method is attractive as it determines the basin recession constant, but it is not at all easy to automate. Furthermore, in basins where the stream goes dry (such as much of Texas), the recession method is difficult to apply as the first storm after the dry period starts a new master recession curve. Observe in Figure 3.5 the storms used for the recession analysis span a period of nearly 40 years, and implicit in the analysis is that the basin recession constant is time invariant and the storms are independent.

The following Figure 3.6 is a multiple peak storm event from Dallas AshCreek station08057320. To automate the rest of data set using this method will be a challenge because of the change of master recession curve for different peaks.

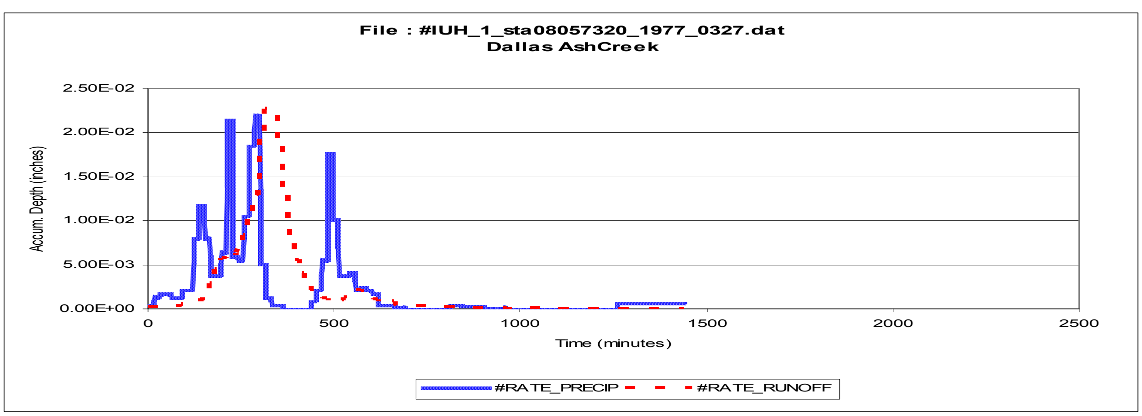

Figure 3.6 Multiple peak storms from Dallas module

Selection of Method to Employ

The principal criterion for method selection was based on the need for a method that was simple to automate because hundreds of events needed processing. Appleby (1970) reports on a base flow separation technique based on a Ricatti-type equation for base flow. The general solution of the base flow equation is a rational functional that is remarkably similar in structure to either a LaPlace transform or Fourier transform. Unfortunately the paper omits the detail required to actually infer an algorithm from the solution, but it is useful in that principles of signal processing are clearly indicated in the model.

Nathan and McMahon (1990) examined automated base flow separation techniques. The objective of their work was to identify appropriate techniques for determination of base flow and recession constants for use in regional prediction equations. Two techniques they studied in detail were a smoothed minima technique and a recursive digital filter (a signal processing technique similar to Appleby's work). Both techniques were compared to a graphical technique that extends pre-event runoff (just before the rising portion of the hydrograph) with the point of greatest curvature on the recession limb (a constant-slope method, but not aimed at the inflection point). They concluded that the digital filter was a fast objective method of separation but their paper suggests that the smoothed minima technique is for all practical purposes indistinguishable from either the digital filter or the graphical method. Furthermore the authors were vague on the constraint techniques employed to make the recursive filter produce non-negative flow values and to produce peak values that did not exceed the original stream flow. Press et.al. (1986) provide convincing arguments against timedomain signal filtering and especially recursive filters. Nevertheless the result for the smoothed minima is still meaningful, and this technique appears fairly straightforward to automate, but it is intended for relatively continuous discharge time series and not the episodic data in the present application.

The constant slope and concave methods are not used in this work because the observed runoff hydrographs have multiple peaks. It is impractical to locate the recession limb inflection point with any confidence. The master depletion curve method is not used because even though there is a large amount of data, there is insufficient data at each station to construct reliable depletion curves. Recursive filtering and smoothed minima were dismissed because of the type of events in the present work (episodic and not continuous). Therefore in the present work the discharge data are treated by the constant discharge method.

The constant discharge method was chosen because it is simple to automate and apply to multiple peaked hydrographs. Prior researchers (e.g. Laurenson and O'Donell, 1969; Bates and Davies, 1988) have reported that unit hydrograph derivation is insensitive to base flow separation method when the base flow is not a large fraction of the flood hydrograph – a situation satisfied in this work. The particular implementation in this research determined when the rainfall event began on a particular day; all discharge before that time was accumulated and converted into an average rate. This average rate was then removed from the observed discharge data, and the result was considered to be the direct runoff hydrograph.

The candidate models will be run in two cases with or without base flow separation, so one can compare how much the separation would effect the runoff prediction.

3.2.2. Effective Precipitation

The effective precipitation is the fraction of actual precipitation that appears as direct runoff (after base flow separation). Typically the precipitation signal (the hyetograph) is separated into three parts, the initial abstraction, the losses, and the effective precipitation.

Initial abstraction is the fraction of rainfall that occurs before direct runoff. Operationally several methods are used to estimate the initial abstraction. One method is to simply censor precipitation that occurs before direct runoff is observed. A second method is to assume that the initial abstraction is some constant volume (Viessman, 1968). The NRCS method assumes that the initial abstraction is some fraction of the maximum retention that varies with soil and land use (essentially a CN based method).

Losses after initial abstraction are the fraction of precipitation that is stored in the watershed (depression, interception, soil storage) that does not appear in the direct runoff hydrograph. Typically depression and interception storage are considered part of the initial abstraction, so the loss term essentially represents infiltration into the soil in the watershed. Several methods to estimate the losses include: Phi-index method, Constant fraction method, and infiltration capacity approaches (Horton’s curve, Green-Ampt model).

Phi-index model

The ϕ-index is a simple infiltration model used in hydrology. The method assumes that the infiltration capacity is a constant ϕ(in/hr). With corresponding observations of a rainfall hyetograph and a runoff hydrograph, the value of ϕ can in many cases be easily guessed. Field studies have shown that the infiltration capacity is greatest at the start of a storm and that it decreases rapidly to a relatively constant rate. The recession time of the infiltration capacity may be as short as 10 to 15 minutes. Therefore, it is not unreasonable to assume that the infiltration capacity is constant over the entire storm duration. When the rainfall rate exceeds the capacity, the loss rate is assumed to equal the constant capacity, which is called the phi (ϕ) index. When the rainfall is less than the value of ϕ, the infiltration rate is assumed to equal to the rainfall intensity.

Mathematically, the phi-index method for modeling losses is described by

$$F(t)= I(t), \text{ for } I(t) < \phi \quad (3.1)$$

$$F(t)= \phi, \text{ for } I(t)> \phi, \quad (3.2)$$

where F(t) is the loss rate, I(t) is storm rainfall intensity, t is time, and ϕ is a constant.

If measured rainfall-runoff data are available, the value of ϕ can be estimated by separating base flow from the total runoff volume, computing the volume of direct runoff, and then finding the value of ϕ that results in the volume of effective rainfall being equal to the volume of direct runoff. A statistical mean phi-index can then be computed as the average of storm event phi values. Where measured rainfall-runoff data are not available, the ultimate capacity of Horton's equation, f_c, might be considered.

Horton's model

Infiltration capacity (f_p) may be expressed as

$$f_p = f_c + (f_o - f_c)\, e^{-\beta t}, \tag{3.3}$$

where f_o = maximum infiltration rate at the beginning of a storm event and reduces to a low and approximately constant rate of f_c as infiltration process continues and the soil is saturated β = parameter describing rate of decrease in f_p.

Factors assumed to be influencing infiltration capacity, soil moisture storage, surface-connected porosity and effect of root zone paths follow the equation

$$f = aS_a^{1.4} + f_c, \tag{3.4}$$

where f = infiltration capacity (in/hr),

a = infiltration capacity of available storage ((in/hr)/(in)1.4)

(Index of surface connected porosity),

S_a = available storage in the surface layer in inches of water equivalent (A-horizon in agricultural soils - top six inches).

Factor f_c = constant after long wetting (in/hr).

The modified Holton equation used by US Agricultural Research Service is

$$f = GIa\ Sa^{1.4} + f_c, \tag{3.5}$$

where GI = Growth index - takes into consideration density of plant roots which assist infiltration (0.0 - 1.0).

<u>Green-Ampt Model</u>

Green & Ampt (1911) proposed the simplified picture of infiltration shown in Figure 3.7.

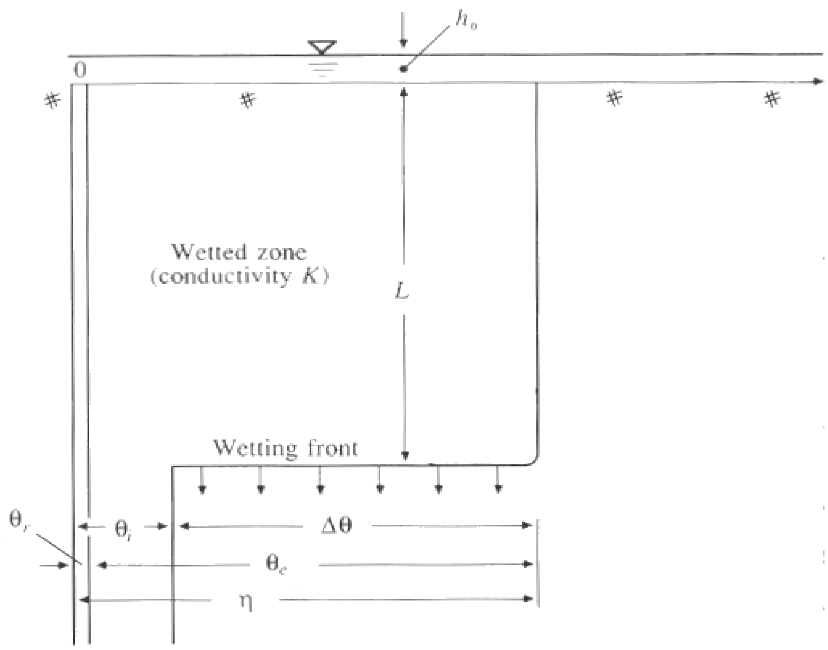

Figure 3.7.Variables in the Green-Ampt infiltration model. The vertical axis is the distance from the soil surface; the horizontal axis is the moisture content of the soil.

(Source: *Applied Hydrology* by Chow/Maidement/Mays 1988)

The *wetting front* is a sharp boundary dividing soil below with moisture content θ_i from saturated soil done with moisture content θ_i above. The wetting front has penetrated to a depth L in time t since infiltration began. Water is ponded to a small depth h_0 on the soil surface. The method computes total infiltration rate at the end of time *t*, with the following equation,

$$F(t) = Kt + \psi\, \Delta\theta \ln\{ 1 + F(t)/(\psi\, \Delta\theta)\}, \qquad (3.6)$$

where

K = Hydraulic conductivity,

t = time in hrs,

F(t) = Total infiltration at the end of time t,

Ψ = Wetting front soil suction head, and

$\Delta\theta$ = increase in moisture content in time t.

Unlike the SCS curve method, this method gives the total amount of infiltration in the soil at the end of a particular storm event. Depending on this value and the total amount of precipitation, we can easily calculate the amount of runoff.

Constant Fraction Model

The constant fraction model simply assumes that some constant ratio of precipitation becomes runoff; the fraction is called a runoff coefficient. At first glance it appears that it is a rational method disguise, but the rational method does not consider storage and travel times. Thus in the rational method, if one doubles the precipitation intensity, and halved the duration, one would expect the peak discharge to remain unchanged, while in a unit hydrograph such changes should have a profound effect on the hydrograph. As a model, the method is simple to apply, essentially

$$\begin{aligned} p_e(t) &= crp * p_{raw}(t) \\ \int Ap_e(t)dt &= \int DRH(t)dt, \end{aligned} \tag{3.7}$$

where crp = the runoff coefficient,

p_e = the effective precipitation,

p_{raw} = the raw precipitation,

A = drainage area.

The first equation states that the effective precipitation is a fraction of the raw precipitation, while the second states that the total effective precipitation volume should equal the total direct runoff volume.

3.3. Summary of Data Preparation

Base flow separation was accomplished using the constant discharge method because it is amenable to automation. We analyzed the data with and without a separation to test whether separation was necessary in our data set. Effective precipitation was always modeled using the constant fraction model, because of the need to automate and also because of the sheer magnitude of the dataset, but the fraction was left as a fitting constant. Ideally, the fitted result should preserve the required mass balance (precipitation volume = runoff volume).

An important detail in this research was the conversion of the original data into “pseudo data” for IUH analysis. The time-step length used in the research was one-minute. This time length was chosen because it is the smallest increment that can be represented in the current DATE_TIME format in the database. It should be noted that there are very few actual one-minute intervals in the original data, so linear interpolation was used to convert the cumulative precipitation into one-minute intervals, then numerical differentiation is performed to obtain the rainfall rates. The resulting units are inches per minute.

Figure 3.8 is a sketch showing the incremental rate and the cumulative depth relationship. The cumulative depth scale is the left vertical scale and the incremental rate scale is the right vertical scale. Mathematically the cumulative rainfall distribution is the integral of the incremental rainfall distribution (or rainfall density) over the entire rainfall event. Equation 3.8 expresses this relationship; integration over the entire number line is intended to indicate the entire lifetime of the individual rainfall event.

$$P(t) = \int_{-\infty}^{\infty} p(t)dt \,. \tag{3.8}$$

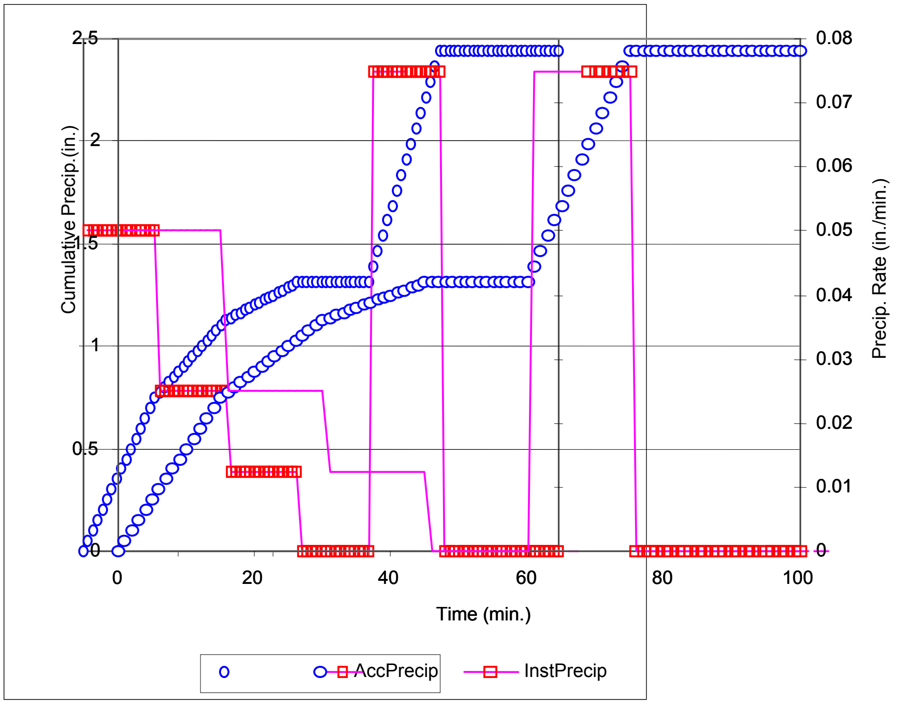

Figure 3.8. Cumulative Precipitation and Incremental Precipitation Relationship

In Figure 3.8 the cumulative precipitation, *P(t),* is indicated by the open circles, while the rate, *p(t)*, is indicated by the open squares. In practice only the cumulative depth is recorded as a function of time; so to determine the rate we simply differentiate the cumulative precipitation.

$$p(t) = \frac{dP(t)}{dt} = \frac{d}{dt}\{\int_{-\infty}^{\infty} p(t)dt\}. \qquad (3.9)$$

The present work used a simple centered differencing scheme, except at the first and last time interval, when forward and backward differencing were used, respectively.

$$p(t) \approx \frac{P(t+\Delta t) - P(t-\Delta t)}{2\Delta t}. \qquad (3.10)$$

Details of the "pseudo data" conversion were reported by Cleveland et. al, (2003). The 1-minute data for roughly 1642 storms are located on a University of Houston server and can be publicly accessed at the URL associated with this citation.

3.4. NRCS Unit Hydrograph

The Natural Resources Conservation Service (NRCS), formerly the Soil Conservation Service, developed a unit hydrograph (UH) in the 1950s. This UH was used to develop storm hydrographs and peak discharges for design of conservation measures on small agricultural watersheds.

Mockus (1956) discussed development of the standard NRCS unit hydrograph and the peak rate equation,

$$q_p=KAQ/T_p, \tag{3.11}$$

where the peak discharge rate q_p is a function of drainage area A, direct runoff volume Q, factor K, and time to peak of the unit hydrograph T_p. He indicated that the peak rate factor (PRF) of K is equal to

$$K=1290.6/(1+H), \tag{3.12}$$

where H is the ratio of the time of recession to the time peak (T_r/ T_p). He also indicated that K was a function of the UH shape and that 3/8 of the storm runoff volume in the rising limb and 5/8 in the recession limb were typical of small agricultural watersheds. K also includes a conversion factor to make the equation dimensionally correct.

Mockus used the triangular UH shape in development of above two equations. It appears that Mockus analyzed many flood hydrographs to justify the selection of the peak rate factor K of 484. A UH with PRF of K of 484 was felt to be representative of small agricultural watersheds in the U.S.

NRCS-DUH (Gamma approximation)

The NRCS Dimensionless Unit Hydrograph (USDA, 1985) used by the NRCS (formerly the SCS) was developed by Victor Mockus in the late 1940's. The SCS analyzed a large number of unit hydrographs for watersheds of different sizes and in different locations and developed a generalized dimensionless unit hydrograph in terms of t/t_p and q/q_p where, t_p is the time to peak. The point of inflection on the unit graph is approximately 1.7 the time to peak and the time to peak was observed to be 0.2 the base time (hydrograph duration) (T_b).

The functional representation is presented as tabulated time and discharge ratios, and as a graphical representation. Table 3.2 is the tabulation of the NRCS DUH from the National Engineering Handbook.

Table3.2. Ratios for dimensionless unit hydrograph and mass curve

Time ratios (t/Tp)	Discharge ratios (q/qp)	Mass Curve Ratios (Q/Qp)
0.0	0.0	0.000
0.1	0.03	0.001
0.2	0.10	0.006
0.3	0.19	0.012
0.4	0.31	0.035
0.5	0.47	0.065
0.6	0.66	0.107
0.7	0.82	0.163
0.8	0.93	0.228
0.9	0.99	0.300
1.0	1.00	0.375
1.1	0.99	0.450
1.2	0.93	0.522
1.3	0.86	0.589
1.4	0.78	0.650
1.5	0.68	0.700
1.6	0.56	0.751
1.7	0.46	0.790

1.8	0.39	0.822
1.9	0.33	0.849
2.0	0.28	0.871
2.2	0.207	0.908
2.4	0.147	0.934
2.6	0.107	0.953
2.8	0.077	0.967
3.0	0.055	0.977
3.2	0.040	0.984
3.4	0.029	0.989
3.6	0.021	0.993
3.8	0.015	0.995
4.0	0.011	0.997
4.5	0.005	0.999
5.0	0.000	1.000

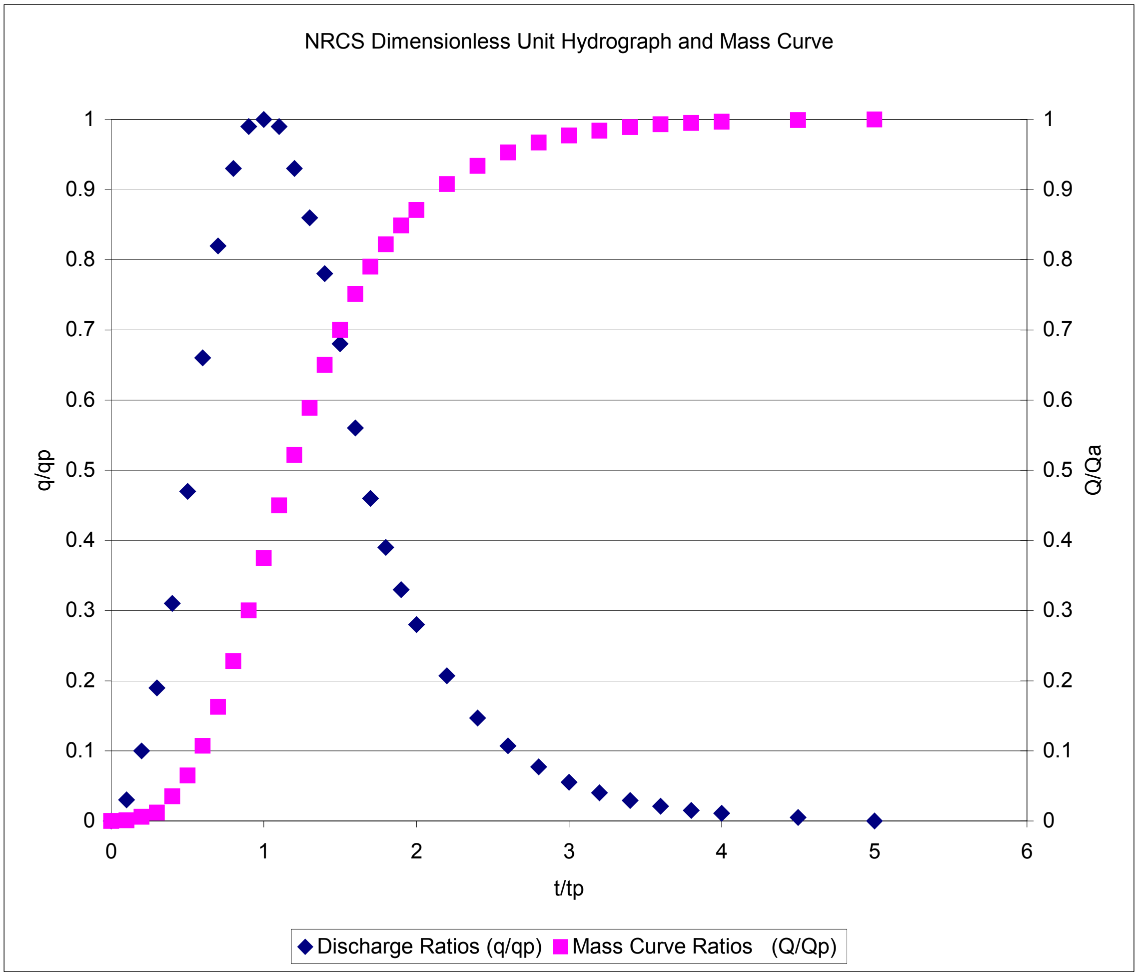

Figure3.9. Plot of DUH and Mass Curve

Figure 3.9 is a plot of these ratios. This figure is identical to Figure 16.1 in the National Engineering handbook (except this figure is computer generated).

The IUH analysis assumed that the hydrograph functions are continuous and the database was analyzed using discrete values calculated from continuous functions. Rather than use the NRCS tabulation in this work, the fit was tested of a function of the same family (the gamma distribution) as the IUH function and this function was used in place of the NRCS tabulation. A similar approach was used by Singh (2000) to express common unit hydrographs (Snyder's, SCS, and Gray's) by a gamma distribution.

The gamma function used to fit the tabulation is

$$P_x(X) = k\lambda^\eta x^{\eta-1} e^{-\lambda x} / \Gamma(\eta). \tag{3.13}$$

The variables λ , η and k are unknown, and were determined by minimization of the sum of squared errors between the tabulation and the model (the function) by selection of numerical values for the unknown parameters. "Excel solver" was used to perform the minimization. The values for parameters λ , η and k were 3.88, 4.81 and 1.29 respectively. So the NRCS DUH approximation is

$$P_x(X) = 1.29 * 3.88^{4.81} x^{3.81} e^{-3.88x} \quad . \tag{3.14}$$

Figure 3.10 is a plot of the model and tabulation, the variable *x* in the equations is the dimensionless time. Qualitatively the fit is good. The maximum residual(s) occur early in dimensionless time and at 60% of the runoff duration, but the magnitudes are quite small, and thus this model of the NRCS DUH is deemed acceptable for use.

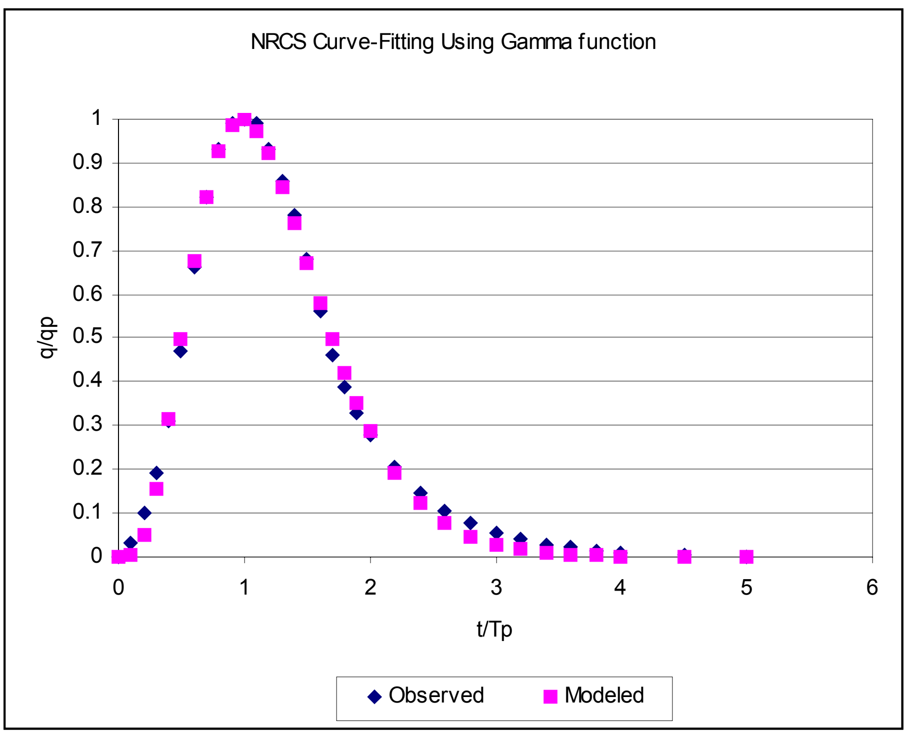

Figure3.10. Plot of Tabulated and Gamma-Model DUH

A Chi-square fitness test was performed to further support the decision to use the model in lieu of the tabulation. The test statistic for the chi-square test was calculated as

$$\chi_c^2 = \sum_{i=1}^{k} (O_i - E_i)^2 / E_i \,. \qquad (3.15)$$

The test statistic is 0.5323. For two degrees of freedom and 90 % confidence limits the value was 10.6 which is greater than the test statistic (0.5323) therefore the hypothesis (model) represent the observed values.

The NRCS DUH as presented in the NEH integrates to a little over 1.4 and thus it is not a true unit hydrograph as presented. It is likely that it originally was a UH; then it was adjusted procedurally so that the peak value of the dimensionless distribution is 1.0 (thus the factor that scales the integral correctly is imbedded in the q_p value). The research assumes that all unit graphs and the accompanying functional representations of IUHs integrate to one; so in this work the NRCS DUH approximation is adjusted by dividing by the integral of the original DUH, in this case the value is 1.2903. Therefore the final approximation to the NRCS DUH as a functional representation useful in IUH analysis is

$$Px(X) = 3.88^{4.81} x^{3.81} e^{-3.88x} \,. \qquad (3.16)$$

Or with all the constants evaluated and simplified and expressed in the NRCS terminology the NRCS DUH (as an IUH function) is

$$\frac{q(t)}{q_p} = 38.5387\left(\frac{t}{t_p}\right)^{3.81} e^{-3.88\frac{t}{t_p}} \,. \qquad (3.17)$$

3.5. Commons Hydrograph

Commons (1942) developed a dimensionless unit hydrograph for use in Texas, but details of how the hydrograph were developed are not reported. The labeling of axes in the original document suggests that the hydrograph is dimensionless. For the sake of completeness in this work, an approximation was produced for treatment as another transfer function by fitting a three-gamma summation model. Essentially there were three integrated gamma models with different peaks and weights to reproduce the shape of Commons' hydrograph. The Commons hydrograph is quite different in shape after the peak than other dimensionless unit hydrographs in current use (i.e. NRCS Dimensionless Unit Hydrograph) – it has a very long time base on the recession portion of the hydrograph.

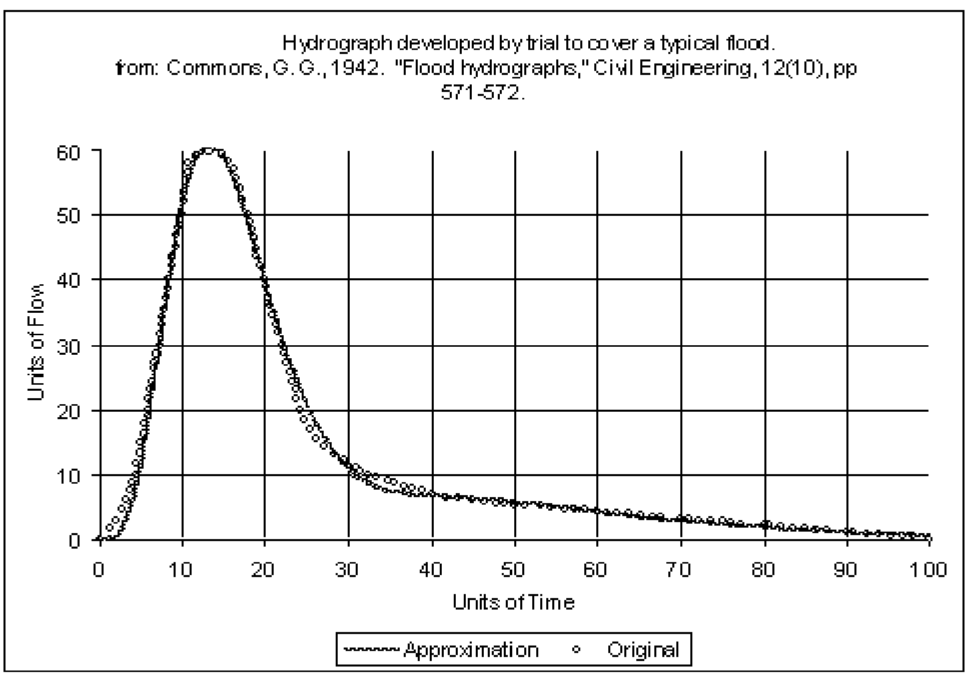

Figure 3.11. Hydrograph developed by trial to cover a typical flood

Circles are tabulation from digitization of the original figure. Curve is a Smooth Curve Approximation. Figure 3.11 is Commons' hydrograph reproduced from a manual digitization. The smooth curve is given by the following equation that was fit by trial and error.

$$\begin{aligned} q(t) = \frac{77.001}{\Gamma(0.118)}\left(\frac{t}{4.707t_p}\right)^{0.176} e^{-\frac{t}{4.707t_p}} \\ + \frac{7.58}{\Gamma(0.925)}\left(\frac{t}{2.694t_p}\right)^{0.965} e^{-\frac{t}{2.694t_p}} \\ - \frac{3.88}{\Gamma(0.288)}\left(\frac{t}{5.641t_p}\right)^{0.132} e^{-\frac{t}{5.641t_p}} \end{aligned} \quad . \qquad (3.18)$$

The numerical values are simply the result of the fitting procedure. The time axis was reconstructed (in the fitting algorithm) so that the t_p parameter could be left variable for consistency with the other hydrograph functions. The tabulated function integrates to approximately 1160; thus the function above is divided by this value to produce a unit hydrograph distribution.

3.6. Gamma Synthetic Hydrographs

The gamma distribution is given in the equation

$$f(x) = Ce^{-x/b}x^a . \qquad (3.19)$$

In the equation C equals $\frac{1}{b^{a+1}\Gamma(a+1)}$ to make the area enclosed by the curve equal to unity. Γ is called the gamma function. Values can be found tabulated in mathematical handbooks. The gamma distribution is similar in shape to the Poisson distribution that is given the form as $f(x) = \frac{m^x e^{-m}}{x!}$.The curve starts at zero when the variable x is zero, rises to a maximum, and descends to a tail that extends indefinitely to the right. The values that the variable x can take on are thus limited by 0 on the left. Values can extend to infinity on the right.

The gamma distribution differs from the Poisson distribution is that it has two parameters instead of the single parameter of the Poisson. This allows the curve to take on a greater variety of shapes than the Poisson distribution. The parameter a is a shape parameter while b is a scale parameter. The shape of the Gamma distribution is similar to the shape of a unit hydrograph, so many researchers started looking for the application of the Gamma distribution into hydrograph prediction. This first started with Edson (1951), who presented a theoretical expression for the unit hydrograph assuming Q to be proportional to $t^x e^{-yt}$

$$Q = \frac{cAy(yt)^x e^{-yt}}{\Gamma(x+1)}, \tag{3.20}$$

where Q= discharge in cfs at time t; A= drainage area in square miles; x and y = parameters that can be represented in terms of peak discharge; and $\Gamma(x+1)$ is the gamma function of (x+1). Nash (1959) and Dooge (1959), based on the concept of n linear reservoirs with equal storage coefficient K, expressed the instantaneous UH (IUH) in the form of a Gamma distribution as

$$q = \frac{1}{K\Gamma(n)}\left(\frac{t}{K}\right)^{n-1} e^{-t/k}, \tag{3.21}$$

in which n and K= parameter defining the shape of the IUH; and q=depth of runoff per unit time per unit effective rainfall. These parameters have been referred to as the Nash model parameters in the subsequent literature.

Previous attempts to fit a Gamma distribution to a hydrograph were by Croley(1980), Aron and White (1982), Hann et al. (1994), and Singh (1998). The procedure given by Croley (1980) to calculate n for known values of q_p and t_p requires programming to iteratively solve for n. Croley also proposed procedures to obtain a UH from other observable characteristics. The method by Aron and White (1992) involves reading the values from a graph, in which errors are introduced. Based on their methods, McCuen (1989) listed a step-by-step procedure to obtain the UH, which maybe briefly described by the following equations,

$$n=1.045+0.5f+5.6f^2+0.3f^3, \quad (3.22)$$

in which $f=\frac{Q_p t_p}{A}$, where Q_p is in cubic feet per second, t_p is in hours, and A is in acres. These two equations require careful attention for the units, and these cannot be used as such when $Q_p t_p$ is required to be computed for a value of n known from other sources. Hann et al. (1994) gave the following expression to calculate n,

$$n=1+6.5(\frac{Q_p t_p}{V})^{1.92}, \quad (3.23)$$

where V=total volume of effective rainfall. An equation provided by Singh (1998) to obtain the value of n may be written,

$$n=1.09+0.164\beta+6.19\beta^2, \quad (3.24)$$

where $\beta=q_p t_p$ (dimensionless), in which q_p is the peak runoff depth per unit time per effective rainfall. Singh observed that the error in n obtained from the equation:

$$n=1.09+0.164\beta+6.19\beta^2 \quad (3.25)$$

is 0.53% when $\beta = 0.25$ and 0.05% when $\beta = 1.0$. The error in n calculated decreases with increasing values of β.

3.7. Weibull Distribution

Historically a two-parameter Weibull distribution is employed to define the configuration of a natural hydrograph of direct runoff and is given in the following forms as (Canavos, 1984)

$$Q = Bt^{n-1}e^{-(t/k)^n}, \tag{3.26}$$

where Q is the discharge ordinate of the natural hydrograph corresponding to the time t after the commencement of direct runoff, n is the dimensionless shape factor, and k is the storage time constant. Both n and k reflect the basin characteristics and are related to the time to peak t_p in the following manner.

$$(t_p / k)^n = (n-1)/n. \tag{3.27}$$

The constant of proportionality B in Equation (3.26) is evaluated as

$$B = \frac{Q_p}{(t_p)^{n-1} e^{-(t_p/k)^n}}, \tag{3.28}$$

where Q_p is the peak discharge and e is the base of the natural logarithms. Combining Equation (3.26), (3.27) and (3.28) yields

$$Q/Q_p = (t/t_p)^{n-1} e^{(n-1)(1-(t/t_p)^n)/n}. \tag{3.29}$$

Equation (3.29) is the desired form of the dimensionless Weibull distribution as used in this study. Analytical formulation of the parameter n can be developed as follows. Designating $q_* = Q/Q_p$, and $t_* = t/t_p$, Equation (3.29) may be written as

$$q_* = (t_*)^{n-1} e^{(n-1)(1-t_*^{\,n})/n}. \tag{3.30}$$

Taking natural logarithms of both sides of the above equation and solving for n, one obtains

$$n = \ln(n \ln t_* + 1 - (n-1)\ln q_* / n) / \ln t_* . \qquad (3.31)$$

The value of n can be obtained from Equation (3.31) through graphical means. Once the value of n has been ascertained properly, the value of k can then be determined from Equation (3.31) conveniently.

3.8. Reservoir Elements

An alternative way to construct the hydrograph functions is to model the watershed response to precipitation as the response from a cascade of reservoirs. The response function is developed as the response to an impulse of input, and the response to a time series of inputs is obtained from the convolution integral. The end result is the same, a function that is a distribution function, but the parameters have a physical interpretation. The kernel (response function) to an impulse in this work is an instantaneous unit hydrograph (IUH). The conceptual approach for a cascade of reservoirs is well studied and works well for many unit hydrograph analyses (e.g. Nash, 1958; Dooge, 1959; Dooge, 1973; Croley, 1980). In this work the cases are examined where a Gamma, Rayleigh, and Weibull distribution govern the individual reservoir element responses, respectively. In addition, we have also converted the NRCS-DUH into its own response model (a special case of gamma).

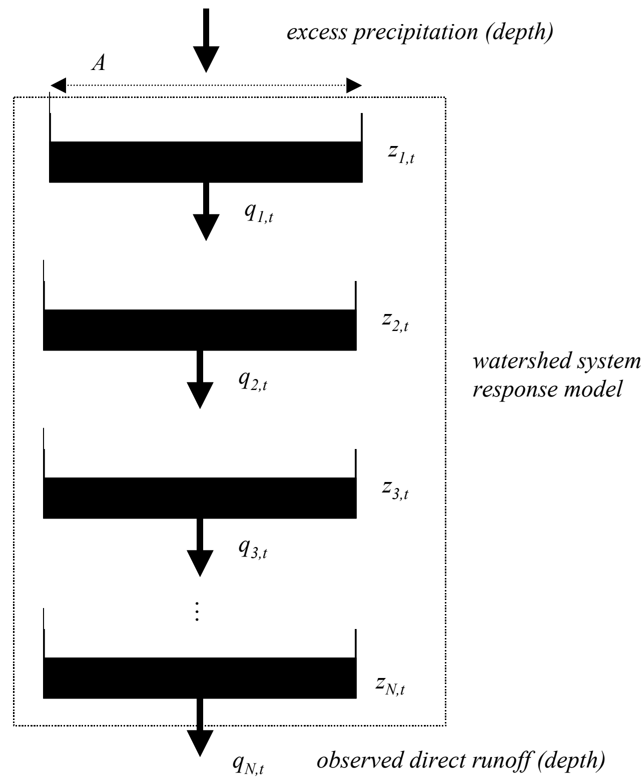

Figure 3.12 Cascade of Reservoir Elements Conceptualization

Figure 3.12 is a schematic of a watershed response conceptualized as a series of identical reservoirs without feedback. The outflow of each reservoir is related to the accumulated storage in the reservoir. The behavior of the individual reservoir elements determines whether the model becomes a Gamma, Rayleigh, or Weibull distribution.

Figure 3.13 is a schematic of a reservoir response element. In the sketch, the element area is A, the accumulated excess storage is z, and the outlet flow area is a.

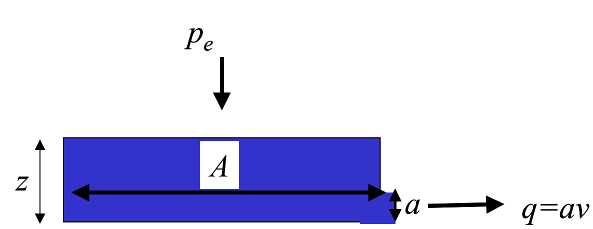

Figure 3.13 Reservoir Element Model

The outlet discharge is the product of the outlet area, *a*, and the flow velocity *v*. The input is p_e.

3.8.1 Linear (Gamma) Reservoir Element

The first response model is a linear reservoir model, where the reservoir discharge is proportional to the accumulated depth of input. The constant of proportionality is *c*. The discharge equation is

$$q = av = acz \; . \tag{3.32}$$

A mass balance of the reservoir is

$$A\frac{dz}{dt} = -acz \; . \tag{3.33}$$

The input p_e is applied over a very short time interval; so the resulting depth, before outflow begins is z_o. The solution to this ODE (Ordinary Differential Equation) is

$$z(t) = z_0 \exp(-\frac{ac}{A}t) \; . \tag{3.34}$$

The ratio *A/ac* is called the residence time of the linear reservoir.

$$\bar{t} = \frac{A}{ac} \, . \tag{3.35}$$

Thus in terms of residence time the accumulated depth in a linear reservoir is

$$z(t) = z_0 \exp(-\frac{t}{\bar{t}}) \; . \tag{3.36}$$

The discharge rate is the product of this function and the constant of proportionality

$$q(t) = acz_0 \exp(-\frac{t}{\bar{t}}) = \frac{A}{\bar{t}} z_0 \exp(-\frac{t}{\bar{t}}) \, . \tag{3.37}$$

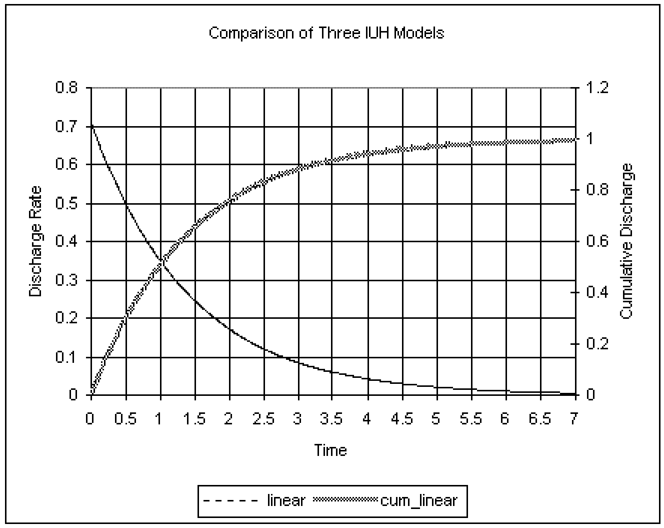

Figure 3.14 Linear Reservoir Model $\bar{t} = \sqrt{2}$, *A*=1, z_0=*1*

This particular watershed model has the following properties:

- Cumulative discharge is related to accumulated time.
- Instantaneous discharge is inversely related to accumulated time.
- The peak discharge is proportional to the precipitation input depth, and occurs at time zero.
- The peak discharge is proportional to the watershed area.

3.8.2 Rayleigh Reservoir Element

The next response model assumes that the discharge is proportional to both the accumulated excess precipitation (linear reservoir) and the elapsed time since the impulse of precipitation was added to the watershed (translation reservoir). The constant of proportionality in this case is *2c*.

$$q = av = a2czt\ . \qquad (3.38)$$

A mass balance for this model is

$$A\frac{dz}{dt} = -a2czt\,. \tag{3.39}$$

The solution (using the same characteristic time re-parameterization as in the linear reservoir model) is

$$z(t) = z_0 \exp(-(\frac{t}{\bar{t}})^2)\,. \tag{3.40}$$

The discharge function is

$$q(t) = a2ctz_0 \exp(-(\frac{t}{\bar{t}})^2) = Az_0 \frac{2t}{\bar{t}^2} \exp(-(\frac{t}{\bar{t}})^2)\,. \tag{3.41}$$

This result is a Rayleigh distribution weighted by the product of watershed area and the initial charge of precipitation (hence the name Rayleigh reservoir). The discharge function for unit area and depth integrates to one; thus it is a unit hydrograph, and it satisfies the linearity requirement, thus it is a candidate IUH function.

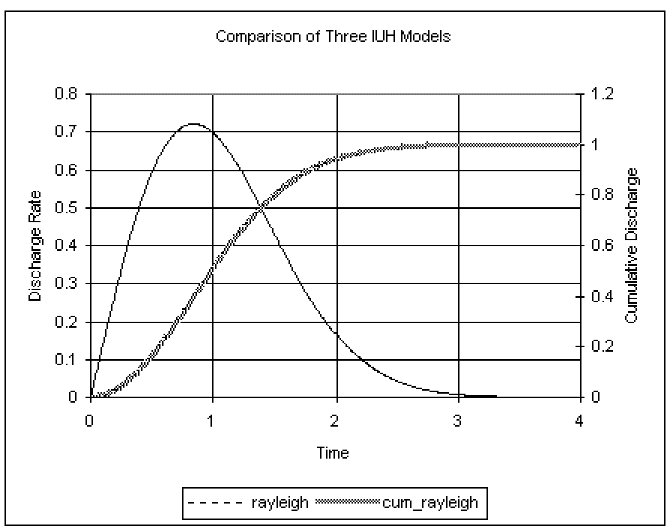

Figure 3.15. Rayleigh Reservoir Watershed Model. $\bar{t} = \sqrt{2}$, A=1, z_0=*1*

Of particular interest, the Rayleigh model qualitatively looks like a hydrograph should, with a peak occurring some time after the precipitation is applied (unlike the linear reservoir) and a falling limb after the peak with an inflection point. Examination of the discharge function includes the following relationships:

- Cumulative discharge is proportional to accumulated time.
- Instantaneous discharge is proportional to accumulated time until the peak, then inversely proportional afterwards.
- The peak discharge is proportional to the precipitation input depth, and occurs at some non-zero characteristic time.
- The peak discharge is proportional to the watershed area.

3.8.3 Weibull Reservoir

The Weibull response model assumes that the discharge is proportional to both the accumulated excess precipitation (linear reservoir) and the elapsed time raised to some non-zero power since the impulse of precipitation was added to the watershed (translation reservoir). The constant of proportionality in this case is *pc*.

$$q = av = apczt^{p-1} . \tag{3.42}$$

A mass balance for this model is

$$A\frac{dz}{dt} = -apczt^{p-1} . \tag{3.43}$$

The solution (using the same characteristic time re-parameterization as in the linear reservoir model) is

$$z(t) = z_0 \exp(-(\frac{t}{\bar{t}})^p) . \tag{3.44}$$

The discharge function is

$$q(t) = apct^{p-1} z_0 \exp(-(\frac{t}{\bar{t}})^p) = Az_0 \frac{pt^{p-1}}{\bar{t}^p} \exp(-(\frac{t}{\bar{t}})^p) . \tag{3.45}$$

This result is a Weibull distribution weighted by the product of watershed area and the initial charge of precipitation (hence the name Weibull reservoir). The discharge function for unit area and depth integrates to one, thus it is a unit hydrograph, and it satisfies the linearity requirement, thus it is a candidate IUH function.

These three models constitute the reservoir element models used in this research.

3.9. Cascade Analysis

Figure 3.12 is the schematic of a cascade model of watershed response. In our research we assumed that the number of reservoirs "internal" to the watershed could range from 0 to $+\infty$. Our initial theoretical development assumed integral values, but others have suggested fractional reservoirs can be incorporated into the theory. To develop the cascade model(s), start with the mass balance for a single reservoir element, and the discharge from this reservoir becomes the input for subsequent reservoirs and we determine the discharge for the last reservoir as representative of the entire watershed response.

3.9.1. Gamma Reservoir Cascade

Equation 3.46, where z_i represents the accumulated storage depth, ac is the reservoir discharge coefficient, q_i is the outflow for a particular reservoir, and A is the watershed area, represents the discharge functions for a cascade of linear reservoirs that comprise a response model. The subscript, i , is the identifier of a particular reservoir in the cascade.

$$Aq_{i,t} = acz_{i,t} \,. \tag{3.46}$$

Equation 3.47 is the mass balance equation for a reservoir in the cascade. In Equation 3.46, the first reservoir receives the initial charge of water, z_o over an infinitesimally small time interval, essentially an impulse, and this impulse is propagated through the system by the drainage functions.

$$A\dot{z}_{i,t} = Aq_{i-1,t} - acz_{i,t}\,. \tag{3.47}$$

The entire watershed response is expressed as the system of linear ordinary differential equations, Equation 3.48, and the analytical solution for discharge for this system for the *N*-th reservoir is expressed in Equation 3.49.

$$\begin{array}{lllllll}
\dot{z}_1 = & z_o & -\frac{1}{\bar{t}}z_1 & & & & \\
\dot{z}_2 = & & \frac{1}{\bar{t}}z_1 & -\frac{1}{\bar{t}}z_2 & & & \\
\dot{z}_3 = & & & \frac{1}{\bar{t}}z_2 & -\frac{1}{\bar{t}}z_3 & & \\
\vdots & & & \vdots & \vdots & & \\
\dot{z}_N = & & & & \frac{1}{\bar{t}}z_{N-1} & -\frac{1}{\bar{t}}z_N &
\end{array} \quad . \tag{3.48}$$

The result in equation 3.48 is identical to the Nash model (Nash 1958) and is incorporated into many standard hydrology programs such as the COSSARR model (Rockwood et. al. 1972). The factorial can be replaced by the Gamma function (Nauman and Buffham, 1983) and the result can be extended to non-integer number of reservoirs.

$$q_{N,t} = Az_0 \cdot \left(\frac{1}{\bar{t}}\right)\left(\frac{t^{N-1}}{(N-1)!\bar{t}^{N-1}}\right)\exp(-\frac{t}{\bar{t}})\,. \tag{3.49}$$

To model the response to a time-series of precipitation inputs, the individual responses (Eq. 3.49) are convolved and the result of the convolution is the output from the watershed. If each input is represented by the product of a rate and time interval ($z_o(t) = q_o(t)\,dt$) then the individual response is (note the Gamma function is substituted for the factorial)

$$dq_{i,\tau} = Aq_0(\tau)\cdot\left(\frac{1}{\bar{t}}\right)\left(\frac{(t-\tau)^{N-1}}{\Gamma(N)\bar{t}^{N-1}}\right)\exp(-\frac{t-\tau}{\bar{t}})d\tau\,. \qquad (3.50)$$

The accumulated responses are given by

$$q_N(t) = \int_0^t Aq_0(\tau)\cdot\left(\frac{1}{\bar{t}}\right)\left(\frac{(t-\tau)^{N-1}}{\Gamma(N)\bar{t}^{N-1}}\right)\exp(-\frac{t-\tau}{\bar{t}})d\tau\,. \qquad (3.51)$$

Equation 3.51 represents the watershed response to an input time series. The convolution integral in Chapter 7 in Chow, *et al* (1988), an overview of that work, is repeated as Equation 3.52,

$$Q(t) = \int_0^t I(\tau)u(t-\tau)d\tau\,. \qquad (3.52)$$

The analogs to our present work are as follows (Chow's variable list is shown on the left of the equalities):

$$\begin{aligned} Q(t) &= q_N(t) \\ I(\tau) &= q_0(\tau) \\ u(t-\tau) &= A\cdot\left(\frac{1}{\bar{t}}\right)\left(\frac{(t-\tau)^{N-1}}{\Gamma(N)\bar{t}^{N-1}}\right)\exp(-\frac{t-\tau}{\bar{t}}) \end{aligned} \qquad (3.53)$$

We call the kernel ($u(t-\tau)$) for the linear reservoir a gamma response because the kernel is essentially a gamma probability distribution. The reason for representing the function as being derived from a cascade is that this derivation provides a "physical" meaning to the distribution parameters.

The analysis is repeated for the Rayleigh and Weibull distributions.

3.9.2. Rayleigh Reservoir Cascade

A Rayleigh response is developed in the same fashion as the gamma, except the Rayleigh reservoir element is used instead of the linear (gamma) response. The discharge and mass balances for the Rayleigh case are given as Equations 3.54 and 3.55, respectively,

$$Aq_{i,t} = 2actz_{i,t}\,, \tag{3.54}$$

$$A\dot{z}_{i,t} = Aq_{i-1,t} - 2actz_{i,t}\,. \tag{3.55}$$

The entire watershed response is expressed as the system of linear ordinary differential equations in Equation 3.56.

$$\begin{aligned}
\dot{z}_1 &= z_o - 2\frac{t}{\bar{t}}z_1 \\
\dot{z}_2 &= 2\frac{t}{\bar{t}}z_1 - 2\frac{t}{\bar{t}}z_2 \\
\dot{z}_3 &= 2\frac{t}{\bar{t}}z_2 - 2\frac{t}{\bar{t}}z_3 \\
&\vdots \\
\dot{z}_N &= 2\frac{t}{\bar{t}}z_{N-1} - 2\frac{t}{\bar{t}}z_N
\end{aligned} \tag{3.56}$$

The analytical solution for any reservoir is expressed in Equation 3.57.

$$q_{N,t} = 2Az_0 \cdot \left(\frac{t}{\bar{t}^2}\right)\left(\frac{(t^2)^{N-1}}{\Gamma(N)(\bar{t}^2)^{N-1}}\right)\exp(-(\frac{t}{\bar{t}})^2)\,, \tag{3.57}$$

Equation 3.58 gives the convolution integral using this kernel.

$$q_i(t) = \int_0^t 2Aq_0(\tau) \cdot \left(\frac{t-\tau}{\bar{t}^2}\right)\left(\frac{((t-\tau)^2)^{N-1}}{\Gamma(N)(\bar{t}^2)^{N-1}}\right)\exp(-\frac{(t-\tau)^2}{\bar{t}^2})d\tau\,, \tag{3.58}$$

This distribution is identical to Leinhard's "hydrograph distribution" (Leinhard, 1972) that he developed from statistical-mechanical analysis.

3.9.3. Weibull Reservoir Cascade

A Weibull response is developed in the same fashion as the gamma by substitution of the Weibull reservoir element in the analysis. The discharge and mass balances are given as Equations 3.59 and 3.60, respectively,

$$Aq_{i,t} = 2actz_{i,t}, \tag{3.59}$$

$$A\dot{z}_{i,t} = Aq_{i-1,t} - 2actz_{i,t}. \tag{3.60}$$

The entire watershed response is again expressed as a system of linear ordinary differential equations; Equation 3.61.

$$\begin{aligned}
\dot{z}_1 &= z_o - p\frac{t^{p-1}}{\bar{t}}z_1 \\
\dot{z}_2 &= p\frac{t^{p-1}}{\bar{t}}z_1 - p\frac{t^{p-1}}{\bar{t}}z_2 \\
\dot{z}_3 &= p\frac{t^{p-1}}{\bar{t}}z_2 - p\frac{t^{p-1}}{\bar{t}}z_3 \\
&\vdots \\
\dot{z}_N &= p\frac{t^{p-1}}{\bar{t}}z_{N-1} - p\frac{t^{p-1}}{\bar{t}}z_N
\end{aligned} \tag{3.61}$$

The analytical solution to this system for any reservoir is expressed in Equation 3.62,

$$q_{N,t} = pAz_0 \cdot \left(\frac{t^{p-1}}{\bar{t}^{p}}\right)\left(\frac{(t^p)^{N-1}}{\Gamma(N)(\bar{t}^p)^{N-1}}\right)\exp(-(\frac{t}{\bar{t}})^p). \tag{3.62}$$

The accumulated responses to a time series of precipitation input are given by Equation 3.63.

$$q_i(t) = \int_0^t pAq_0(\tau) \cdot \left(\frac{(t-\tau)^{p-1}}{\bar{t}}\right)\left(\frac{((t-\tau)^p)^{N-1}}{\Gamma(N)\bar{t}^{N-1}}\right)\exp(-\frac{(t-\tau)^p}{\bar{t}^p})d\tau. \tag{3.63}$$

The utility of the Weibull model is that both the linear cascade (exponential) and the Rayleigh cascade are special cases of the generalized Weibull model, thus if we program a Weibull-type model as the IUH, we can investigate other models by restricting parameter values. The parameters have the following impacts on the discharge function:

1. The power term controls the decay rate of the hydrograph (shape of the falling limb). If p is greater than one, then decay is fast (steep falling limb); if p is less than one then the decay is slow (long falling limb).
2. The $\bar{t}$ term controls the scale of the hydrograph. It simultaneously establishes the location of the peak and the magnitude of the peak.
3. The reservoir number, N, controls the lag between the input and the response, as well as the shape of the hydrograph.

The next chapter describes how the distribution parameters are determined from observations.

CHAPTER 4

DISCUSSION OF RESULTS

Parameter Estimation Procedure(s)

The IUH parameters are estimated by simulating the DRH from the effective rainfall signal and adjusting values until some merit function is minimized. The minimization algorithms used in this research is the downhill simplex method of Nelder and Mead (Nelder and Mead, 1965) as implemented by Press et al (Press et. al., 1986). This method, while slow, is quite robust and faster than a grid search technique. Refinement of values from the simplex algorithm is accomplished using Powell's direction set method (Powell, 1964) for minimizing functions without calculating derivatives, again as implemented by Press et. al. (Press et. al., 1986).

The principal effort in this research was to adapt the two programs (as presented in Numerical Recipes) to function with the hydrologic data, adjust some of the stopping criteria (mostly reduced iteration exits), and manage the file I/O operations to keep data in memory during the optimization procedures rather that reading and writing to files. This effort required about a year of programming and testing before the formal production runs were commenced.

Merit Functions

The functions considered are the classic sum of squared errors (SSE), the root mean squared error (RMSE), and the maximum absolute deviation (MAD).

Mathematically these merit functions are:

$$SSE = \sum_{i=1}^{N} (Q_S - Q_O)_i^2 \tag{4.1}$$

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (Q_S - Q_O)_i^2} \tag{4.2}$$

$$MAD = \max \left| Q_S - Q_O \right|_i \tag{4.3}$$

where Q is the discharge (L^3/T),the subscripts O and S represent observed and simulated discharge, respectively, and N is the total number of values in a particular storm event.

Evaluating the Results

Figure 4.1 is an illustration of the result of such automated parameter estimation for a particular storm on the Ash Creek station in the Dallas area. The other storms in other modules produce similar results. In the figure, the model runoff and cumulative model runoff differ from the observed values, although qualitatively this particular example is not a bad model of the observed data.

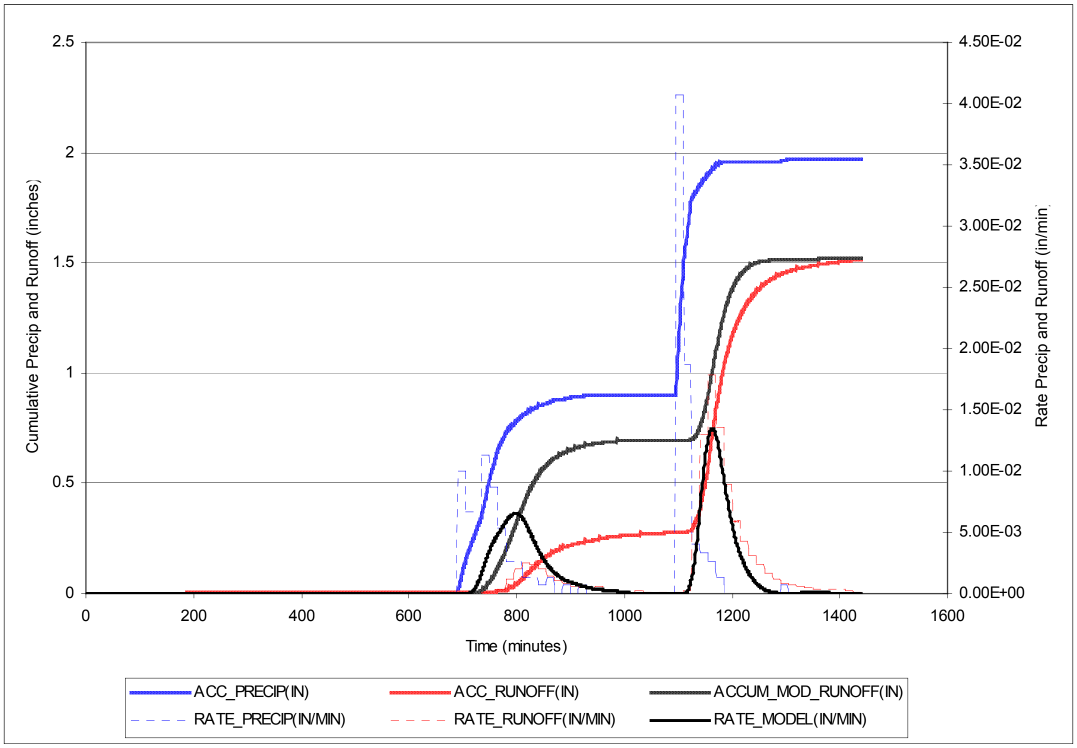

Figure 4.1 Plot of Observed and Model Runoff

Quantitative Measures

To quantify how well the model has represented the data we examined several measures (beyond the merit function) of acceptability. These measures are evaluated after the minimization step and are not used in the fitting procedure, except to suggest manual restart adjustments.

Bias

Bias is the mean error defined as the residual between the observed and model value of runoff rate. It is calculated using

$$Bias = \overline{Q_o - Q_m} = \frac{1}{N}\sum_{i=1}^{N} Q_{o,i} - Q_{m,i} \; . \tag{4.4}$$

Fractional Bias

Fractional bias (FB) is a normalized mean error between the observed and predicted runoff rates (or cumulative values). Fractional bias will be zero if the model and observed values are identical and will always fall in the range [-2,2]. The fractional bias is calculated using

$$FB = 2\left(\frac{\overline{Q}_o - \overline{Q}_m}{\overline{Q}_o + \overline{Q}_m}\right), \qquad (4.5)$$

where $\overline{Q}_o$ is the arithmetic mean of the observed runoff values, and $\overline{Q}_m$ is the arithmetic mean of the model runoff values.

Fractional Variance

Fractional variance (FV) is a normalization of the mean bias of the sample variances of the observed and predicted values. FV is calculated using

$$FV = 2\left(\frac{\sigma^2{}_{qo} - \sigma^2{}_{qm}}{\sigma^2{}_{qo} + \sigma^2{}_{qm}}\right), \qquad (4.6)$$

where $\sigma^2{}_{qo}$ is the sample variance of the observed runoff values, and $\sigma^2{}_{qm}$ is the sample variance of the model runoff values.

Normalized Mean Square Error

Normalized mean square error emphasizes scatter in a data set. NMSE is not biased towards models that over- or under-predict. Smaller values of NMSE indicate better performance. NMSE is calculated using

$$NMSE = \frac{\overline{(Q_o - Q_m)^2}}{\overline{Q}_o \overline{Q}_m}. \qquad (4.7)$$

Geometric Mean Bias

The geometric mean bias is calculated using

$$MG = \exp(\overline{\ln Q_o} - \overline{\ln Q_m}) . \tag{4.8}$$

Geometric Mean Variance

The geometric mean variance is calculated using

$$VG = \exp[\overline{(\ln Q_o - \ln Q_m)^2}] . \tag{4.9}$$

In addition to these measures there also was quantified a peak discharge relative error (QB) and peak temporal bias (TB).

Peak Relative Error

The peak discharge error is the %-difference in magnitude between the observed and model peak rate. It is calculated from

$$QB = (Q_{Po} - Q_{Pm}) / Q_{Po} . \tag{4.10}$$

Peak Temporal Bias

The peak temporal bias (TB) is the difference in arrival time of the largest runoff rate in the model results as compared to the observed results. It is calculated for each storm from

$$TB = t_{Po} - t_{Pm} . \tag{4.11}$$

A TB less than zero indicate that the model predicts a late peak (i.e. real peak comes sooner). While a positive value indicates that the model predicts an early peak. We assume that a TB in the range [-30,30] (minutes) is a desirable value.

--
0.4%
0.3%
0.1%
98.9%
99.9%
0.0%
11.8%

--
78.9%
70.3%
78.1%
99.9%
99.9%
18.8%
27.4%

Acceptance Criteria

The performance of an exact model (that is faithful reproduction of observations) is that the FB, FV and NMSE should all be zero, and the MG and VG should both be one. In the present work the following acceptance criteria were adopted, and other measures are simply reported. The model (and its parameterization) is deemed acceptable if:

1. $NMSE \leq \frac{1}{2}$
2. $-\frac{1}{2} \leq FB \leq \frac{1}{2}$
3. $-\frac{1}{2} \leq FV \leq \frac{1}{2}$
4. $\frac{3}{4} \leq MG \leq \frac{5}{4}$
5. $\frac{3}{4} \leq VG \leq \frac{5}{4}$

These particular criteria were adopted using the reliability criteria suggested by Hanna and Heinhold (1985), Patel and Kumar (1998), and Kumar et al. (1999). The meaning of acceptance is that the model is qualitatively useful (the picture) and meets these quantitative criteria. The criteria are expected to identify if a particular model is useless, but they cannot choose among different models, except that if a particular model comes closer to the ideal measure values that all other models considered, then that is a non-inferior model. Because all the models in this research are related to the gamma probability distribution, it is expected that they will all meet the acceptance criteria (after fitting), and our goal will be to select a sub-class (Gamma, Raleigh, Weibull, NRCS-DUH) as a most appropriate model for the Central Texas Data.

For the case presented in Figure 4.1 the values of the measures are listed in Table 4.1.

Table 4.1. Acceptance analysis for Ash Creek, 1973, October 30 storm event.

IUH Analysis for: sta08057320_1973_1030.dat			
Measure	Value	Utility/Meaning	Acceptable
SSE	2.66×10^{-3}	Merit function value at exit	N/A
NMSE	3.8×10^{-5}		Yes
FB	-6.15×10^{-3}		Yes
FV	0.26		Yes
MG	0.99		Yes
GV	1.00		Yes
QB	24%	Model peak is smaller than observed.	N/A
TB	11	Model peak occurs later than observed.	N/A

The IUH model for this storm is

$$\frac{Q(t)}{A} = (0.772)\cdot(1.13)\cdot z_0 \cdot \left(\frac{t^{0.13}}{\bar{t}}\right)\left(\frac{(t^{1.13})^{6.05}}{\Gamma(7.05)\cdot(14.10)^{6.05}}\right)\exp\left(-\frac{t^{1.13}}{14.10}\right), \quad (4.12)$$

where A is the watershed area (in appropriate units) and z_o is the precipitation input depth for one time interval (in this case a one minute interval).

The model hydrograph is obtained by the convolution of the above equation for each precipitation interval (~1400 minutes) with each input lagged. The numerical values are determined from the simplex minimization algorithm followed by a Powell refinement. Some arithmetic is left incomplete for clarity to correspond with the parameters that appear in the generic equation for the hydrograph.

Comparison among Different IUH models

The utility of the acceptance testing is apparent when the different IUH models are considered. Figures 4.2 through 4.6 are examples of the same storm analyzed using the five different models: Gamma, Rayleigh, Weibull, NRCS, and Commons, respectively. Qualitatively speaking, all the models perform about the same; each captures the peak times reasonably well and each predicts the magnitude of the smaller peaks about the same. The two large peaks are under-predicted in all the models but the degree of difference is different in each model.

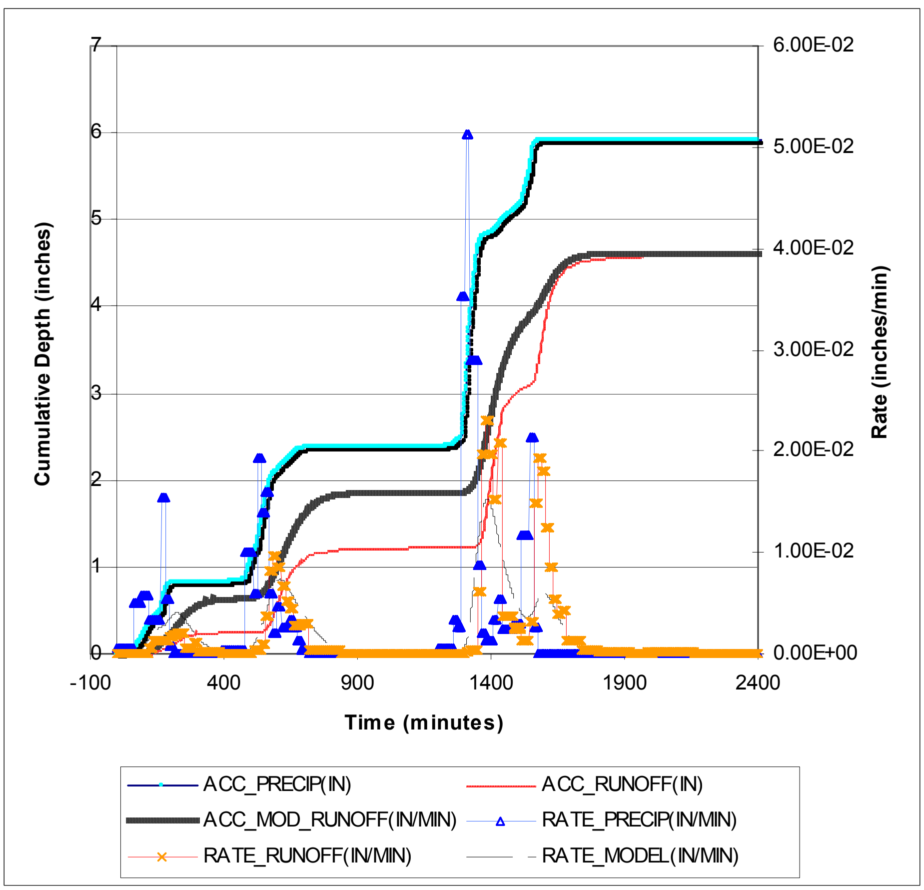

Figure 4.2 Plot of Observed and Model Runoff, Ash Creek, June 3, 1973 storm using

the Gamma IUH model.

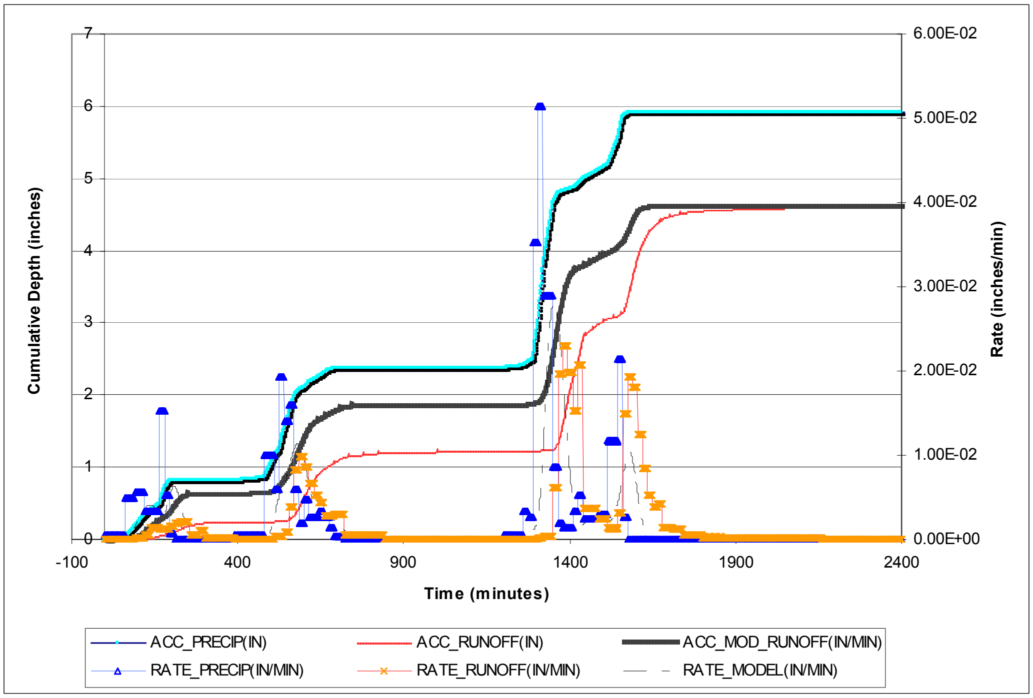

Figure 4.3 Plot of Observed and Model Runoff, Ash Creek, June 3, 1973 storm using the Rayleigh IUH model.

The Rayleigh model, in contrast to the gamma model has much shorter decay times, thus shorter tails after the peak discharges. In this particular storm it over-predicts the biggest peak somewhat, but certainly is a better estimate of peak magnitude than the gamma model.

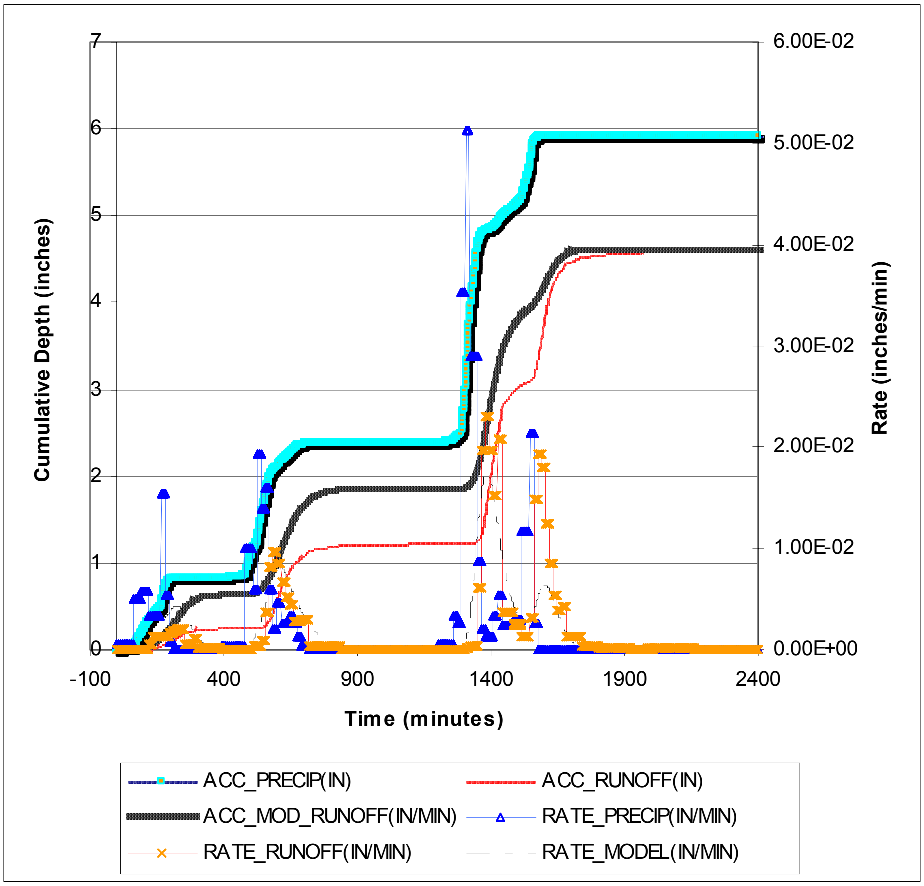

Figure 4.4 Plot of Observed and Model Runoff, Ash Creek, June 3, 1973 storm using the Weibull IUH model.

The Weibull model performs more like the Rayleigh model with regards to peak discharge prediction as well as capture the decay behavior of the hydrographs after the peaks pass. Like the other two models it has difficulty with the second large peak, but otherwise is not a qualitatively bad model.

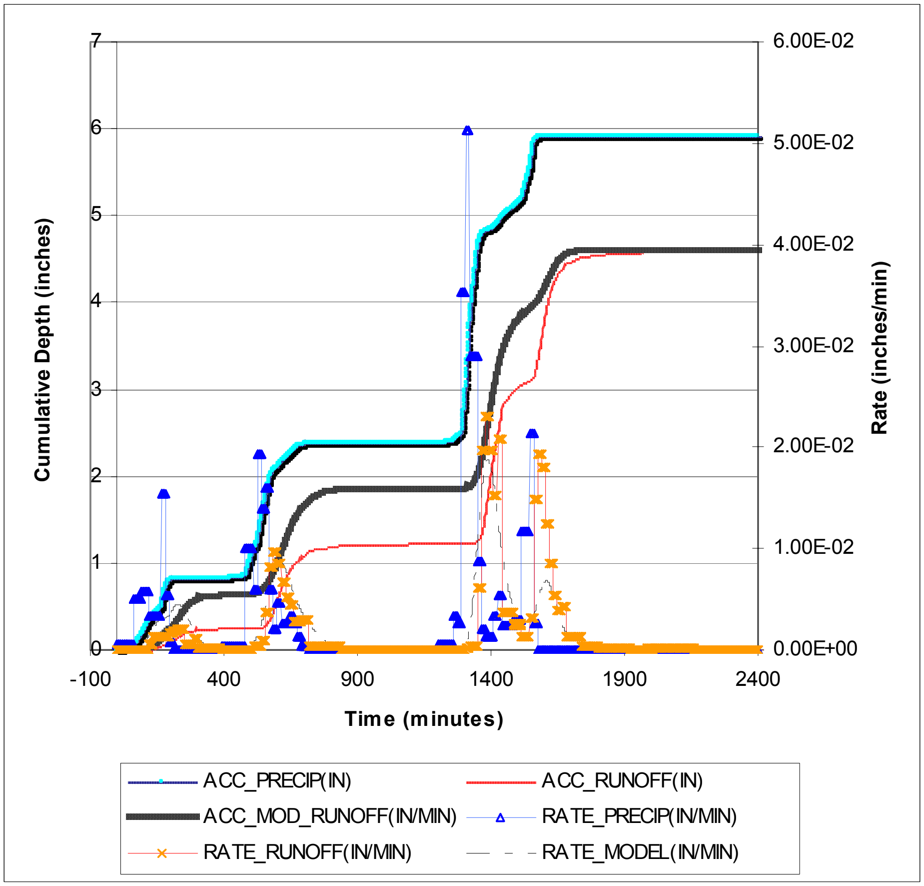

Figure 4.5 Plot of Observed and Model Runoff, Ash Creek, June 3, 1973 storm using the NRCS-IUH model.

The NRCS model is nearly indistinguishable from the Weibull model for this storm. It too does a decent job of modeling both the peak magnitudes, times, and the decay of the hydrograph after the peaks. Like the Weibull model, it too has a hard time with the second large peak.

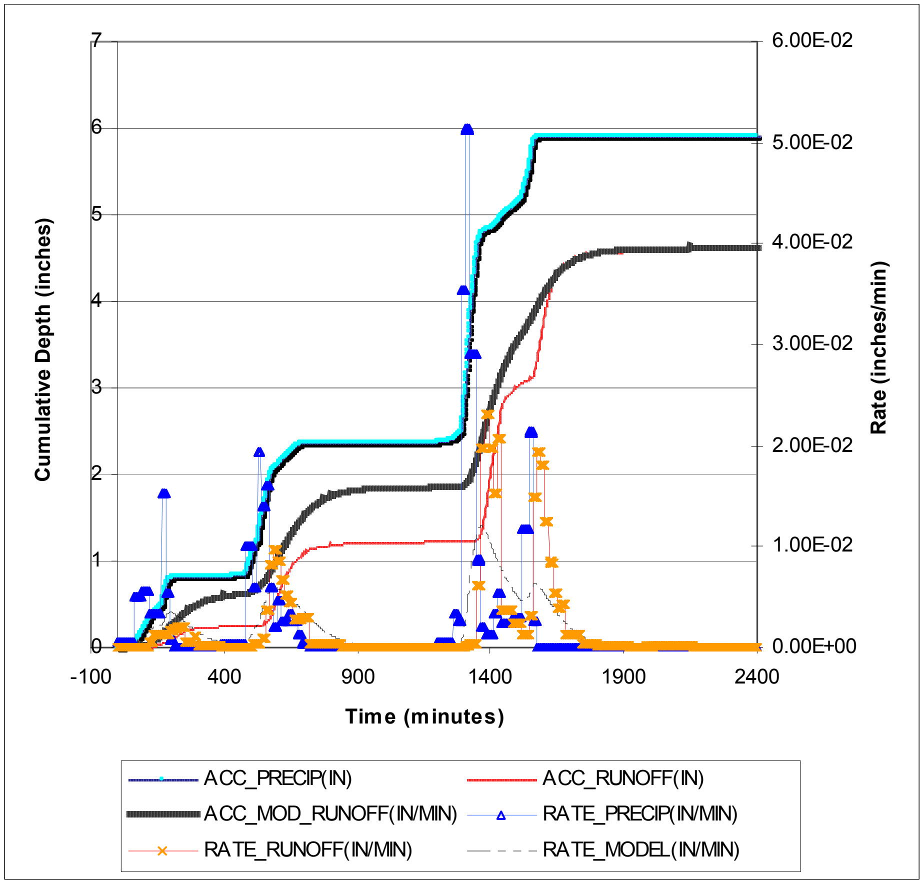

Figure 4.6 Plot of Observed and Model Runoff, Ash Creek, June 3, 1973 storm using the Commons IUH model.

The Commons model behaves a lot like the gamma model for this storm, again with the longer tails.

From the qualitative comparison it is difficult to select one model over another, but when the acceptance criteria are examined in this particular storm we can make a selection. Table 4.2 is a list of the acceptance criteria (along with the parameter values which are of use for future storms). From this table we can conclude that the Weibull and NRCS model have a lower NMSE by at least one order of magnitude over the other models and their QB and TB values are favorable (good prediction of time of peak as well as an acceptable flow bias). For this particular storm it is a good model choice, and the NRCS model is probably the best. Such comparisons are made for the entire data base.

Table 4.2 List of the acceptance criteria (along with the parameter values which are of use for future storms).

	Model					
	Gamma	Rayleigh	Weibull	NRCS	Commons	Acceptance Criteria Parameters
SSE	0.016609	0.04274	0.014026	0.012969	0.020403	
CRP	0.781416	0.781416	7.81E-01	7.81E-01	7.81E-01	
CTP	40	1091	216	65	19	
CQP	2.4	1.56812	2.108175	0.015383	0.000967	
CEP	1	2	1.387482	1	1	
NMSE	1.29E-05	8.54E-05	9.19E-06	7.86E-06	1.95E-05	
BIAS	6.42E-09	-9.4E-11	-7.23E-11	7.38E-11	-4.79E-11	
FB	3.99E-06	-5.85E-08	-4.5E-08	4.59E-08	-2.98E-08	
FV	0.734308	0.012489	0.487123	0.431675	0.875127	
QB	40.60%	-21.86%	23.53%	18.37%	47.40%	
TB	-3	28	-5	-8	14	

Model Acceptance Testing

The five different candidate models were used to analyze the rainfall-runoff data in the database. Two principle research questions were asked in this analysis:

Is base flow separation necessary in these data? Is one particular model preferable to the other models for these data?

Once the models were used to analyze the data, a set of IUH results for each condition was created (about 11 GB of data in 10 high-level directories). Table 4.3 is a summary table of the acceptance criteria used to select candidate models.

The base flow separation showed only a small improvement for the Gamma model, Rayleigh model and Weibull model. But the NRCS and Commoms model results showed better prediction without base flow separation which was not expected technically. In the future application base flow separation will still be used as a process of data preparation although for our data set did not make too much difference. With the base flow separation, the Commons model has the lowest acceptance, and also the NRCS model did not have a good performance over all the dataset. Compared with other four models, the Weibull model has the best acceptable prediction. Therefore the Weibull model will be the main approach to analyze the Texas dataset.

Table 4.3 Acceptance Criteria for IUH Models

Baseflow Separated	Gamma		Rayleigh		Weibull		NRCS		Commons
# INFO Storm Count =	1642	--	1642	--	1642	--	1642	--	1642
# INFO Num Accept CR1 =	1586	96.6%	1445	88.0%	1569	95.6%	347	21.1%	7
# INFO Num Accept CR2 =	1570	95.6%	1388	84.5%	1545	94.1%	297	18.1%	5
# INFO Num Accept CR3 =	1358	82.7%	1022	62.2%	1471	89.6%	251	15.3%	2
# INFO Num Accept CR4 =	1642	100.0%	1642	100.0%	1642	100.0%	1642	100.0%	1624
# INFO Num Accept CR5 =	1642	100.0%	1642	100.0%	1642	100.0%	1642	100.0%	1641
# INFO Num Accept CR_QB =	259	15.8%	394	24.0%	551	33.6%	93	5.7%	0
# INFO Num Accept CR_TB =	607	37.0%	384	23.4%	911	55.5%	579	35.3%	194
# INFO Max NMSE	65.4		3080		225		18100		1810000
# INFO Min NMSE	1.78E-09		6.28E-10		2.33E-09		2.14E-08		0.000423
# INFO Mean NMSE	0.148572		8.112503261		0.575904		34.60697		2582.34
# INFO Median NMSE	0.0122		0.0168		0.01345		4.45		165

Baseflow Included	Gamma		Rayleigh		Weibull		NRCS		Commons
# INFO Storm Count =	1642	--	1642	--	1642	--	1642	--	1642
# INFO Num Accept CR1 =	1541	93.9%	915	55.6%	1500	90.7%	833	50.7%	1295
# INFO Num Accept CR2 =	1494	91.0%	765	46.4%	1467	88.7%	760	46.3%	1154
# INFO Num Accept CR3 =	1374	83.7%	1084	65.8%	1226	74.1%	776	47.2%	1282
# INFO Num Accept CR4 =	1635	99.6%	1642	100.0%	1638	99.8%	1641	99.9%	1641
# INFO Num Accept CR5 =	1635	99.6%	1642	100.0%	1638	99.8%	1641	99.9%	1641
# INFO Num Accept CR_QB =	499	27.3%	699	29.6%	520	31.7%	210	12.8%	309
# INFO Num Accept CR_TB =	516	31.5%	466	28.4%	699	42.6%	162	9.9%	450
# INFO Max NMSE	282		1.9E+15		333000		934000		1180
# INFO Min NMSE	1.87E-09		6.08E-08		4.85E-10		6.41E-08		4.15E-09
# INFO Mean NMSE	0.638562		1.15924E+12		216.0147		1014.895		2.311689
# INFO Median NMSE	0.0119		0.324		0.0177		0.45		0.0828

--
0.4%
0.3%
0.1%
98.9%
99.9%
0.0%
11.8%

--
78.9%
70.3%
78.1%
99.9%
99.9%
18.8%
27.4%

CHAPTER 5

CONCLUSIONS

Models are used to analyze 1642 storm events from 88 selected watersheds located in central Texas varying in size from 0.33 to 166 square miles. Using a set of acceptance criteria we can examine how well the model represented the data. Each model is run twice per storm in order to compare the effect of base flow separation, as well as between model performances. The Weibull model, Gamma and Rayleigh model show a little improvement for the result with separation, but for NRCS and Commons model, the results should be showed even a lower acceptance with the base flow separation. Since it is a basic process of the unit hydrograph methodology, we will keep the process to make sure the results more reliable for the future precipitation of un-gaged watersheds.

The candidate models show very acceptable results under all the criteria conditions except for the Commons model which failed to predict runoff compared with the observed data. Among other four models, the Weibull model shows the best fit with the observed data, so it will be the main approaches in the future study of Texas general data. The Gamma model has better results than the Rayleigh model.

Future work

Prior work on the same dataset assumed the Weibull model was proper model and analyzed for module difference as well as tested the possibility of regionalization. With this work we have demonstrated that the original conjecture of Jonalagadda(2003) and Lazarescu (2003) theses were supportable. The next task is a formal regionalization analysis to produce a method to synthesis model parameters for use an ungaged watershed.

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