CORRELATION OF GEOMETRIC PROPERTIES OF SMALL WATERSHEDS IN CENTRAL TEXAS WITH OBSERVED INSTANTANEOUS UNIT HYDROGRAPHS

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ABSTRACT

The unit hydrograph is a widely used method to represent the causal relationship between rainfall and runoff. The method is used when observed rainfall and runoff data is available.

The purpose of the thesis is to analyze the existing data on small watersheds in Central Texas and to determine some correlations between the parameters of the hydrograph and some identifiable and readily obtainable physical characteristics of the watersheds. The correlations will serve as the basis of a synthetic procedure for generating design hydrographs for un-gaged watersheds in Texas.

In this research, instantaneous unit hydrographs (IUH) from nearly 1600 runoff-producing storms for 88 gaged watersheds in Central Texas were analyzed.

Data for storm hydrographs, recorded from early 1960's to the middle of 1970's, was obtained from USGS studies. After digitizing the data, a database containing all the recorded events of rainfall and runoff was constructed. The database was then used to derive instantaneous unit hydrographs (IUH's) for every station monitored by USGS.

Another database was constructed to analyze the watershed characteristics. The watersheds were delineated on the topographic maps and tables containing the areas, perimeters, aspect ratios, stream lengths, etc were completed.

Analysis revealed in general a good correlation between hydrograph parameters and watershed characteristics within a module but not for the entire database.

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CHAPTER 1: Introduction

1.1 RESEARCH BACKGROUND

This research is part of the project funded by Texas Department of Transportation, TxDOT Project 0-4194, titled "Regional Characteristics of Unit Hydrographs".

The unit hydrograph is a widely used method to represent the causal relationship between rainfall and surface runoff. Unit hydrographs are used when observed rainfall and runoff data are available; synthetic hydrographs are used on un-gaged watersheds. Unit hydrographs assumed a linear system excitation-response relationship, but despite this apparent limitation they have been successfully used to predict surface runoff and are used in hydrologic design procedures.

The Texas Department of Transportation (TxDOT) uses hydrograph methods to estimate the magnitude and duration of discharges for design of drainage structures when watershed drainage area exceeds 200 acres, but is less than about 20 square miles. Typically, Natural Resources Conservation Service (NRCS) methods are used.

1.2 RESEARCH OBJECTIVES

The objectives of the research in this thesis are:

- To review current TxDOT unit hydrograph procedures and prepare a supporting literature review.
- To assemble a database of measured rainfall-runoff responses for selected Texas watersheds.

- To develop unit hydrograph functions from the database.

The work in this thesis will eventually support:

- Comparisons of the Natural Resources Conservation Service (NRCS) dimensionless unit hydrograph to observed unit hydrographs in Texas, and the
- Regionalization of the observed unit hydrographs for purposes of estimating unit hydrographs for ungaged watersheds.

1.3 ORGANIZATION OF THE THESIS

This thesis is organized into different chapters each dealing with a specific task of the research. Chapter 2, the problem statement explains the purpose of the project; it explains the necessity of a central database showing the rainfall-runoff data for Texas.

Chapter 3 explores the currently applied methods in Texas region, with an emphasis on the methods suitable for the watershed with drainage areas ranging from 250 acres to 10 square miles. Chapter 4 discusses the available data for the database construction; it presents the different formats of the collected data. It gives a detailed explanation of the organization of the data in the database. Chapter 5 explains existing methods of developing unit hydrographs from the available rainfall-runoff data organized in Chapter 4. This chapter explains the Instantaneous Unit Hydrograph (IUH) method, and provides a derivation from simple linearized physics. Chapter 6 is an analysis of the database using the IUH model(s). The parameters of the IUH model for each watershed are analyzed for their dependencies on watershed identity such as an area. The analysis is the first step to a regionalization. Chapter 7 presents the results of the analysis and the associated conclusions. Chapter 8 gives the list of references that have been cited in the Thesis. The thesis includes the tables with the parameters of the suitable model for Texas. The tables are organized into different modules, each showing the event specific parameters for all the events. Appendix.A contains the tables with the model parameters for each module.

CHAPTER 2 : Problem Statement

TxDOT currently uses the Natural Resources Conservation Service (NRCS) dimensionless unit hydrograph procedure for its design applications. This dimensionless unit hydrograph was derived from a large number of natural unit hydrographs from watersheds varying widely in size and geographical locations. The dimensionless unit graph characteristics represent values that have been adopted for an "average" watershed.

But general use of the NRCS procedure without consideration of actual regional or site characteristics can conduct to a poor correlation with statistical expectation, inadequate design, or inefficient over-designed structures.

CHAPTER 3: Literature Review

3.1 INTRODUCTION

Runoff, or surface runoff, refers to all the waters flowing on the surface of the earth, either by overland sheet flow or by channel flow in rills, gullies, streams, or rivers (ASCE, 1996). Runoff is usually expressed in terms of volume, depth or flow rate. Flow rate or discharge at a cross-section or gaging station usually varies in time; therefore its value at any time is the instantaneous or local discharge. Instantaneous values of the discharge can be integrated over a period of time to give the runoff volume for the entire period.

3.1.1 RAINFALL-RUNOFF PROCESS

Depending upon the rate at which the rain falls, the water may either infiltrate into the soil or accumulate and flow from the area as surface runoff. For the practical purpose or runoff analysis, the total runoff in a stream channel is generally classified as direct runoff and base flow. The direct runoff is the part of the runoff, which enters the stream promptly after the rainfall. It is the sum of surface runoff, prompt subsurface runoff, and the channel precipitation. The base flow is the sustained runoff. It is composed of groundwater runoff and delayed subsurface runoff. The total rainfall may be considered to consist of rainfall excess and abstractions. The rainfall excess is the fraction of rainfall that contributes directly to the surface runoff. The abstractions are the remaining parts that eventually becoming surface runoff such as interception, evaporation, transpiration, depression storage, and infiltration. Figure 3.1 is a block diagram showing different components involved in the rainfall-runoff process.

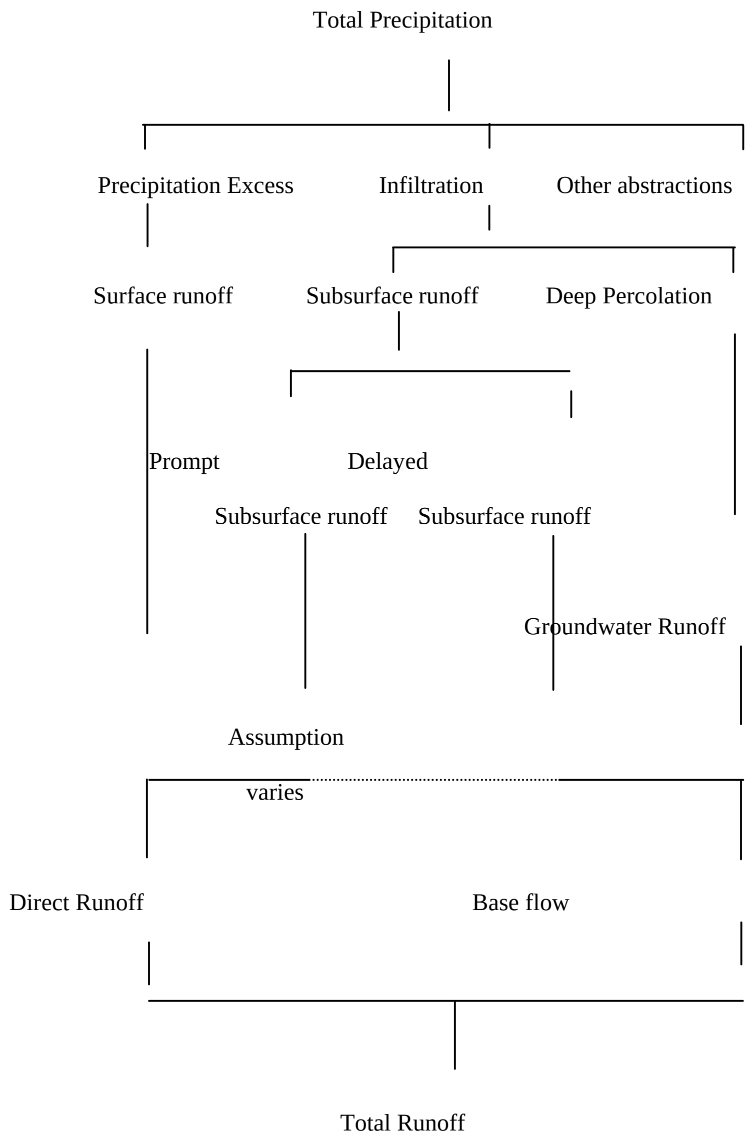

Figure 3.1. Block diagram showing the rainfall-runoff process.

(Reproduced from Chow V. T, 1964)

3.2 HYDROGRAPH

A hydrograph is a representation of the variation of a water flow quantity (discharge or stage) as a function of time. Depending on the represented flow quantity, the hydrograph is designated as Surface runoff, base flow, stream flow, low flow, flood flow, or stage hydrograph (ASCE, 1996). The runoff hydrograph is a response of the contributing watershed to a specific rainfall event. The response of a watershed is governed by a specific transfer function (unit hydrograph). The transfer function converts the hyetograph; a time-series of rainfall, to a hydrograph; time-series of runoff in this thesis. For design purposes engineers use hydrographs to understand the time-rate distribution of surface runoff volumes.

3.2.1 HYDROGRAPH COMPONENETS

A typical runoff hydrograph consists of three distinct components at depicted in Figure 3.2. AB is the approach limb, BD is the rising limb, and DH is the receding limb. A complete stream flow hydrograph is obtained by the addition of a fourth component, which is the base flow. Other major components of the hydrograph involve surface runoff, interflow, groundwater flow and channel precipitation (Linsley, 1949).

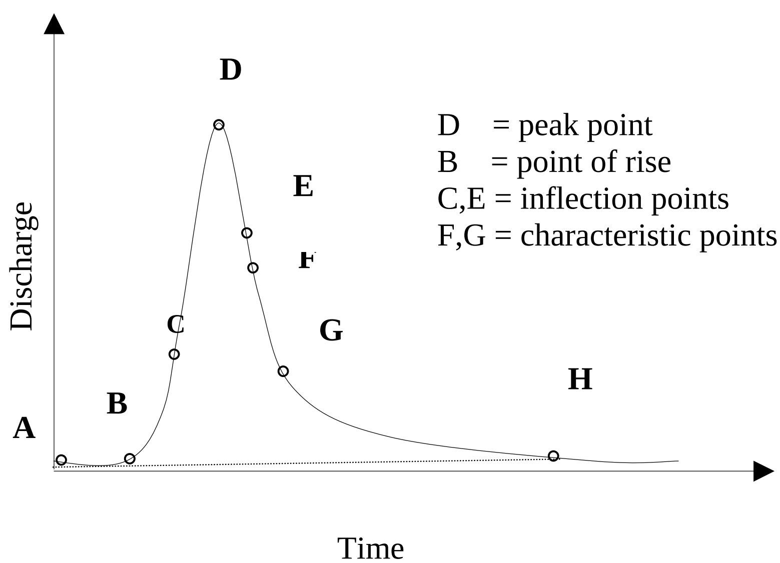

Figure 3.2. Typical Runoff Hydrograph.

The discharge hydrograph is not measured directly, it is inferred from the stage hydrograph (using a rating curve). Direct runoff hydrograph is obtained from a total runoff hydrograph by separating the base flow.

3.2.2 BASE FLOW SEPARATION

As mentioned earlier, the total runoff hydrograph can be viewed as two parts, direct runoff and the base flow. Base flow is the water discharged from extensive groundwater aquifers to the stream. There are different methods to separate the base flow component of a hydrograph.

3.2.3 METHODS OF BASE FLOW SEPARATION

Base flow separation methods involved in the separation of direct runoff hydrograph (DRH) from the total runoff hydrograph are given below. The following list of different methods to separate the base flow component of a hydrograph..

1. Constant Discharge Method: The base flow is assumed to be constant regardless of the discharge. A minimum value immediately prior to the beginning of the storm is considered as the constant value that is subtracted from the ordinates of total runoff hydrograph to obtain DRH.
2. Constant Slope Method: In this method, base flow separation line is drawn between the inflection points on receding limb of storm hydrograph to beginning of storm hydrograph. Inflection point is that point on the recession limb where the direct runoff ends. For large watersheds the infection point is assumed at N=$A^{0.2}$, where N is number of days after hydrograph peak, A (mi^2) is watershed area.
3. Concave Method: The starting and end points for the base flow separation line are the same as the constant slope method. However, base flow continues to decrease

until the time of the peak discharge (t_p) of the storm hydrograph, from that point (q_m) it is a straight line till the inflection point (t_r) on the recession limb.

4. Master Depletion Curve Method: A general recession model is obtained by combining data from several recessions. From the model an equation of the form $q_t = q_o e^{-kt}$ is derived which gives q_t at any time t after discharge q_o is measured.

Figure 3. is a graph illustrating the above explained methods of base flow separation.

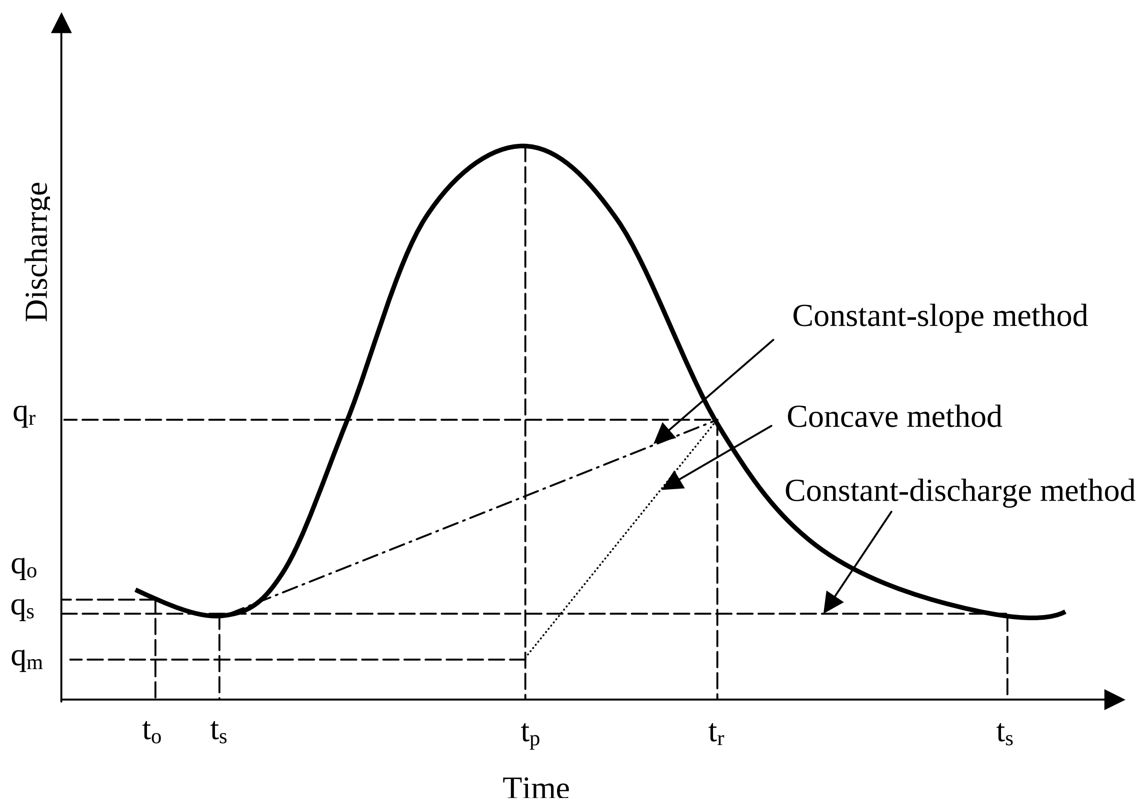

Figure 3.3. Base flow separation techniques (After McCuen, 1998)

The constant slope and concave methods (McCuen, 1998) are not used in this work because the observed runoff hydrographs have multiple peaks and it is impractical to locate the recession limb inflection point with any confidence. The master depletion curve method (McCuen, 1998) is not used because even though there is a large amount of data, there is insufficient data at each station to construct reliable depletion curves, and the time scale is inadequate. Therefore in the present work the discharge data are treated by the constant discharge method.

The constant discharge method was chosen because it is simple to automate and apply to multiple peaked hydrographs. Prior researchers (e.g. Laurenson and O'Donell, 1969; Bates and Davies, 1988) have reported that unit hydrograph derivation is insensitive to baseflow separation method when the baseflow is not a large fraction of the flood hydrograph – a situation satisfied in this work. The particular implementation in this research determined when the rainfall event began on a particular day, all discharge before that time was accumulated and converted into an average rate. This average rate was then removed from the observed discharge data, and the result was considered to be the direct runoff hydrograph.

3.3 RUNOFF ESTIMATION

One of the first concerns in the design of engineering projects is to not only to obtain the total volume of the runoff but also to know the peak flow and the flow regime of the river through out the proposed life of the project.

3.3.1 RATIONAL METHOD

Rational method is the one of the earliest methods used for determining flood peak discharges from measurements of rainfall depths. The concept of rational method owes its origin to T. J. Mulvaney (Chow, 1964). In the early 1900s empirical methods have been used in the estimation of hydrograph. All these method related rainfall intensity, duration and watershed characteristics to the runoff. The use of the rational method is predominant for its simplicity to use. According to rational method the formula to give the peak flow Q_p is,

$$Q_p = C \cdot i_{t_c} \cdot A \tag{1.1}$$

Where C is the coefficient of runoff (dependent on watershed characteristics), i is the intensity of rainfall in time t_c, time of concentration, which is the time required for rain falling from the farthest point of the watershed to flow to its outlet, and A is the area of the watershed. The value of i, the mean intensity, assumes that the rate of rainfall is constant during t_c, and that all the rainfall measured over the area contributes to the flow. The peak flow Q_p occurs after the period t_c. However when the rational method leads to over design, more precise methods need to be examined to optimize the designs and thereby reduce the construction costs.

3.4 UNIT HYDROGRAPH

A major step forward in hydrological analysis was the concept of the unit hydrograph introduced by Sherman (Sherman, 1932). It was for the first time that an entire hydrograph can be predicted instead of just the peak (Todini,1988).A unit hydrograph (UH) is defined as the runoff hydrograph that results from one unit of excess rainfall depth uniformly distributed over the entire watershed over one unit of time. In his paper Sherman illustrated a procedure to construct direct runoff hydrographs from a sequence of rainfall “units” by addition of ordinates of unit hydrographs lagged by the duration of the individual rainfall durations. For about 25 years unit hydrograph methods were used in applied hydrology without recognition of the essential assumption involved, namely that the relationship between rainfall excess and surface runoff was that of a linear time-invariant system (Note that unit hydrograph and unit graph are used interchangeably in this thesis).

3.4.1 UNIT HYDROGRAPH ASSUMPTIONS AND LIMITATIONS

The following are the assumptions that limit the application of the unit hydrograph.

a) For a given watershed, rainfall excesses of equal duration are assumed to produce hydrographs with equivalent time bases regardless of the intensity of the rain (system is time-invariant).

b) For a given watershed, surface runoff ordinates for a storm of given duration are assumed directly proportional to rainfall excess volumes. Thus, twice the rainfall produces a doubling of hydrograph ordinates (system is linear-superposition applies).

c) For a given watershed, the time distribution of direct runoff is assumed independent of antecedent precipitation.

d) For a given watershed, rainfall is assumed to be the same for all storms of equal duration, spatially and temporally (lumped behavior).

Based on the above assumptions the property of proportionality and the principle of superposition both apply to the unit hydrograph.

3.4.2 UNIT HYDROGRAPH-PROPOSITIONS

The three basic propositions of unit graph theory that refer to surface-runoff hydrograph stated by Johnstone and Cross (Dooge, 1973) are.

1. For a given drainage basin, the duration of surface runoff is essentially constant for all uniform-intensity storms of the same length, regardless of differences in the total volume of the surface runoff.
2. For a given drainage basin two uniform-intensity storms of the same length produce different total volumes of surface runoff, then the rates of surface runoff at corresponding times t, after the beginning of two storms, are the same proportion to each other as the total volume of the surface runoff.
3. The time distribution of surface runoff from a given storm period is independent of concurrent runoff from antecedent storm periods.

The classic statement of unit hydrograph theory quoted above can be summarized as, the system is linear and time-invariant. Proposition 1 and proposition 2 together make up the property of proportionality. If, the length of input remains constant but the volume of input increases, then the base length of the outflow is not altered, but the ordinates of the outflow are raised in proportion to the volume of input. Proposition 3 is the principle of superposition, which allows us to decompose the input into separate parts and then superimpose on one another the separate outputs to obtain the total output (Dooge, 1973).

The procedure to develop a unit hydrograph for a storm with a single peak is fairly simple. Direct runoff hydrograph can be obtained from the total runoff hydrograph by removing the base flow. The total runoff volume is determined by integrating the direct runoff hydrograph. In order to obtain the unit hydrograph, each ordinate of the direct runoff hydrograph is divided by the runoff volume. In theory, unit hydrographs developed from different storms should be same, however this is rarely the case in practice. An average response can be obtained by developing unit hydrographs from at least five storm events (Ponce, 1989). An average response may be determined by calculating the average peak flow rate and time to peak, then sketching a hydrograph shape such that it contains 1 unit of runoff, passes through the average peak, and has a shape similar to the unit hydrograph developed from the individual storm events (Linsley, 1975).

The unit hydrograph is assumed to be a constant response function of the watershed as long as there are no major changes in the land use. Conventional unit hydrograph models cannot explain differences in the watershed response due to seasonal condition. Some important features defining the applicability of the UH theory are,

Area: Though it is stated that unit hydrograph is only applicable for a limited range of watershed sizes (Chow, 1964), it is not clearly defined to what extent. Sherman (1932) used the unit hydrograph theory on watersheds ranging from 1300 km^2 to 8000 km^2. Brater in 1940 stated that unit hydrograph theory can be applied for watersheds ranging between 4 acres and 10 sq. mi. (Chow, 1964) showing that the UH theory is also applicable for small watersheds. Linsley (1975) recommended that the unit hydrograph can be used for watersheds with drainage area less than 5000 km^2.

Watershed Linearity: Due to the assumption of linearity, the unit hydrograph method is not applicable for watersheds that have appreciable storage effects (Gray, 1973). In addition, the unit hydrograph theory may not be applicable to small watersheds because they tend to exhibit a nonlinear response more than larger areas.

Superposition: The unit hydrograph theory is based on the idea of superposition. The hydrograph ordinates for a complex storm event are the sum of the ordinates of the incremental hydrographs that are developed for each period of rainfall excess.

3.4.3 UNIT HYDROGRAPH –CHRONOLOGY OF DEVELOPMENT

Sherman (1932) first proposed the unit hydrograph theory. Commons (1942) suggested that a dimensionless hydrograph, the so-called basic hydrograph, would give an acceptable approximation of the flood hydrograph on any Texas basin. This hydrograph was developed from flood hydrographs in Texas. It is divided so that the base time is expressed as 100 units, the peak discharge as 60 units and the area as a constant of 1,196.5 units. Snyder (1956) applied the method of least squares to compute a unit graph from observations of rainfall and runoff. He applied the method to ten storms from two different watersheds. Many research workers throughout the world have studies extensions of the unit hydrograph principles. One of the most searching and fundamental contributions was made by Dooge (Dooge, 1959). Concentrating on the Linear mechanisms, he suggested that the response of a watershed could be modeled by combining storage effects (Nash, 1959) with translation effects. Further simplifications of the Dooge approach using linear theory were also made by Diskin, who modeled the watershed response with two series of equal linear reservoirs in parallel in 1964 (Shaw, 1988). Singh developed another linear watershed model by routing the time-area curve through two different linear storages in series, and showed that in practice it is possible to use dimple geometrical forms in place of the real time-area curve (Chow, 1964). Kulandaiswamy produced a non-linear watershed response function using a non-linear storage expression and hence incorporated non-linear relationships that have long been recognized as being more realistic in the description of watershed behavior (Chow, 1964). Eagleson, et al.(1966) examined unit hydrograph extraction from measured rainfall-runoff series. Subsequent efforts by many other authors codified these ideas, and

UH theory today is essentially the application of linear-systems theory to the rainfall runoff process (Dooge, 1973; Chow, et al, 1988). Madsley and Tagg (1981) used the deconvolution method to derive unitgraphs from multiple events.

3.5 S-CURVE METHOD

When a unit hydrograph of given effective-rainfall duration is available, the unit hydrographs of other durations can be derived. If other durations are integral multiples of the given duration the resulting unit hydrograph can be easily computed by the application of the principle of superposition. However, to obtain a unit hydrograph of any required duration. from a given UH, S-curve method may be used. S-hydrograph or S-Curve is not a unit hydrograph but results form a rainfall intensity of $1/t_r$ where, t_r is the duration of UH. To obtain another UH of duration t_r', the s-curve has to be lagged by the duration t_r'. Subtract ordinates from original S-Curve. The resulting hydrograph has a duration t_r' and intensity $1/t_r$. The UH of required duration is obtained by multiplying the ordinates by (t_r/t_r'). The derivative of the S-Hydrograph is the instantaneous unit hydrograph (IUH).

3.6 INSTANTANEOUS UNIT HYDROGRAPH

The unit hydrograph from an effective precipitation of infinitesimally small duration is called an Instantaneous Unit Hydrograph (IUH). Generally the IUH is represented by u(t) in the literature. For an IUH the effective precipitation is applied to the drainage basin over a very short duration of time (impulse). The major advantage of the IUH over the unit hydrograph is that the IUH is independent of the duration of the effective rainfall reducing the number of variables in the hydrograph analysis. The idea of applying an IUH to derive a unit hydrograph was originally attributed to Clark in 1945 (Clark, 1945).

Nash in 1957, instead of characterizing the runoff as translation followed by storage in a single reservoir as Clark did, viewed the watershed as a series of n identical linear storage reservoirs. The convolution integral provides the hydrograph as a function of the precipitation input and the impulse response function (the IUH).

$$Q(t) = \int_0^{\tau} I(\tau)u(t-\tau)d\tau, \qquad (3.2)$$

where: $Q(t)$ = output time function,

$I(\tau)$ = input time function,

$u(t-\tau)$ = impulse response function

$(t-\tau)$ = time lag between time the impulse is applied and t, and

t = time.

In discrete time, the pulse response function is

$$Q_n = \sum_{m=1}^{n \le M} P_m U_{n-m+1} \qquad (3.3)$$

where: U_n = unit response function (unitgraph; L^2/T), and

P_m = effective precipitation (L) for period m.

If one has the functional form of the IUH, a DRH for any duration precipitation signal can be obtained by simply convolving the input signal with the IUH function. It is the analyst preference as S-hydrograph, time lagging, or the IUH to choice though all contain the same information about the rainfall-runoff process.

3.7 SYNTHETIC UNIT HYDROGRAPHS

Sherman's unit hydrograph is based on observed rainfall and runoff data, it is only applicable for gauged basins. Unfortunately, the majority of watersheds are ungauged.

Synthetic unit hydrographs can be applied for watersheds with no or with short hydrologic records based on the watershed characteristics.

According to Chow et al. (1988) three are three major types of synthetic unit hydrographs. They are:

1. Based on hydrograph characteristics such as peak discharge and time to peak (Snyder, 1938);
2. Based on a dimensionless unit hydrograph (SCS, 1972);
3. Based on watershed storage (Clark, 1945).

Snyder (1938) proposed the first unit hydrograph technique that was applicable to ungauged areas, based on a study of watersheds located in the Appalachian Mountains. In his approach, the time to peak is estimated from watershed length, the distance form the outlet to the watershed centroid, and a regional coefficient, while the predicted peak flow rate is calculated using the watershed area, the time to peak, and a storage flow rate is calculated using the watershed area, the time to peak, and a storage coefficient. In order to sketch the hydrograph shape, the hydrograph width is estimated at 50% and 75% of the peak discharge. The widths are generally distributed such that 1/3 is placed before the peak and 2/3 is placed after the peak.

The SCS unit hydrograph is based on a dimensionless hydrograph. the time to peak is estimated based on the duration of effective rainfall anf the lag time between the centroid of the excess rainfall and the time to peak. The lag time is calculated using the watershed length, the average slope, and a factor based on watershed storage. The peak flow rate is based on the watershed area and the time to peak. A triangle is commonly used to estimate the unit hydrograph shape.

Clark (1945) developed a unit hydrograph model that combined a watershed time-area diagram with a linear reservoir at the basin outlet. The shape of Clark's (1945) unit hydrograph is developed from the travel time through the basin, as well as the watershed shape and storage characteristics. In order to create a unit hydrograph, Clark divided the basin into isochrones, then developed a time-area histogram. The time- area curve was assumed to be the inflow into a hypothetical reservoir. The direct runoff hydrograph at the watershed outlet can be predicted by routing the inflow hydrograph through a reservoir that has the same storage characteristics as the watershed.

Regional regression equations are the most commonly used method for establishing peak flows at larger ungaged sites in Texas. Regression equations have been developed for Texas and were categorized according to the urban area for which they were developed. Regression equations were developed for three urban areas in Texas: Austin, Dallas-Fort Worth, and Houston. Also, statewide regional equations for rural watershed were developed.

3.8 TOPOGRAPHIC FACTOR AND THE HYDROGRAPH

The surface-runoff hydrograph for a watershed represents the integrated effect of all the basin physical characteristics and their modifying influence on the translation and storage of a rainfall-excess volume. Sherman suggested that the dominant factors having a major influence on the rainfall-runoff phenomena are drainage-area size and shape; distribution of the watercourses; slope of the valley sides or general land slope; slope of the main stream; and pondage resulting from surface or channel obstructions forming natural detention reservoirs (Gray, 1962). Model parameters can be related to the physical characteristics of the watershed like storage (Nash, 1959), drainage area (Gray, 1962) etc., and can be used to analyze the watersheds with no or short record of rainfall-runoff data (ungauged watersheds).

CHAPTER 4 : Database Construction

4.1 INTRODUCTION

The Texas district USGS conducted studies in Texas during the period between early 1960's to the middle 1970's. Unfortunately no data pertaining to unitgraph research from these studies was digitally available and the USGS reports were the sole data source. Most of these watersheds were unregulated (no dams). These data are critical for the unit hydrograph investigation in Texas. Pair-wise records of rainfall and direct runoff exist for each storm and often more than one precipitation gage exists in the watershed. Some of the runoff record documented in the reports often-included pre-computed direct runoff values.

It is desirable to derive the unit hydrographs from several floods for which hydrograph and recording gauge data are available as it represents the long-term variations. Figure 4.1 depicts the geographic region, for which rainfall-runoff data has been obtained from the USGS small watershed studies. In addition to the strictly rural data, the urban watershed records for Austin, Dallas, Fort Worth, and San Antonio were considered. A total of 1631 storm events are available from the USGS reports.

4.2 USGS Reports

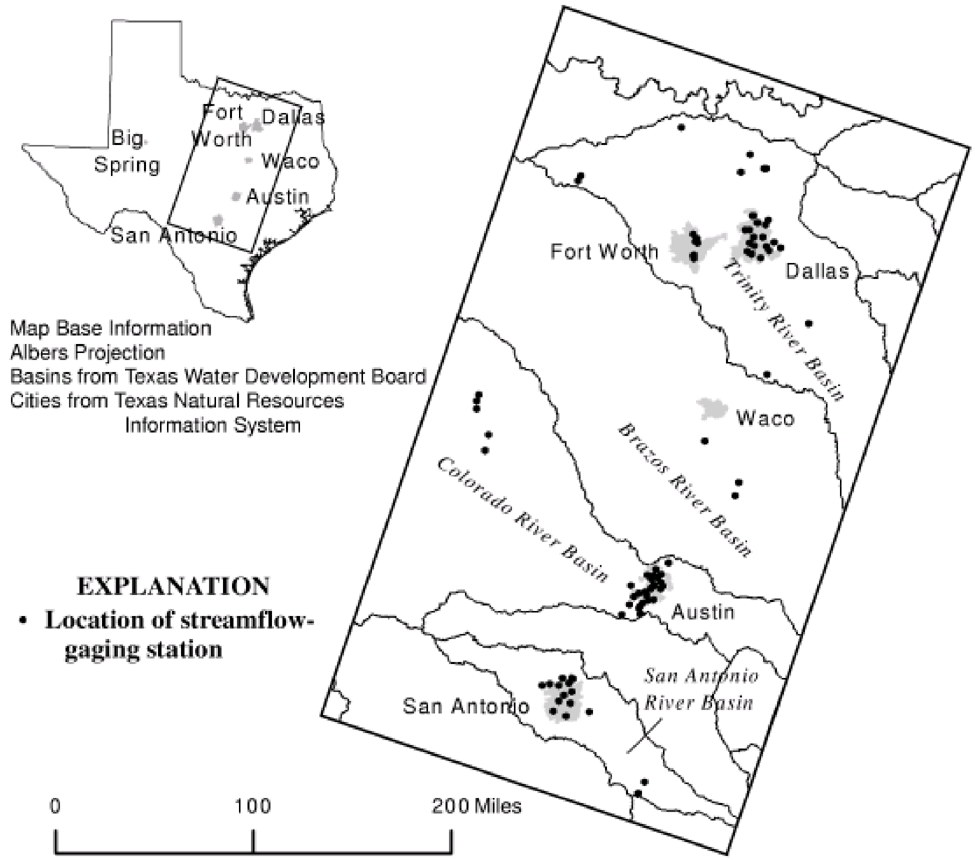

Figure 4.1. Study area with the gaging stations.
(Asquith, 2003)

The USGS reports contained pair-wise records of rainfall-runoff in two different forms:

1. Dot-matrix printout.

2. Handwritten format.

Figures 4.2 and 4.3 are typical examples of the two forms. The precipitation is obtained by appropriate aerial weighting of the precipitation values from network of gaging stations in each watershed.

Figure 4.2 is a typical dot matrix printout for a given storm event. In the dot matrix print out the time intervals for rainfall and runoff measurements varied from five minutes to one hour. The consecutive columns report precipitation depths from different rain gages, identified by a gage number with the respective rainfall measurements. Figure 4.3 is a typical hand handwritten format of rainfall runoff measurements. Each storm event had an observed hydrograph plotted with it in the report. Figure 4.4 shows such a hydrograph. In a typical hydrograph from the USGS report, the x-axis represents the time in hours and the primary y-axis represents the accumulated rainfall and runoff, in inches and the secondary y-axis represents the discharge in cfs (cubic feet per second).

The hydrologic data is organized into five different principal database modules, the names are selected to reflect geographic location of watershed type. The names of the five modules are:

1. Dallas Module
2. Fort Worth Module
3. San Antonio Module
4. Austin Module
5. Small Rural Sheds Module

The USGS reports for different modules were assigned to different participating institutions in the project. University of Houston keypunched all data in the Dallas module.

STA. NO. 08057130 STORM RAINFALL AND RUNOFF RECORD 1974 WATER YEAR

RUSH BRANCH AT ARAPAHO ROAD, DALLAS, TEX. STORM OF OCT. 30-31, 1973

DATE & TIME	GAGE NUMBER: 1-RH							ACCUM. WEIGHTED PRECIP. IN.	DISCHARGE IN FT³/S	ACCUM. RUNOFF IN.
OCT. 30										
1530	0.97							0.97	4.4	0.1136
1600	0.97							0.97	3.2	0.1151
1615	0.97							0.97	2.6	0.1160
1630	0.97							0.97	2.1	0.1170
1700	0.97							0.97	1.7	0.1180
1730	0.97							0.97	1.1	0.1185
1740	0.97							0.97	1.1	0.1187
1745	0.98							0.98	1.1	0.1188
1750	0.98							0.98	1.1	0.1190
1800	1.33							1.33	1.7	0.1192
1805	1.42							1.42	2.1	0.1195
1810	1.51							1.51	20.0	0.1216
1815	1.81							1.81	45.0	0.1263
1820	2.05							2.05	83.0	0.1351
1825	2.24							2.24	159.0	0.1520
1830	2.36							2.36	260.0	0.1795
1835	2.48							2.48	350.0	0.2165
1840	2.51							2.51	400.0	0.2589
1845	2.53							2.53	412.0	0.3025
1850	2.55							2.55	420.0	0.3469
1855	2.56							2.56	392.0	0.3884
1900	2.57							2.57	368.0	0.4274
1905	2.58							2.58	336.0	0.4629
1910	2.58							2.58	304.0	0.5112
1920	2.60							2.60	254.0	0.5650
1930	2.62							2.62	221.0	0.6001
1935	2.64							2.64	203.0	0.6215
1940	2.65							2.65	188.0	0.6514
1950	2.67							2.67	167.0	0.6868
2000	2.67							2.67	151.0	0.7267
2015	2.67							2.67	143.0	0.7721
2030	2.67							2.67	132.0	0.8140
2045	2.67							2.67	119.0	0.8518
2100	2.68							2.68	119.0	0.8896
2115	2.69							2.69	117.0	0.9206
2125	2.69							2.69	109.0	0.9436
2135	2.69							2.69	102.0	0.9652
2145	2.69							2.69	95.0	0.9904
2200	2.69							2.69	83.0	1.0187

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Figure 4.2: Dot matrix printout of measurements from a USGS watershed.

TX-65
(Rev.6-68)

UNITED STATES DEPARTMENT OF THE INTERIOR

GEOLOGICAL SURVEY-TEXAS DISTRICT

RUNOFF COMPUTATIONS

Station 8-1786.90 Salado Creek trib. at Bitters Road, San Antonio, Tex

Period of Record Jan. 16, 1969 Drainage Area 0.26 sq mi.

Time	G. Ht. Feet	Sh. Adj.	Discharge c.f.s.	Discharge Inc.	Discharge In/hr.	Runoff Inches	Runoff Acc. In.
Jan 16, 1969							
0000	—		0	1		0.0000	0.0000
0005	—		(5.0)	2	0.0298	.0015	.0025
0010	3.32	0	22	2	.1312	.0109	.0134
0015	3.57		32	2	.1908	.0159	.0293
0080	3.66		36	2	.2146	.0179	.0472
0025	3.51		29	2	.1729	.0144	.0616
0030	3.34		23	2	.1371	.0114	.0730
0035	3.21		17	2	.1014	.0084	.0814
0040	3.12		14	2	.0835	.0070	.0884
0045	3.05		12	2	.0715	.0060	.0944
0050	2.96	0	9.1	3	.0543	.0068	.1012
0100	—		(5.5)	5	.0328	.0068	.1080
0115	—		(2.6)	6	.0155	.0039	.1119
0130	—		(1.0)	9	.0060	.0022	.1141
0200	—		(0)	18	0	.0000	.1141
0300	—		(0)	24	0	.0000	.1141
2400	—		(0)	252	0	.0000	.1141
			459.4)	576			
			0.80				
Total	Runoff =		1.58	ac-ft			
( )	Estimated	data					

Computed by LEL Date 2/18/70 Checked by W.E.R. Date 02/19/70

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Figure 4.3. Handwritten runoff computations from a USGS watershed.

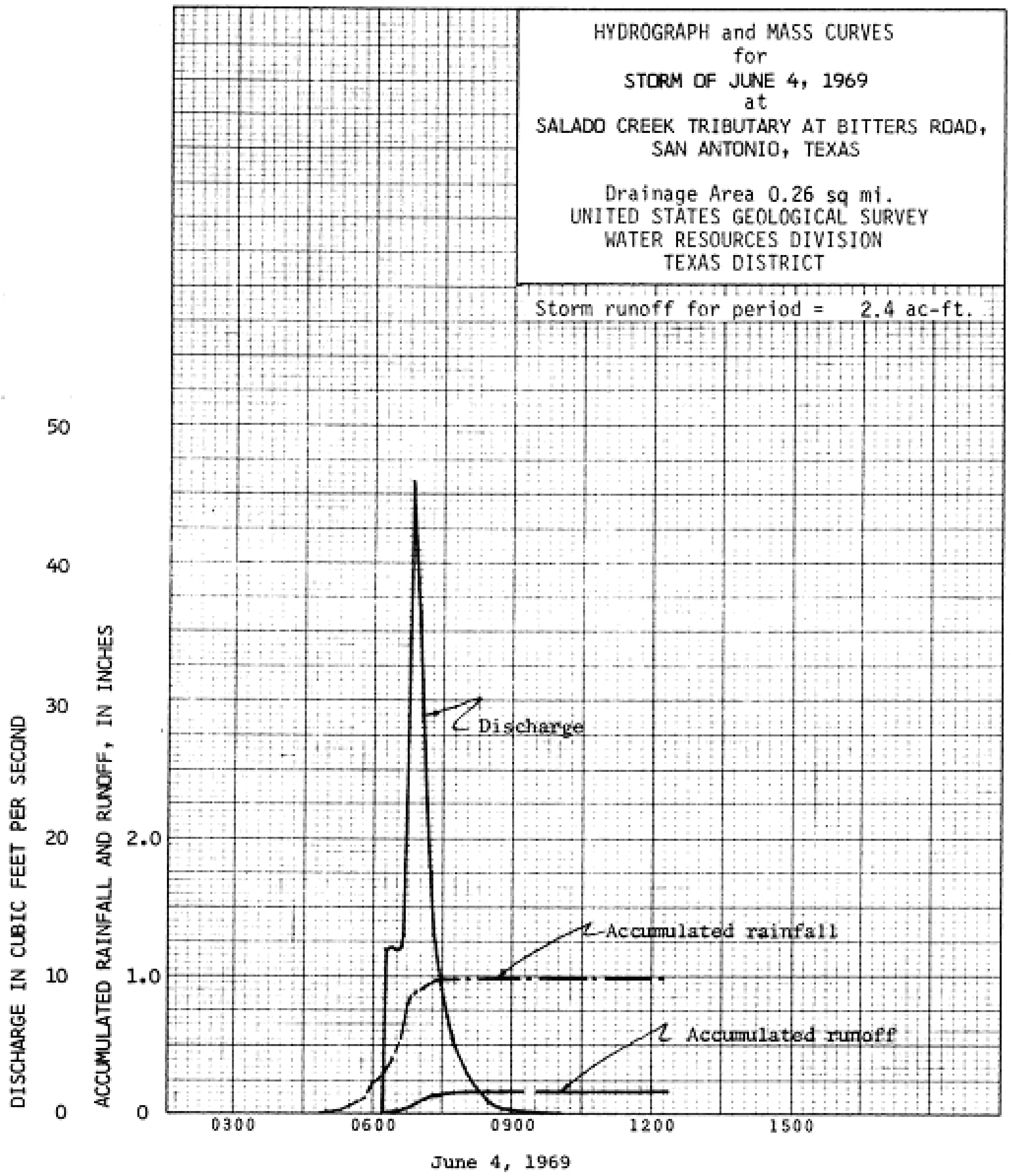

Figure 4.4. Hydrographs from a typical USGS report.

4.3 Database

The database that has been developed with all the recorded rainfall-runoff events consisted of two parts:

1. The first part of the database consisted of rainfall-runoff events and the corresponding model parameters for each rainfall runoff event.
2. The second part of the database consists of all the measured physical parameters of the watershed.

4.3.1 PART I. DATABASE WITH MODEL PARAMETERS

The rainfall-runoff data was keypunched into a standard format agreed upon by the different institutions involved in the work. All the data that has been entered is stored in a central repository at Texas Tech University and was copied onto a federal repository operated by the USGS. During the process, each data set was entered and checked by at least two different persons. The database was tested and updated after performing checksums and correcting errors, the database was declared correct in late July 2002. The resulting database has about 1631 storms over the entire set of gaging stations with a minimum of two storms to over 30 storms at each station that is considered. Table 4.1 is a list of data in the different modules, the number of stations, and the total number of storms associated with each module.

The model parameters for each rainfall-runoff event were calculated and stored in the database. Also the database contained the modeled hydrographs plotted with the observed hyetographs and hydrographs for each storm event. The database of model parameters led to further analysis. Part II is the outcome of the extended analysis of the database of Part I, development of a database with the governing parameters of the model.

Table 4.1. Summary of number of watersheds and storms used in the study

Module	Number of watersheds	Number of subwatersheds	Number of stations	Number of storm events
Dallas	18	---	21	243
Fort Worth	3	---	8	193
San Antonio	4	---	12	208
Austin	14	---	29	392
Small Rural Sheds	4	12	20	595
Total	43	12	90	1631

4.3.2 PART-II. MEASURED WATERSHED CHARACTERISTICS DATABASE

Part II of the database construction was to develop a database with all the measured watershed characteristics.

The following is the list of the watershed parameters that have been measured from the topographic maps.

- Drainage area (A) of the watershed, measured directly from the topographic maps with a planimeter. It is expressed in square miles.
- Perimeter (P) of the watershed.
- Maximum distance (Dm) represents the distance from the outlet to the farthest point on the watershed.

- Stream length (S_L). The length of the main stream is measured with a map measurer along the main stream as shown on the topographic maps.
- Aspect ratio (f). Differences in shape of the watershed will result in different time of concentration and hence will affect the shape of the hydrograph. It is equal with ratio between dimensions on NE direction and EW direction of the watershed.
- Difference in elevation (H) is the difference between the highest and the lowest elevation of the watershed points.

The gaging stations were located on the USGS quadrangle maps, in accordance with their latitude and longitude given in the USGS reports. For each gaging station located on the topographic maps, the watershed is delineated. The area of the delineated watershed is measured using a planimeter and compared with the area of the respective watershed in the USGS reports. The process of delineation and the area measurement were repeated till the measured area was comparable to the respective area from the reports with a tolerance of ± 1%. Once the area is of the watershed is determined, other parameters (perimeter, elevation etc.,) of the watershed are measured. This process is repeated for every gaging station in the database from Part I. There were 76 gaging staions for which the physical characteristics were measured.

All the measurements were entered into a database. The database is divided into five modules as in Part I, and the measurements were reported in the form of tables.

Tables 4.2 to 4.6 list the median values of the hydrograph parameters, lag time and residence time, for each of the five modules.

Tables 4.7 to 4.11 show the geometric characteristics of the watersheds measured on the topographic maps, for the specified modules.

Table 4.2. Instantaneous Unit Hydrograph (IUH) parameters for Dallas module

Watershed number	**Station_ID**	**Lag time median t_lag (min)**	**Residence time median t_mean (min)**
1	sta08057320_d	37.0	37.0
2	sta08055700_d	54.0	54.5
3	sta08057050_d	33.0	14.0
4	sta08057020_d	27.0	36.0
5	sta08057140_d	26.5	51.0
6	sta08061620_d	30.0	90.0
7	sta08057415_d	9.5	17.5
8	sta08057418_d	24.0	54.0
9	sta08057420_d	46.0	58.5
10	sta08057160_d	36.5	37.5
11	sta08055580_d	7.0	25.0
12	sta08057445_d	98.5	185.5
13	sta08057130_d	19.0	40.0
14	sta08057120_d	45.0	70.0
15	sta08056500_d	27.0	43.0
16	sta08057425_d	30.5	42.5

Table 4.3. Instantaneous Unit Hydrograph (IUH) parameters for Fort Worth module

Watershed number	**Station_ID**	**Lag time median t_lag (min)**	**Residence time median t_mean (min)**
17	sta8048550_d	26.0	44.0
18	sta8048600_d	17.0	121.0
19	sta8048820_d	70.0	234.5
20	sta8048850_d	67.0	223.0
21	sta8048520_d	43.0	158.0
22	sta8048530_d	6.0	16.5
23	sta8048530_d	7.5	15.0
24	StaSSSC_d	5.0	10.0

Table 4.4. Instantaneous Unit Hydrograph (IUH) parameters for San Antonio module

Watershed number	Station_ID	Lag time median t_lag (min)	Residence time median t_mean (min)
25	sta8178300_d	14.0	21.5
26	sta8181000_d	42.0	45.0
27	sta8181400_d	106.0	171.0
28	sta8181450_d	58.0	64.0
29	sta8177600_d	34.0	40.0
30	sta8178555_d	74.0	92.5
31	sta8178600_d	103.0	38.0
32	sta8178620_d	188.0	53.0
33	sta8178640_d	82.0	14.0
34	sta8178645_d	129.0	55.0
35	sta8178690_d	4.0	23.0
36	sta8178736_d	6.5	39.0

Table 4.5. Instantaneous Unit Hydrograph (IUH) parameters for Austin module

Watershed number	Station_ID	Lag time median t_lag (min)	Residence time median t_mean (min)
37	sta8155200_d	228.5	356.0
38	sta8155300_d	342.5	613.5
39	sta8158810_d	75.0	92.0
40	sta8158820_d	378.0	624.5
41	sta8158825_d	97.5	25.5
42	sta8158050_d	67.5	85.5
43	sta8158880_d	23.0	62.0
44	sta8154700_d	49.0	101.0
45	sta8158380_d	39.0	45.5
46	sta8158700_d	192.0	594.5
47	sta8158800_d	339.0	977.5
48	sta8156650_d	11.0	64.0
49	sta8156700_d	16.0	46.0
50	sta8156750_d	19.0	57.0
51	sta8156800_d	56.0	75.0
52	sta8158840_d	39.0	112.0
53	sta8158860_d	156.5	87.0
54	sta8157000_d	24.0	48.0
55	sta8157500_d	11.0	44.0
56	sta8158100_d	67.0	131.0
57	sta8158200_d	51.0	112.0
58	sta8158400_d	18.5	49.5
59	sta8158500_d	30.0	97.0
60	sta8158600_d	99.0	166.0
61	sta8155550_d	22.0	70.0
62	sta8159150_d	92.0	96.0
63	sta8158920_d	20.0	97.0
64	sta8158930_d	62.5	123.0
65	sta8158970_d	152.5	190.0

Table 4.6. Instantaneous Unit Hydrograph (IUH) parameters for

Small Rural Sheds module

Watershed number	**Station_ID**	**Lag time median t_lag (min)**	**Residence time median t_mean (min)**
66	sta8182400_d	101.0	131.0
67	sta8187000_d	45.0	67.0
68	sta8187900_d	41.0	129.0
69	sta8050200_d	31.0	86.5
70	sta8057500_d	14.0	78.0
71	sta8058000_d	30.5	43.0
72	sta8052630_d	53.0	137.0
73	sta8052700_d	339.5	926.5
74	sta8042650_d	68.5	92.0
75	sta8042700_d	105.0	193.0
76	sta8063200_d	144.0	373.0

Table 4.7. Watershed characteristics for Dallas module

Watershed Number	Area A	Perimeter P		Maximum distance Dm		Stream length S_L		Aspect ratio f	Difference in elevation H	
	(sq mile)	(ft)	(mile)	(ft)	(mile)	(ft)	(mile)		(ft)	(mile)
1	6.92	58200.0	11.0	20190.0	3.8	17400.0	3.3	1.1748	160	0.0303
2	10.00	89700.0	17.0	34764.0	6.6	19800.0	3.7	2.2364	110	0.0208
3	9.42	71400.0	13.5	25561.8	4.8	24000.0	4.5	1.5534	220	0.0417
4	4.75	59400.0	11.2	25561.8	4.8	21600.0	4.1	1.2381	220	0.0417
5	8.50	75000.0	14.2	32876.0	6.2	35400.0	6.7	2.2951	240	0.0455
6	8.05	68400.0	13.0	23595.5	4.5	22500.0	4.3	1.1674	110	0.0208
7	1.25	25800.0	4.9	10617.9	2.0	4800.0	0.9	1.3483	80	0.0152
8	7.65	66000.0	12.5	23202.0	4.4	24600.0	4.7	1.0757	290	0.0549
9	13.20	94200.0	17.8	32247.1	6.1	39000.0	7.4	1.5285	280	0.0530
10	4.17	58800.0	11.1	23516.0	4.5	18600.0	3.5	1.7898	140	0.0265
11	7.51	57600.0	10.9	24775.0	4.7	18600.0	3.5	1.6570	205	0.0388
12	9.03	75600.0	14.3	33191.0	6.3	31800.0	6.0	2.5389	180	0.0341
13	1.22	26400.0	5.0	11089.0	2.1	14157.3	2.7	1.0588	120	0.0227
14	6.77	67640.4	12.8	23516.9	4.5	22200.0	4.2	1.4817	220	0.0417
15	7.98	77400.0	14.7	29730.0	5.6	22800.0	4.3	1.4764	190	0.0360
16	11.50	86400.0	16.4	27528.0	5.2	25200.0	4.8	1.5079	305	0.0578

Table 4.8. Watershed characteristics for Fort Worth module

Watershed Number	**Area A**	**Perimeter P**		**Maximum distance Dm**		**Stream length S_L**		**Aspect ratio f**	**Difference in elevation H**	
	(sq mile)	(ft)	(mile)	(ft)	(mile)	(ft)	(mile)		(ft)	(mile)
17	1.08	21000.0	4.0	7400.0	1.4	7100.0	1.3	1.1176	40	0.008
18	2.15	34500.0	6.5	14100.0	2.7	14800.0	2.8	1.2157	70	0.013
19	5.64	60500.0	11.5	26580.0	5.0	27750.0	5.3	1.0439	170	0.032
20	12.3	98200.0	18.6	41500.0	7.9	44000.0	8.3	0.9023	260	0.049
21	17.7	87600.0	16.6	29000.0	5.5	39000.0	7.4	1.1250	165	0.031
22	0.97	18850.0	3.6	6900.0	1.3	4500.0	0.9	0.8358	110	0.021
23	1.35	24750.0	4.7	9700.0	1.8	7500.0	1.4	0.6649	130	0.025
24	0.38	17650.0	3.3	8300.0	1.6	5500.0	1.0	0.6533	140	0.027

Table 4.9. Watershed characteristics for San Antonio module

Watershed Number	**Area A**	**Perimeter P**		**Maximum distance Dm**		**Stream length S_L**		**Aspect ratio f**	**Difference in elevation H**	
	(sq mile)	(ft)	(mile)	(ft)	(mile)	(ft)	(mile)		(ft)	(mile)
25	3.26	42000	8.0	18200	3.4	17750.0	3.4	1.7979	300	0.0568
26	5.57	56000	10.6	20900	4.0	25250.0	4.8	1.5075	460	0.0871
27	15.00	118750	22.5	34150	6.5	45800.0	8.7	0.9828	590	0.1117
28	1.19	27150	5.1	6250	1.2	6250.0	1.2	0.4955	20	0.0038
29	0.33	13650	2.6	6100	1.2	3900.0	0.7	1.8226	110	0.0208
30	2.43	41250	7.8	17750	3.4	14250.0	2.7	0.8963	55	0.0104
31	9.54	80500	15.2	29100	5.5	34800.0	6.6	1.1828	515	0.0975
32	4.05	44400	8.4	15400	2.9	17200.0	3.3	1.1197	230	0.0436
33	2.45	36700	7.0	13900	2.6	13500.0	2.6	1.6163	330	0.0625
34	2.33	39750	7.5	17250	3.3	17000.0	3.2	2.2143	315	0.0597
35	0.26	13000	2.5	5500	1.0	1000.0	0.2	0.4273	22	0.0042
36	0.45	14250	2.7	4100	0.8	4075.0	0.8	0.7917	65	0.0123

Table 4.10. Watershed characteristics for Austin module

Watershed Number	**Area A**	**Perimeter P**		**Maximum distance Dm**		**Stream length S_L**		**Aspect ratio f**	**Difference in elevation H**	
	(sq mile)	(ft)	(mile)	(ft)	(mile)	(ft)	(mile)		(ft)	(mile)
37	89.70	516964.8	97.9	88809.6	16.8	296313.6	56.1	0.5755	723	0.1369
38	116.00	601656.0	113.9	94382.0	17.9	379473.6	71.9	1.3557	953	0.1805
39	12.2	90129.6	17.1	23548.8	4.5	22915.2	4.3	1.2170	324	0.0614
40	24.00	150585.6	28.5	51849.6	9.8	65313.6	12.4	0.5421	574	0.1087
41	21.00	112675.2	21.3	39388.8	7.5	60720.0	11.5	0.5626	400	0.0758
42	13.10	79411.2	15.0	29620.8	5.6	36590.4	6.9	1.7672	264	0.0500
43	3.58	53275.2	10.1	18585.6	3.5	18216.0	3.4	0.5514	198	0.0375
44	22.30	115051.2	21.8	33264.0	6.3	40972.8	7.8	0.9392	490	0.0928
45	5.22	45988.8	8.7	16368.0	3.1	14361.6	2.7	1.3306	140	0.0265
46	124.00	310939.2	58.9	92505.6	17.5	184430.4	34.9	0.5688	760	0.1439
47	166.00	425198.4	80.5	135854.4	25.7	255340.8	48.4	1.6846	930	0.1761
48	2.79	38808.0	7.3	12038.4	2.3	10612.8	2.0	1.2194	190	0.0360
49	7.03	59875.2	11.3	19219.2	3.6	18110.4	3.4	1.3584	230	0.0436
50	7.56	64996.8	12.3	21964.8	4.2	21278.4	4.0	1.4552	250	0.0473
51	12.30	106392.0	20.1	44193.6	8.4	46516.8	8.8	2.5213	410	0.0777
52	8.24	61881.6	11.7	21278.4	4.0	19694.4	3.7	0.7201	231	0.0437
53	23.10	136382.4	25.8	49104.0	9.3	65419.2	12.4	0.7681	531	0.1006
54	2.31	41395.2	7.8	18585.6	3.5	15787.2	3.0	2.2208	200	0.0379
55	4.13	50846.4	9.6	22228.8	4.2	19694.4	3.7	1.8447	220	0.0417

Table 4.10. Watershed characteristics for Austin module (contd.)										
Watershed Number	**Area A**	**Perimeter P**		**Maximum distance Dm**		**Stream length S_L**		**Aspect ratio f**	**Difference in elevation H**	
	(sq mile)	(ft)	(mile)	(ft)	(mile)	(ft)	(mile)		(ft)	(mile)
56	12.60	78988.8	15.0	20116.8	3.8	21278.4	4.0	0.8265	240	0.0455
57	26.20	110721.6	21.0	39969.6	7.6	47678.4	9.0	1.0216	330	0.0625
58	5.57	49051.2	9.3	18427.2	3.5	16737.6	3.2	1.4132	160	0.0303
59	12.10	90235.2	17.1	35164.8	6.7	28934.4	5.5	1.8144	290	0.0549
60	51.30	189921.6	36.0	67056.0	12.7	87489.6	16.6	1.3398	480	0.0909
61	3.12	40128.0	7.6	17582.4	3.3	14414.4	2.7	1.2231	229	0.0434
62	4.61	40180.8	7.6	15998.4	3.0	17740.8	3.4	0.9930	180	0.0341
63	6.30	55176.0	10.4	20644.8	3.9	23812.8	4.5	0.5515	276	0.0523
64	19.00	107606.4	20.4	42504.0	8.0	50001.6	9.5	0.5636	426	0.0807
65	27.60	170385.6	32.3	64944.0	12.3	91608.0	17.3	0.4858	600	0.1136

Table 4.11. Watershed characteristics for Small Rural Sheds module

Watershed Number	**Area A**	**Perimeter P**		**Maximum distance Dm**		**Stream length S_L**		**Aspect ratio f**	**Difference in elevation H**	
	(sq mile)	(ft)	(mile)	(ft)	(mile)	(ft)	(mile)		(ft)	(mile)
66	7.01	65102.4	12.3	19588.8	3.7	20856	3.9	0.9818	160	0.0303
67	3.29	39811.2	7.5	12619.2	2.4	13411.2	2.5	1.4278	165	0.0312
68	8.43	67003.2	12.7	20908.8	4.0	24024	4.5	0.8312	140	0.0265
69	0.77	21489.6	4.1	12196.8	2.3	9081.6	1.7	2.4409	140	0.0265
70	2.14	35059.2	6.6	9715.2	1.8	9873.6	1.9	1.4737	112	0.0212
71	1.26	25027.2	4.7	8923.2	1.7	9926.4	1.9	1.8713	110	0.0208
72	2.1	36273.6	6.9	13886.4	2.6	15787.2	3.0	1.6711	110	0.0208
73	75.5	232848	44.1	102432	19.4	124925	23.7	1.5701	301	0.0570
74	6.82	22440	4.2	19324.8	3.7	22862.4	4.3	1.2688	320	0.0606
75	21.6	106392	20.1	36801.6	7.0	67161.6	12.7	0.8153	350	0.0663
76	17.6	100690	19.1	32841.6	6.2	42926.4	8.1	0.9529	190	0.0360

CHAPTER 5 : Method of Analysis

5.1. INTRODUCTION

Unit hydrographs are developed for a specific watershed using two basic approaches. If unit rainfall-runoff data are available, then numerous techniques can be applied to estimate a unit hydrograph from the measurements. If no data are available, then methods of synthetic hydrology must be applied. For the purposes of this thesis, attention will be focused on examination of methods for developing a unit hydrograph from measurements of rainfall and runoff.

There are various mathematical methods for the determination of an Unit Hydrograph (UH) from the given Effective Rainfall Hyetograph (ERH) and Direct Runoff Hydrograph (DRH). The methods vary widely, from common and easy to understand as least-squares fitting method to complex methods as harmonic analysis or nonlinear programming.

Three different methods have been used for the present project:

- Least squares fitting method (applied by Texas Tech University)
- Linear programming method (applied by Lamar University)
- Instantaneous unit hydrograph (applied for this thesis).

Deconvolution, the inverse process of convolution, is the process of extracting the unit response function (unit hydrograph) from a direct runoff hydrograph and the generating precipitation sequence. The process is to solve Equation (5.1) for each U_n sequentially in a process that amounts to the back-substitution solution of a matrix equation. This procedure is documented in Chapter 7 of Chow, *et al* (1988).

$$Q_n = \sum_{m=1}^{n \le M} P_m U_{n-m+1} \tag{5.1}$$

where: Q_n = pulse response function (L^3/T)

U_n = unit response function (unitgraph; L^2/T)

P_m = effective precipitation (L)

$n = 1,2, \ldots, N$ = direct runoff hydrograph time interval

$m = 1,2, \ldots, n$ = precipitation time interval

Suppose there are M pulses of excess rainfall and N pulses of direct runoff in the storm considered; then N equations can be written for Q_n, $n = 1,2, \ldots, N$, in terms of $N-M+1$ unknown values of the unit hydrograph.

Once the unit hydrograph has been determined by solving the system of equations, it may be applied to find the direct runoff and streamflow hydrographs for given storm inputs.

5.2. INSTANTANEOUS UNIT HYDROGRAPH

A unit hydrograph is the runoff hydrograph that results from one unit of excess rainfall depth uniformly distributed over the entire watershed over one unit of time. An instantaneous unit hydrograph (IUH) is the unit hydrograph produced when the excess rainfall is applied over a very short time period. The development of an IUH requires assumptions about how the watershed converts rainfall into runoff (a transfer function) - here a simplified conceptual model based on a series of connected reservoirs was used to simulate how a watershed converts rainfall into runoff. The use of a cascade of reservoirs is a well-studied conceptual model that has been used for many unit hydrograph analyses (e.g. Nash, 1958; Dooge, 1959; Dooge, 1973; Croley, 1980).

5.3. METHODOLOGY

A schematic of a watershed conceptualized as a series of identical reservoirs without feedback is presented in Figure 5.1:

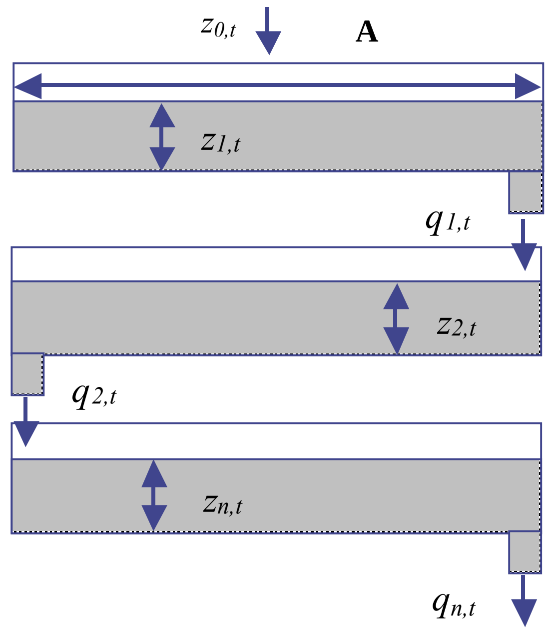

Figure 5.1. Watershed Conceptual Model

The first reservoir receives the initial charge of water, z_0 over a very short time interval.

The outflow of each reservoir is assumed to be proportional to the accumulated storage in the reservoir:

$$A\, q_i(t) = \alpha\, z_i(t) \tag{5.2}$$

α = reservoir discharge characteristic; incorporates properties of flow resistance and storage

z_i = accumulated storage depth (depth of water in reservoir at any instant time)

q_i = outflow for a particular reservoir

A = watershed area

Single-Reservoir Model

The simplest system one might envision is to represent the entire watershed as a single reservoir with rainfall input and discharge output. The goal is to relate the input hyetograph to the output hydrograph.

A volume balance equation over this model is:

$$A\frac{dz(t)}{dt} = A\,z_0(t) - \alpha\,z(t) \tag{5.3}$$

Dividing both sides by the watershed area $\rightarrow$ $\frac{dz(t)}{dt} = z_0(t) - \frac{\alpha}{A}z(t) = z_0(t) - \frac{1}{\bar{t}}z(t)$

The parameter $\bar{t} = \frac{A}{\alpha} =$ mean residence time of rainfall in the watershed has dimensions of time as required by the volume balance.

For a pulse input, rate of change in z_0, z_0 (t) =0.

$$\Rightarrow \frac{dz(t)}{dt} = -\frac{1}{\bar{t}}z(t) \;\Rightarrow\; \frac{1}{z(t)}dz = -\frac{1}{\bar{t}}dt\,.$$

Integrating, we have $\int_{z_o}^{z(t)} \frac{1}{z(t)} = -\int_0^t \frac{1}{\bar{t}}dt$, ($z(t) = z_o$ for $t = 0$)

$$\Rightarrow\; z(t) = z_o \exp(-\frac{t}{\bar{t}}) \tag{5.4}$$

Dividing both sides of equation (5.4) by $\bar{t}$, we have $q(t) = \frac{z_0}{\bar{t}}\exp(\frac{-t}{\bar{t}})$

The discharge Q(t) for the outflow hydrograph is given by:

$$Q(t) = Aq(t) = Az_o\frac{1}{\bar{t}}\exp(-\frac{t}{\bar{t}}) \tag{5.5}$$

Equation (5.5) represents an instantaneous unit hydrograph (IUH).

By definition, a unit hydrograph is a linear and time-invariant system response function. Thus we need to check if the volume balance and the linearity are conserved.

- *Check volume balance*

$$\int_{-\infty}^{\infty} Q(t)dt = \int_{-\infty}^{\infty} Az_0 \frac{1}{\bar{t}} \exp(\frac{-t}{\bar{t}})dt = Az_0 \frac{1}{\bar{t}} \int_{-\infty}^{\infty} \exp(\frac{-t}{\bar{t}})dt = Az_0 \frac{1}{\bar{t}} \bar{t} = Az_0$$

$\Rightarrow$ volume balance is conserved

- *Linearity* can be checked by determining if two rainfall charges applied simultaneously produce the same result as a single charge with a magnitude equal with the sum of two rainfall magnitudes.

$$f(z_0) + f(z_1) = Az_0 \frac{1}{\bar{t}} \exp(\frac{-t}{\bar{t}}) + Az_1 \frac{1}{\bar{t}} \exp(\frac{-t}{\bar{t}}) = A(z_0 + z_1) \frac{1}{\bar{t}} \exp(\frac{-t}{\bar{t}}) = f(z_0 + z_1)$$

$\Rightarrow$ superposition applied and linearity is preserved.

<u>*Continuous input at constant rate*</u>

Effect of continuous rainfall at constant rate is examined next. These continuous input functions can be used to extend the time base for practical application.

Let rainfall rate be z(t) so that z_0 = z(t)dt' $\Rightarrow$ $Q(t,t') = Az(t)\frac{1}{\bar{t}}\exp(\frac{-(t-t')}{\bar{t}})dt'$ and $\tau = t - t'$, $-\mathrm{d}\tau = \mathrm{dt'}$.

By convolution, we have:

$$Q(t) = \int_t^0 -Az(t)\frac{1}{\bar{t}}\exp(\frac{-\tau}{\bar{t}})d\tau = Az(t)\int_t^0 -\frac{1}{\bar{t}}\exp(\frac{-\tau}{\bar{t}})d\tau = Az(t)\left[\exp(\frac{-\tau}{\bar{t}})\right]_t^0 = Az(t)\left[1-\exp(\frac{-t}{\bar{t}})\right]$$

(5.6)

Finite length rainfall events (duration = t_{lag}) are modeled by convolution using equation (5.6) with a lag-time to represent the cessation of rainfall.

$$Q(t) = Az(t)[\{1-\exp(\frac{-t}{\bar{t}})\} - \{1-\exp(\frac{t_{lag}-t}{\bar{t}})\}] \quad (5.7)$$

Figure 5.2 is a plot of a single reservoir response, for a unit input and unit area. In figure 5.2 the instantaneous unit hydrograph produces runoff immediately upon application of rainfall. This behavior is contrary to practical experience but the unit hydrographs are only an approximation of reality. When we examine the continuous rainfall hydrograph the graph makes sense. Rainfall starts at time zero and there is no runoff, as rainfall continues runoff begins and the discharge increases over time until it reaches an asymptotic value. At the asymptote, the discharge should equal the volumetric input (product of area and rainfall rate). The elapsed time required to reach the asymptotic value is related to the residence time. The finite duration graph also makes sense and looks somewhat like the storm hydrograph with a rising limb, peak, and falling limb.

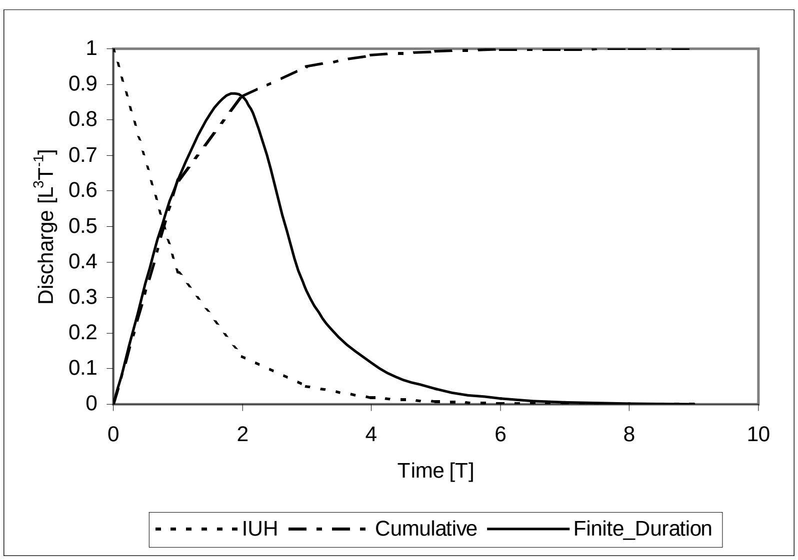

Figure 5.2. Instantaneous Unit Hydrograph (IUH), Continuous Rainfall Hydrograph and Finite-Duration Rainfall Hydrograph for a single reservoir case.

Two-Reservoir Model

In the two-reservoir model has been assumed that each reservoir has the same area and same residence time. The accumulated depth cascades through the system until the “last” reservoir where the observed discharge occurs.

Important characteristic of the cascade conceptual model is that rainfall applies only to the top reservoir.

Volume balance equations:

$$\frac{dz_1(t)}{dt} = z_0(t) - \frac{1}{\bar{\mathrm{t}}} z_1(t) \tag{5.8}$$

$$\frac{dz_2(t)}{dt} = \frac{1}{\bar{t}} z_1(t) - \frac{1}{\bar{\mathrm{t}}} z_2(t) \tag{5.9}$$

From (5.8) $\Rightarrow$ $z_1(t) = z_o \exp(-\frac{t}{\bar{t}})$

Substituting in (5.9) $\Rightarrow$ $\dfrac{dz_2(t)}{dt} + \dfrac{1}{\bar{t}} z_2(t) = z_0 \dfrac{1}{\bar{\mathrm{t}}} \exp\left(\dfrac{-t}{\bar{t}}\right)$

The solution can be found by applying the general solution to a linear ODE:

$$z_2(t) = \exp\left(-\int \frac{1}{\bar{t}} dt\right)\left[\int \exp(\int \frac{1}{\bar{t}} dt) \frac{z_0}{\bar{t}} \exp\left(\frac{-t}{\bar{t}}\right) dt + C\right]$$

Observe that this solution contains the solution to a single-reservoir case. For any cascade system with the assumptions used in this development, the solution to N-1 case will be part of the N-th reservoir solution.

But @ time t=0, z_2(t)=0 $\Rightarrow$ $z_2(t) = z_o \dfrac{t}{\bar{t}} \exp(-\dfrac{t}{\bar{t}})$ $\Rightarrow$ $q_2(t) = \dfrac{1}{\bar{t}} \dfrac{z_0 t}{\bar{t}} \exp(\dfrac{-t}{\bar{t}})$

The discharge Q(t) for the outflow hydrograph is given by:

$$Q_2(t) = A q_2(t) = \frac{A}{\bar{t}} z_o t \frac{1}{\bar{t}} \exp(-\frac{t}{\bar{t}}) \tag{5.10}$$

Equation (5.10) represents an instantaneous unit hydrograph (IUH) for two-reservoir case. If one integrates this solution for all time (use integration by parts and apply l'Hopital's rule), one will conclude that the function is indeed a unit hydrograph (volume is preserved). The function also has the required linearity property.

Corresponding continuous rainfall discharge hydrograph is given by:

$$Q(t) = Az(t)[1 - \frac{t}{\bar{t}}\exp(-\frac{t}{\bar{t}}) - \exp(-\frac{t}{\bar{t}})] \qquad (5.11)$$

Finite length rainfall events (duration=t_{lag}) are modeled by convolution using equation (5.11) with a lag-time to represent the cessation of rainfall.

$$Q(t) = Az(t)[\{1 - \frac{t}{\bar{t}}\exp(-\frac{t}{\bar{t}}) - \exp(-\frac{t}{\bar{t}})\} - \{1 - \frac{t}{\bar{t}}\exp(\frac{t_{lag} - t}{\bar{t}}) - \exp(\frac{t_{lag} - t}{\bar{t}})\}] \qquad (5.12)$$

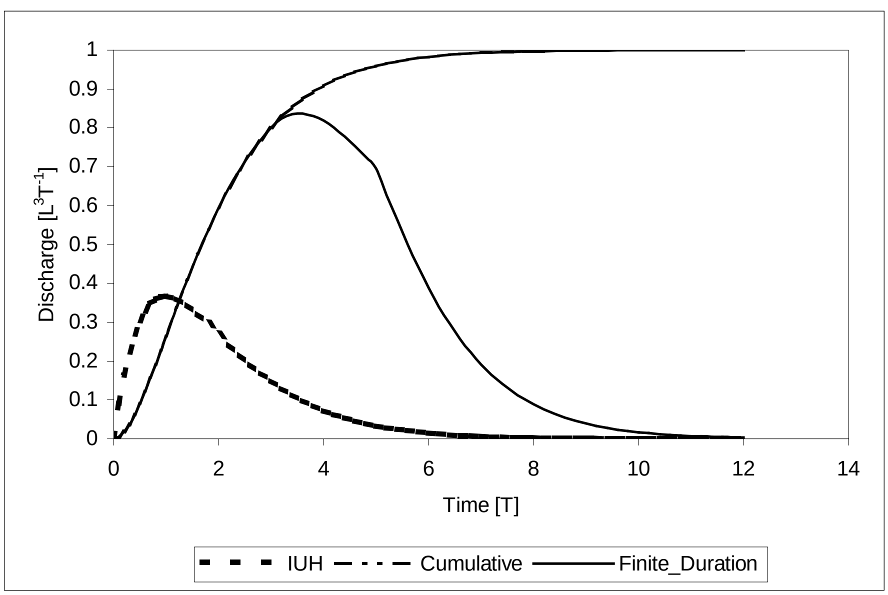

Figure 5.3. Instantaneous Unit Hydrograph (IUH), Continuous Rainfall Hydrograph and Finite-Duration Rainfall Hydrograph for a two-reservoir case.

N- Reservoir Model

By mathematical induction (Swokowski, 1979) the IUH for an N-reservoir cascade system is:

$$Q_N(t) = \frac{A}{t} z_o t^{N-1} \frac{1}{(N-1)!\bar{t}^{N-1}} \exp(-\frac{t}{\bar{t}}) \qquad (5.13)$$

The factorial can be replaced by the Gamma function and the result can be extended to a conceptual model with a non-integer number of reservoirs. To model a time-series of precipitation inputs, the individual responses are convolved and the result of the convolution is the output from the watershed. If each input is represented by the product of a rate and time interval ($z_o(t) = q_o(t)\, dt$) then the individual response is (note the Gamma function is substituted for the factorial)

$$dq_{i,\tau} = q_0(\tau) \cdot \left(\frac{t-\tau}{\bar{t}}\right)^i \left(\frac{1}{\Gamma(i)}\right)\left(\frac{1}{t-\tau}\right) \exp(-\frac{t-\tau}{\bar{t}}) d\tau \qquad (5.14)$$

The accumulated responses are given by

$$q_i(t) = \int_0^t q_0(\tau) \cdot \left(\frac{t-\tau}{\bar{t}}\right)^i \left(\frac{1}{\Gamma(i)}\right)\left(\frac{1}{t-\tau}\right) \exp(-\frac{t-\tau}{\bar{t}}) d\tau \qquad (5.15)$$

In addition to the reservoir number and residence time, it is observed in real data that there is a lag in time between the input sequence and the output sequence. The physical explanation of this lag time is to observe that the cascade model does not account for travel time between the reservoirs representing the watershed. A simple approach to account for the observed time lag is to include a time delay related to some mean travel time in the watershed. Figure 5.4 is a schematic including this delay in response. The linear system is unchanged except time from the input is shifted by the amount *t_lag* which is assumed to be proportional to the ratio of a characteristic length x_c and some characteristic velocity v_c.

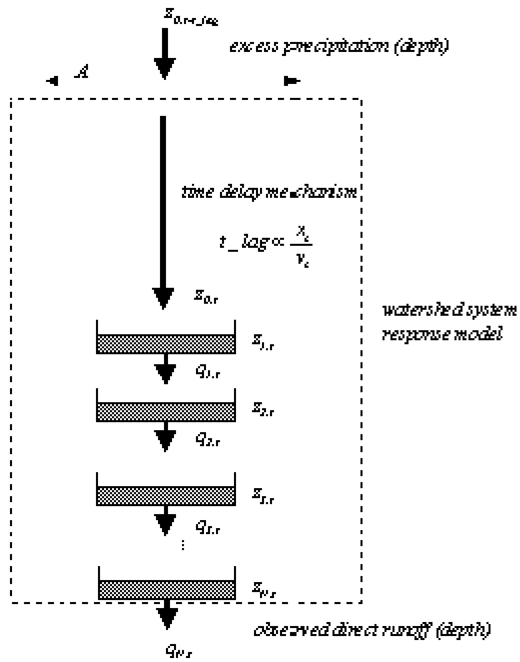

Figure 5.4. Watershed Conceptual Model (with Time-Delay)

The analytical solution for this conceptual model is

$$q_i(t) = \int_0^t q_0(\tau - t\_lag) \cdot \left(\frac{t-\tau}{\bar{t}}\right)^i \left(\frac{1}{\Gamma(i)}\right)\left(\frac{1}{t-\tau}\right) \exp\left(-\frac{t-\tau}{\bar{t}}\right) d\tau \qquad (5.16)$$

where the input sequence in the integrand (q_0) has zero value at times smaller than or equal to zero. These two models are identical except that in Equation (5.16) the inputs are lagged *t_lag* units – that is if elapsed time is smaller than the lag time, the input depths are zero, otherwise the input depths are those at time (*t - t_lag)*. Thus a precipitation event at time zero will not produce an output until time *t_lag*, and so on.

Equation (5.16) was coded into a computer program to predict the watershed response to a time-series of rainfall inputs. The unknown watershed characteristics are the residence time, the reservoir number, and the lag time.

Figure 5.5 is a plot of the IUH for different values of *N*. As the *N* value increased, the peak occurs at later times and the flows are spread over a greater time.

Figure 5.6 is a plot of the variation of $\bar{t}$ for an *N*-value of 3. For lower values of $\bar{t}$, the peak of the hydrograph is concentrated, but as $\bar{t}$ increases the peak is spreading and also the tail is asymptotic to the x-axis.

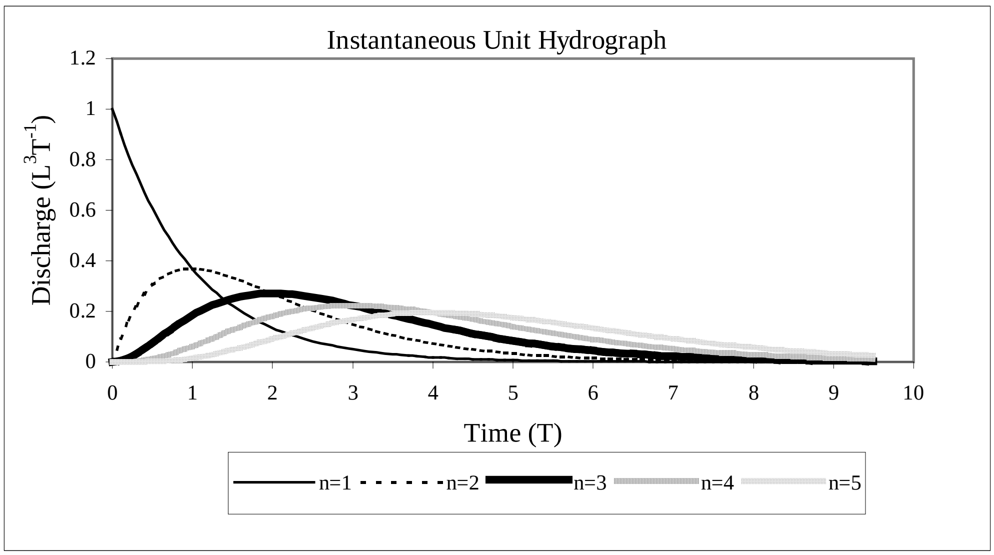

Figure 5.5. Plot of response functions, IUH's, for different values of *N* and $\bar{t}$ =1.

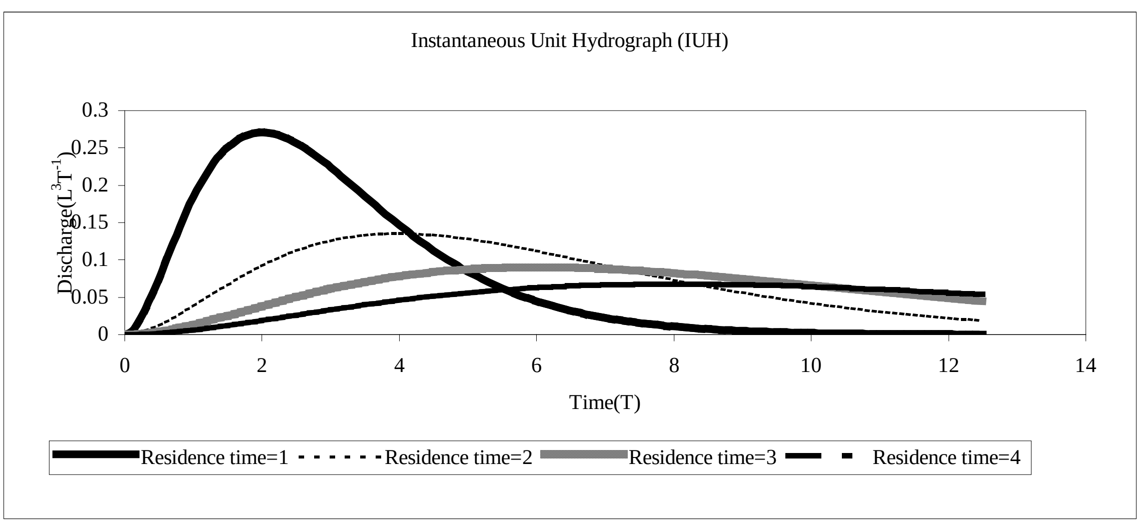

Figure 5.6. Plot of response functions for different values of $\bar{t}$ and constant *N*.

5.4. CONVOLUTION

The IUH's obtained in the previous sections were used in constructing direct runoff hydrographs by convolving a sequence of rainfall inputs over time.

Convolution is superposition in a sequence of responses from a sequence of inputs each occurring at a different location in the time domain to produce a direct runoff hydrograph. Figure 5.7 illustrates the relationship to an impulse of rainfall (expressed as a rate) and the corresponding output hydrograph. If this pulse of rain is the only pulse in a particular storm, then the depicted hydrograph is the direct runoff hydrograph.

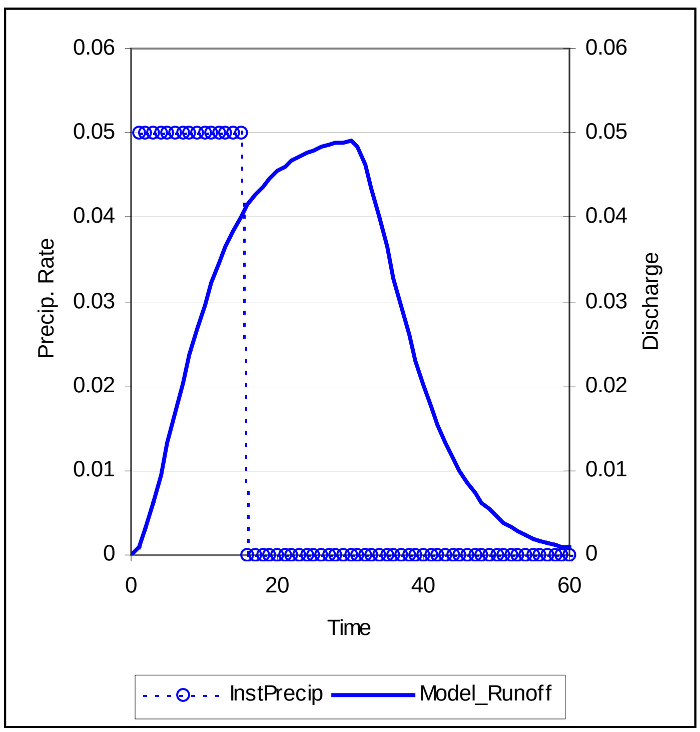

Figure 5.7. Plot of Finite-Duration Rainfall Impulse and Corresponding Outflow Hydrograph.

Figure 5.8 now illustrates a second pulse of rain at the same intensity, starting when the first pulse ends. The hydrograph resulting from the second pulse is now plotted along with the first pulse. The sum (addition of ordinates) of the two hydrographs produces the expected direct runoff hydrograph. The mathematical operation describing this process is superposition in time or convolution.

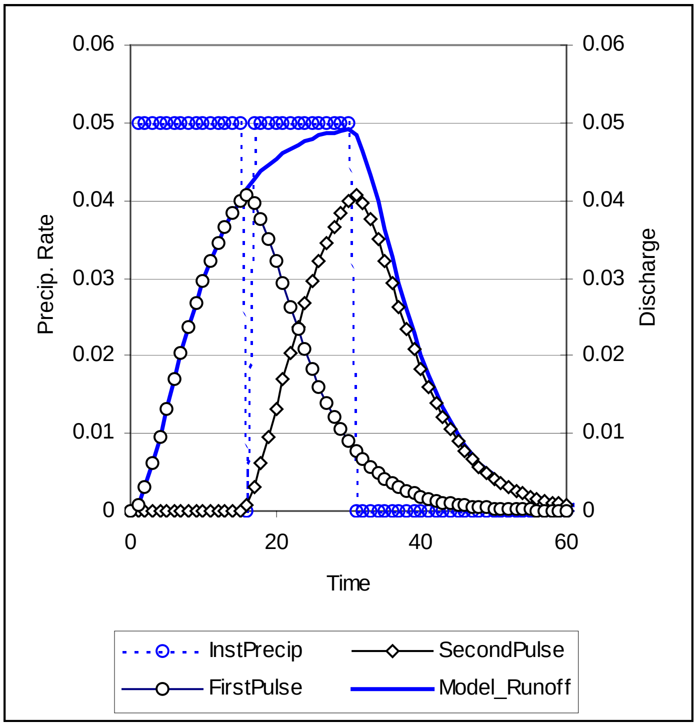

Figure 5.8. Two rainfall-runoff sequences convolved into the direct runoff hydrograph.

The rainfall pulses need not be the same length, nor the same intensity.

But the most important concept is that the underlying unit hydrograph is the same for each pulse, i.e., the parameters (*N,t-bar*) are constant

The modeled hydrographs can be compared with the observed hydrographs for the gaged stations. The approach can be extended to ungaged stations by deriving the governing parameters from the watershed characteristics.

5.5 DECONVOLUTION

Deconvolution is the inverse of convolution process. Deconvolution determines the individual response characteristics from a convolved (the observed hydrograph) direct runoff hydrograph. The IUH parameters from the observed hydrograph and the observed hyetograph can be determined using the deconvolution process.

Numerical differentiation can be coupled with the deconvolution process to determine incremental rainfall rates from the cumulative rainfall data. Figure 5.9 is a sketch showing the incremental rate and the cumulative depth relationship.

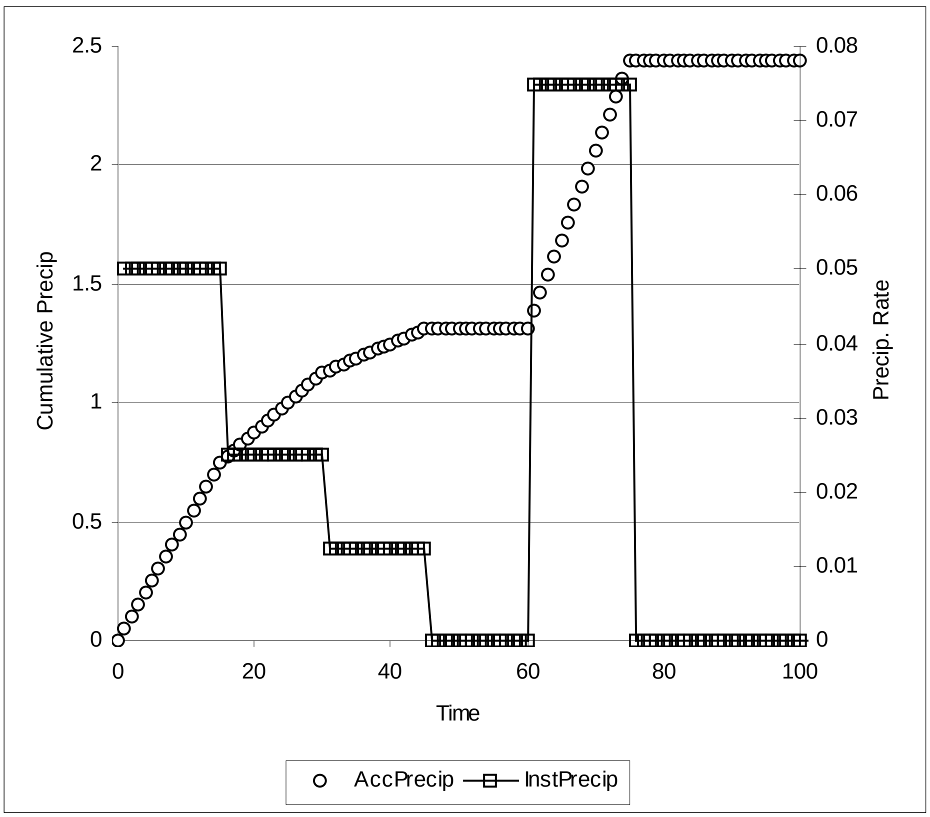

Figure 5.9. Cumulative Precipitation and Incremental Precipitation Relationship.

The cumulative rainfall distribution is the integral of the incremental rainfall distributionover the entire rainfall event. Equation (5.17) expresses this relationship.

$$P(t) = \int_{-\infty}^{\infty} p(t)dt \tag{5.17}$$

In the present work a simple first-order, forward differencing scheme is used to obtain the precipitation rate from the cumulative precipitation *P(t).*

$$p(t) \approx \frac{P(t+\Delta t) - P(t)}{\Delta t} \tag{5.18}$$

The time-step length used in the research was one-minute intervals. This time length was chosen because it is the smallest increment that can be represented in the current DATE_TIME format in the database. Linear interpolation was used to convert the cumulative precipitation into one-minute intervals, and then the numerical differentiation is performed to obtain the rainfall rates. The typical units were inches per minute.

5.6. PARAMETER ESTIMATION

Initial estimates of the hydrograph characteristics are made by graphical analysis of selected data sets in each watershed. Because the watershed characteristics are supposed to be invariant, only one event pair needs to be analyzed to get initial estimates of the *t-lag*, and the mean residence time. Figure 5.8 is a plot of rainfall and runoff for a particular event at a particular station. The vertical axes are in inches of depth, the left axis is the runoff depth, and the right axis is the rainfall depth. The step function appearance is an artifact of the linear interpolation scheme used to represent the data on one-minute intervals. The transit lag time initial estimate is the time between the arrival of the peak rainfall and the peak runoff. The mean residence time initial estimate is the time between the first 1% of cumulative flow (start of runoff) and the time to peak flow.

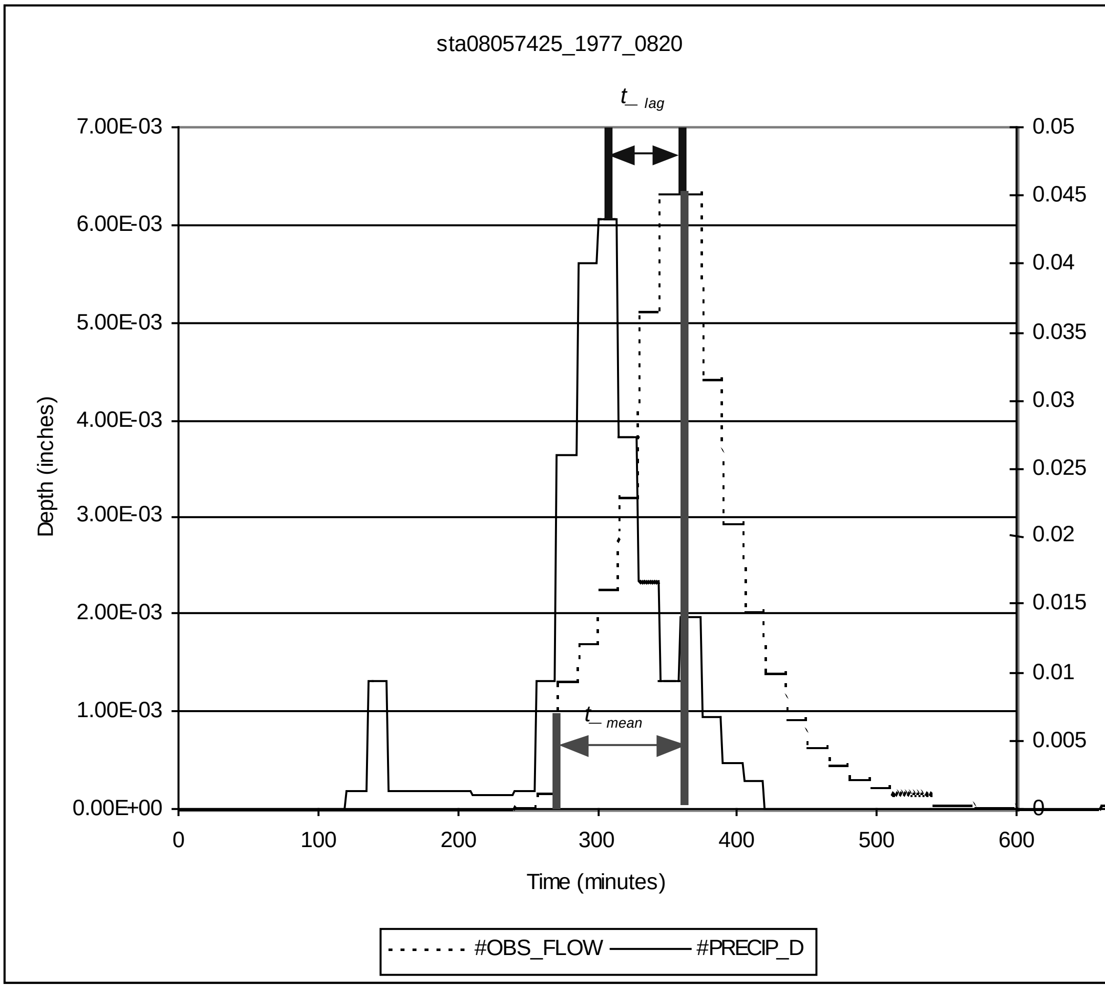

Figure 5.10. Initial parameter estimations using the graphical observation.

For hydrographs with multiple peaks the time to 50% runoff was used as the initial estimate of mean residence time.

The IUH parameters are estimated by simulating the DRH from the effective rainfall signal and adjusting values until some merit function is minimized. The functions considered are the classic sum of squared errors (SSE), the root mean squared error (RMSE), and the maximum absolute deviation (MAD). Mathematically these merit functions are:

$$SSE = \sum_{i=1}^{N} (Q_S - Q_O)_i^2$$

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N} (Q_S - Q_O)_i^2} \ ;$$

$$MAD = \max\left|Q_S - Q_O\right|_i$$

where Q is the discharge (L^3/T); the subscripts $_O$ and $_S$ represent observed and simulated discharge, respectively, and *N* is the total number of values in a particular storm event. The search procedure used initial estimates determined by graphing a single storm at each station and guessing a reasonable value for *t_bar*, *t_lag*, and *i(res. number)=1*.

A search routine that systematically adjusted these guesses by increments of 1.0 was employed. The algorithm was programmed to continue adjustment(s) as long as an adjustment improved the merit function. When no further improvement could be detected, the algorithm then randomly selected 100 adjustments from a uniform distribution centered on the last best guess as a check that a local minima was not stopping the processing. If the program could still not improve the merit function, then processing for that storm stopped, and the algorithm moved onto the next storm

The model has been used to obtain the hydrographs using the parameters estimated. Figures 5.11 and 5.12 are the plots with the observed and modeled hydrographs.

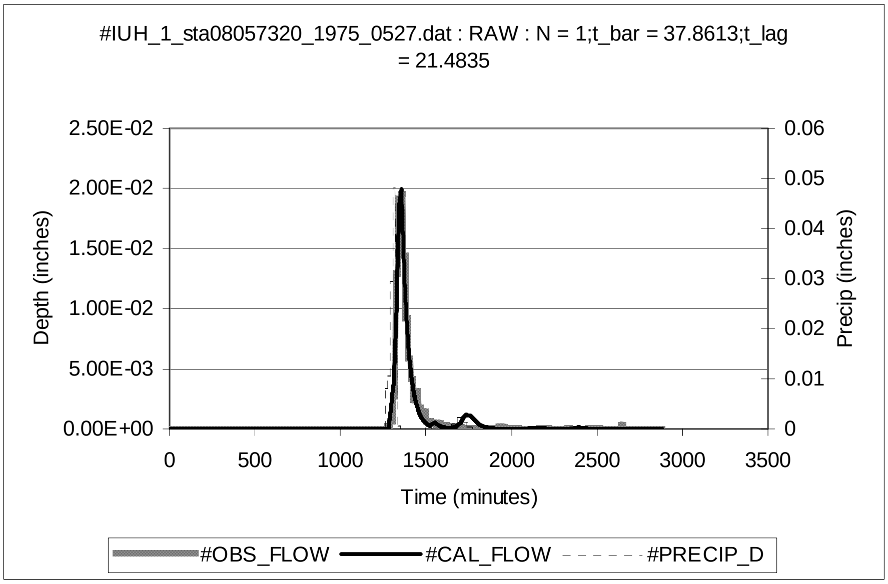

Figure 5.11. Observed and Modeled hydrographs for a typical station.

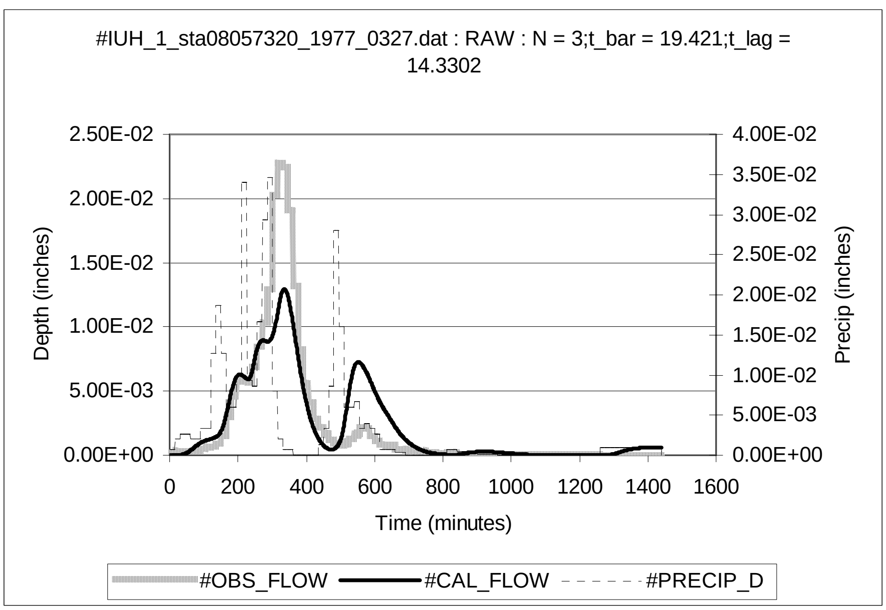

Figure 5.12. Observed and Modeled hydrographs for a typical station.

CHAPTER 6 : Data Analysis

Instantaneous unit hydrographs were estimated as a function of the parameters *N, t-bar* and *t-lag*. Each station of every module has IUH different parameters for each storm event. Median values of *t-bar* and *t-lag* were considered for each station.

The purpose is to determine the relations between the IUH parameters and some identifiable and readily obtainable watershed characteristics.

The following six parameters were evaluated for analysis:

- Drainage area (A)
- Perimeter (P)
- Maximum distance (Dm)
- Stream length (S_L)
- Aspect ratio (f)
- Difference in elevation (H)

Multiple correlations were performed between the dependent variables *t-bar* and *t-lag* and the independent variables A, P, Dm, S_L, f and H.

6.1 DATA STRUCTURE

Seventy-six small watersheds distributed in Central Texas were selected for the hydrograph study. They were organized in five modules: Dallas, Fort Worth, San Antonio, Austin and Small Rural Sheds.

Tables 6.1 to 6.5 list the names of the watersheds, the corresponding gaging stations with their locations and the drainage areas.

Table 6.1 - Studied watersheds for Dallas module

No.	Watershed	Station_ID	Latitude	Longitude	Drainage area in sq miles
1	Ash Creek	sta08057320_d	32°48'18"	96°43'04"	6.92
2	Bachman Branch	sta08055700_d	32°51'26"	96°50'12"	10.00
3	Cedar Creek	sta08057050_d	32°44'50"	96°47'44"	9.42
4	Coombs Creek	sta08057020_d	32°46'01"	96°50'07"	4.75
5	CottonwoodCreek	sta08057140_d	32°54'33"	96°45'54"	8.50
6	Duck Creek	sta08061620_d	32°55'53"	96°39'55"	8.05
7	Elam Creek	sta08057415_d	32°44'14"	96°41'36"	1.25
8	FivemileCreek	sta08057418_d	32°42'19"	96°51'32"	7.65
9	FivemileCreek	sta08057420_d	32°41'15"	96°49'22"	13.20
10	FloydBranch	sta08057160_d	32°54'33"	96°45'34"	4.17
11	JoesCreek	sta08055580_d	32°53'43"	96°41'36"	7.51
12	PrairieCreek	sta08057445_d	32°42'17"	96°40'11"	9.03
13	RushBranch	sta08057130_d	32°57'45"	96°47'44"	1.22
14	SpankyCreek	sta08057120_d	32°57'58"	96°48'11"	6.77
15	TurtleCreek	sta08056500_d	32°48'26"	96°48'08"	7.98
16	WoodyBranch	sta08057425_d	32°40'58"	96°49'22"	11.50

Table 6.2 - Studied watersheds for Fort Worth module

No.	Watershed	Station_ID	Latitude	Longitude	Drainage area in sq miles
17	Dry Branch	sta08048550_d	32° 47' 19"	97° 18' 22"	1.08
18	Dry Branch	sta08048600_d	32° 47' 19"	97° 18' 22"	2.15
19	Little Fossil	sta08048820_d	32° 50' 22"	97° 19' 22"	5.64
20	Little Fossil	sta08048850_d	32° 48' 33"	97° 17' 28"	12.30
21	Sycamore	sta08048520_d	32° 39' 55"	97° 19' 16"	17.70
22	Sycamore	sta08048530_d	32° 41' 08"	97° 19' 44"	0.97
23	Sycamore	sta08048540_d	32° 41' 18"	97° 19' 11"	1.35
24	Sycamore	staSSSC	-	-	0.38

Table 6.3 - Studied watersheds for San Antonio module

No.	Watershed	Station_ID	Latitude	Longitude	Drainage area in sq miles
25	Alazan Creek	sta08178300_d	29° 27' 29"	98° 32' 59"	3.26
26	Leon Creek	sta08181000_d	29° 35' 14"	98° 37' 40"	5.57
27	Leon Creek	sta08181400_d	29° 34' 42"	98° 41' 29"	15.00
28	Leon Creek	sta08181450_d	29° 23' 12"	98° 36' 00"	1.19
29	Olmos Creek	sta08177600_d	29° 34' 35"	98° 32' 45"	0.33
30	Olmos Creek	sta08178555_d	29° 21' 05"	98° 29' 32"	2.43
31	Salado Creek	sta08178600_d	29° 37' 31"	98° 31' 06"	9.54
32	Salado Creek	sta08178620_d	29° 35' 24"	98° 27' 47"	4.05
33	Salado Creek	sta08178640_d	29° 37' 23"	98° 26' 29"	2.45
34	Salado Creek	sta08178645_d	29° 37' 04"	98° 25' 41"	2.33
35	Salado Creek	sta08178690_d	29° 31' 36"	98° 26' 25"	0.26
36	Salado Creek	sta08178736_d	29° 26' 37"	98° 27'13"	0.45

Table 6.4 - Studied watersheds for Austin module

No.	Watershed	Station_ID	Latitude	Longitude	Drainage area in sq miles
37	Barton Creek	sta08155200_d	30°17'46"	97°55'31"	89.70
38	BartonCreek	sta08155300_d	30°14'40"	97°48'07"	116.00
39	BearCreek	sta08158810_d	30°09'19"	97°56'23"	12.20
40	BearCreek	sta08158820_d	30°08'25"	97°50'50"	24.00
41	BearCreek	sta08158825_d	30°07'31"	97°51'43"	21.00
42	BoggyCreek	sta08158050_d	30°15'47"	97°40'20"	13.10
43	BoggySouthCreek	sta08158880_d	30°10'50"	97°46'55"	3.58
44	BullCreek	sta08154700_d	30°22'19"	97°47'04"	22.30
45	LittleWalnutCreek	sta08158380_d	30°21'15"	97°41'52"	5.22
46	OnionCreek	sta08158700_d	30°04'59"	98°00'29"	124.00
47	OnionCreek	sta08158800_d	30°05'09"	97°50'52"	166.00
48	ShoalCreek	sta08156650_d	30°21'55"	97°44'11"	2.79
49	ShoalCreek	sta08156700_d	30°20'50"	97°44'41"	7.03
50	ShoalCreek	sta08156750_d	30°20'21"	97°44'50"	7.56
51	ShoalCreek	sta08156800_d	30°16'35"	97°45'00"	12.30
52	SlaughterCreek	sta08158840_d	30°12'32"	97°54'11"	8.24
53	SlaughterCreek	sta08158860_d	30°09'43"	97°49'55"	23.10
54	WallerCreek	sta08157000_d	30°17'49"	97°43'36"	2.31
55	WallerCreek	sta08157500_d	30°17'08"	97°44'01"	4.13
56	WalnutCreek	sta08158100_d	30°24'35"	97°42'41"	12.60
57	WalnutCreek	sta08158200_d	30°22'30"	97°39'37"	26.20
58	WalnutCreek	sta08158400_d	30°20'57"	97°41'34"	5.57
59	WalnutCreek	sta08158500_d	30°18'34"	97°40'04"	12.10
60	WalnutCreek	sta08158600_d	30°16'59"	97°39'17"	51.30
61	WestBouldinCreek	sta08155550_d	30°15'49"	97°45'17"	3.12
62	WilbargerCreek	sta08159150_d	30°27'16"	97°36'02"	4.61
63	WilliamsonCreek	sta08158920_d	30°14'06"	97°51'36"	6.30
64	WilliamsonCreek	sta08158930_d	30°13'16"	97°47'36"	19.00
65	WilliamsonCreek	sta08158970_d	30°11'21"	97°43'56"	27.60

Table 6.5 - Studied watersheds for SmallRuralSheds module

No	Watershed	Subshed	Station_ID	Latitude	Longitude	Drainage area in sq miles
66	SanAntonioBasin	Calaveras	sta08182400_d	29°22'49"	98°17'33"	7.01
67	SanAntonioBasin	Escondido	sta08187000_d	28°46'41"	97°53'41"	3.29
68	SanAntonioBasin	Escondido	sta08187900_d	28°51'39"	97°50'39"	8.43
69	TrinityBasin	ElmFork	sta08050200_d	33°37'13"	97°24'15"	0.77
70	TrinityBasin	Honey	sta08057500_d	33°18'12"	96°41'22"	2.14
71	TrinityBasin	Honey	sta08058000_d	33°18'20"	96°40'12"	1.26
72	TrinityBasin	LittleElm	sta08052630_d	33°24'33"	96°48'41"	2.10
73	TrinityBasin	LittleElm	sta08052700_d	33°17'00"	96°53'33"	75.50
74	TrinityBasin	North	sta08042650_d	33°14'52"	98°19'19"	6.82
75	TrinityBasin	North	sta08042700_d	33°16'57"	98°17'53"	21.60
76	TrinityBasin	PinOak	sta08063200_d	31°48'01"	96°43'02"	17.60

The storm events recorded are associated with their respective station name. The hydrographs observed for each storm event were modeled as explained in chapter 4. It can be noted that each modeled hydrograph is dependent on three parameters (*N, t_bar, t_lag*).

6.2 DATA ANALYSIS

The purpose of the data analysis is to find the correlation coefficients between the observed Instantaneous Unit Hydrograph (IUH) parameters and watershed characteristics. The analysis is focused towards comparison of data among a module. This analysis is done for all the five modules using the database constructed in chapter 4.

In order to perform the analysis, two multiple regression models were considered: a linear model (denoted as Model 1) and a power law model (denoted as Model 2).

Model 1 has the following expression:

$$Y = \beta_1 + \beta_2 X_2 + ... + \beta_m X_m + E \qquad (6.1)$$

where Y= independent variable

X_i, i=2,…,m, represent the m independent variables

β_1= intercept (value when all the independent variables are 0)

β_i, i=2,…,m, represent the correspondent regression coefficients

E is the random error, usually assumed to be normally distributed with mean zero and variance σ^2

Formally, a coefficient β_i in a multiple regression model is defined as a partial regression coefficient, whose interpretation is the change in the mean response associated with a unit change in X_i, holding constant all other variables.

In contrast, if m separate simple regressions had performed, the regression coefficient for the simple linear regression involving, say, X_i, would be interpreted as the change in the mean response associated with a unit change in X_i, ignoring the effect of any other variables.

To estimate the regression coefficients, a set of n observed values (specific for each module) and the least squares method are used to obtain the following equation for estimating the mean of Y:

$$\overline{Y} = \overline{\beta_1} + \overline{\beta_2} X_2 + ... + \overline{\beta_m} X_m$$

The least square principle specifies that the estimates, $\overline{\beta_i}$, minimize the error sum of squares:

$$SSE = \sum (\mathrm{Y} - \overline{\beta_1} - \overline{\beta_2} X_2 - ... - \overline{\beta_m} X_m)^2$$

For convenience we redefine the model:

$Y = \beta_1 X_1 + \beta_2 X_2 + ... + \beta_m X_m + E$

where X_1 is a variable that has the value 1 for all observations.

The error sum of squares to be minimized is now written:

$$SSE = \sum (\mathrm{Y} - \overline{\beta_1} X_1 - \overline{\beta_2} X_2 - ... - \overline{\beta_m} X_m)^2$$

The least squares estimates are provided by the solution to the following set of (m+1) linear equations in the (m+1) unknown parameters, $\beta_1, \beta_2, \ldots, \beta_m$.

The solutions to these normal equations provide the least squares estimates of the coefficients, which were already denoted by $\overline{\beta}_1, \overline{\beta}_2, ..., \overline{\beta}_m$.

$$\beta_0 n + \beta_1 \sum X_1 + \beta_2 \sum X_2 + ... + \beta_m \sum X_m = \sum Y$$

$$\beta_0 \sum X_1 + \beta_1 \sum X_1^2 + \beta_2 \sum X_1 X_2 + ... + \beta_m \sum X_1 X_m = \sum X_1 Y$$

$$\beta_0 \sum X_2 + \beta_1 \sum X_2 X_1 + \beta_2 \sum X_2^2 + ... + \beta_m \sum X_2 X_m = \sum X_2 Y$$

……………………………………………………………………

$$\beta_0 \sum X_m + \beta_1 \sum X_m X_1 + \beta_2 \sum X_m X_2 + ... + \beta_m \sum X_m^2 = \sum X_m Y$$

The system equation was solved using Microsoft Excel – Solver tool.

The correlation coefficients were determined by applying the least square errors method that minimizes the sum of squares of deviations of the computed and observed values of timing parameters t_lag and t_bar.

Model 2 has the following expression:

$$Y^* = \beta_1 X_2^{*\beta 2} X_3^{*\beta 3} X_4^{*\beta 4} X_5^{*\beta 5} \ldots X_m^{*\beta m} + \mathrm{E} \qquad (6.2)$$

By applying the natural logarithm operator to the expression (6.2), it is obtained:

$$LN(Y^*) = LN(\beta_1) + \beta_2 LN(X_2^*) + \beta_3 LN(X_3^*) + \beta_4 LN(X_4^*) + \ldots + \beta_m LN(X_m^*) \qquad (6.3)$$

Substituting in expression (6.3) $LN(Y^*)=Y$, $LN(\beta_1)=\beta_1^*$ and Xi^*, $i=1,\ldots,m$ by Xi, the following expression is derived:

$$Y = \beta_1^* + \beta_2 X_2 + \ldots + \beta_m X_m + \mathrm{E} \qquad (6.4)$$

Equation (6.4) is identical with equation (6.1). So, correlation coefficients $\beta_1, \beta_2, \ldots, \beta_m$ will be determined following the same algorithm as it was explained for the previous model.

Tables 6.6 and 6.7 present the results of the multiple correlations, performed for each of the five modules, using linear model (Model 1) and power law model (Model 2).

Table 6.6. Correlation between IUH parameters and watersheds characteristics for

Model 1: $T\_lag = \beta_1 + \beta_2 A + \beta_3 P + \beta_4 Dm + \beta_5 S_L + \beta_6 f + \beta_7 H$

$T\_bar = \beta_1 + \beta_2 A + \beta_3 P + \beta_4 Dm + \beta_5 S_L + \beta_6 f + \beta_7 H$

Module	IUH parameter	Watershed characteristics	Correlation coefficients							Regression coefficient
			β1	β2	β3	β4	β5	β6	β7	R^2
Dallas	T_lag	A,P,Dm,S_L,f,H	-8.137	-0.892	4.656	-7.647	7.952	12.064	-641.744	0.495
	T_bar	A,P,Dm,S_L,f,H	-7.256	1.485	5.242	-27.069	22.203	40.237	-1017.56	0.5145
Fort Worth	T_lag	A,P,Dm,S_L,f,H	7.052	0.914	-13.123	23.653	14.985	9.368	-439.243	0.8681
	T_bar	A,P,Dm,S_L,f,H	-360.678	-46.849	37.491	-239.857	192.855	262.802	10337.292	0.9865
San Antonio	T_lag	A,P,Dm,S_L,f,H	-26.819	-28.483	22.745	-33.218	39.607	22.052	-739.677	0.3495
	T_bar	A,P,Dm,S_L,f,H	-16.558	-1.781	22.229	-32.833	9.215	37.254	-1776.05	0.91
Austin	T_lag	A,P,Dm,S_L,f,H	-38.573	-0.446	-0.608	2.688	1.751	-13.838	1848.989	0.7651
	T_bar	A,P,Dm,S_L,f,H	-30.003	5.121	-2.585	-7.769	-0.374	-1.165	2685.577	0.8224
SmalRuralsheds	T_lag	A,P,Dm,S_L,f,H	-98.525	-4.369	6.965	36.682	-14.332	12.415	1218.670	0.9668
	T_bar	A,P,Dm,S_L,f,H	-96.114	-8.884	4.930	95.746	-12.576	14.321	-1741.79	0.979

Table 6.6. Correlation between IUH parameters and watersheds characteristics for

Model 2: $T\_lag = \beta_1 A^{\beta 2} P^{\beta 3} Dm^{\beta 4} S_L^{\beta 5} f^{\beta 6} H^{\beta 7}$

$T\_bar = \beta_1 A^{\beta 2} P^{\beta 3} Dm^{\beta 4} S_L^{\beta 5} f^{\beta 6} H^{\beta 7}$

Module	IUH parameter	Watershed characteristics	Correlation coefficients							Regression coefficient
			β1	β2	β3	β4	β5	β6	β7	R^2
Dallas	T_lag	A,P,Dm,S_L,f,H	206.183	0.041	3.733	-5.529	1.651	2.745	-0.985	0.6708
	T_bar	A,P,Dm,S_L,f,H	207.069	0.334	0.294	-2.391	2.730	1.482	-1.770	0.6351
Fort Worth	T_lag	A,P,Dm,S_L,f,H	185.655	0.460	-3.868	3.459	-0.008	3.002	1.064	0.903
	T_bar	A,P,Dm,S_L,f,H	2985.365	0.809	-4.698	5.654	-0.956	3.432	0.055	0.9998
San Antonio	T_lag	A,P,Dm,S_L,f,H	2950.315	0.678	0.307	-0.929	0.347	0.870	-0.421	0.35
	T_bar	A,P,Dm,S_L,f,H	1576.295	3.530	6.166	-1.078	-5.270	5.279	-2.500	0.8028
Austin	T_lag	A,P,Dm,S_L,f,H	0.001	-0.246	0.763	0.151	-0.792	-0.184	2.431	0.8162
	T_bar	A,P,Dm,S_L,f,H	1576.556	0.893	-1.220	-0.621	0.283	0.145	2.113	0.8359
SmalRuralsheds	T_lag	A,P,Dm,S_L,f,H	177.719	0.724	-0.076	0.144	-0.177	0.340	-0.225	0.9562
	T_bar	A,P,Dm,S_L,f,H	205.503	0.556	-0.460	-0.015	1.151	0.355	-1.524	0.982

CHAPTER 7: Results and Conclusions

7.1 RESULTS

The multiple correlation analysis shows the relationship between the IUH parameters, lag time (t_lag) and residence time (t_bar), and the watersheds characteristics; therefore, t_lag and t_bar can be calculated using linear model or power law model, with the coefficients determined in chapter 6.

Analysis of all the watersheds in the database was not considered feasible or necessary, selected groups being considered sufficient to indicate the trend.

Since the watersheds used in this research range in area from 0.26 to 166 square miles, three groups of watersheds for each module were selected. The selection of these watersheds was accomplished by ranking them from smallest to largest drainage area.

Three different stations having “small”, “median” and “large” areas were analyzed for each of the five modules: Dallas, Fort Worth, San Antonio, Austin and Small Rural Sheds.

Table 7.1 lists the stations numbers, storms numbers, areas and IUH timing parameters (t_lag and t_bar) for the watersheds considered for result analysis.

Table 7.1. IUH parameters for selected watersheds

Module	**Station No.**	**Storm No.**	**Area**	**T_lag**		**T_bar**	
				Model 1	Model 2	Model 1	Model 2
Dallas	sta 8057130	*_1979_0330	1.22	18	21	43	46
	sta 8057418	*_1979_0330	7.65	24	13	42	18
	sta 8057420	*_1979_0330	13.20	49	47	77	70
Fort Worth	sta SSSC	*_1973_0603	0.38	11	8	16	9
	sta 8048600	*_1973_0603	2.15	34	26	140	122
	sta 8048520	*_1973_0603	17.7	43	44	158	158
San Antonio	sta 8178690	*_1979_0601	0.26	1	10	14	22
	sta 8178555	*_1979_0601	2.43	89	73	82	88
	sta 8181400	*_1981_0612	15.0	125	114	162	167
Austin	sta 8157000	*_1978_0511	2.31	10	22	32	14
	sta 8158100	*_1978_0410	12.60	37	34	86	53
	sta 8158800	*_1983_0520	166.0	295	315	865	955
Small Rural Sheds	sta 8050200	*_1961_0325	0.77	49	24	105	41
	sta 8042650	*_1978_0409	6.82	63	70	73	88
	sta 8052700	*_1976_0530	75.5	340	341	934	932

Typical results of the analysis are presented on Figure 7.1 to 7.6, which are plots of both the instantaneous (rate) precipitation and runoff as well as the cumulative effective precipitation and runoff.

In the plots, dashed lines represent the instantaneous values, while the solid lines are cumulative values. The solid black line is the result of convolving the instantaneous effective precipitation using the IUH with the parameter values shown on the charts. Each storm produced a different result and the utility of the IUH concept is that the median values of these parameters can be estimated from the geometric watershed properties.

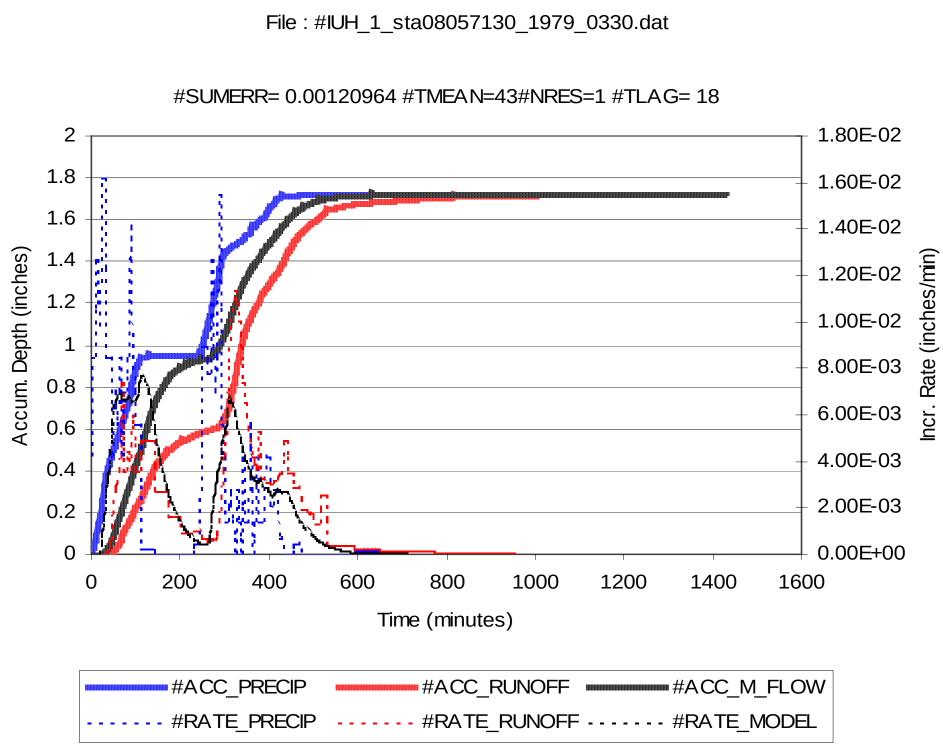

Figure 7.1. IUH parameters estimated by Linear correlation with watershed properties Dallas Module – "small" area

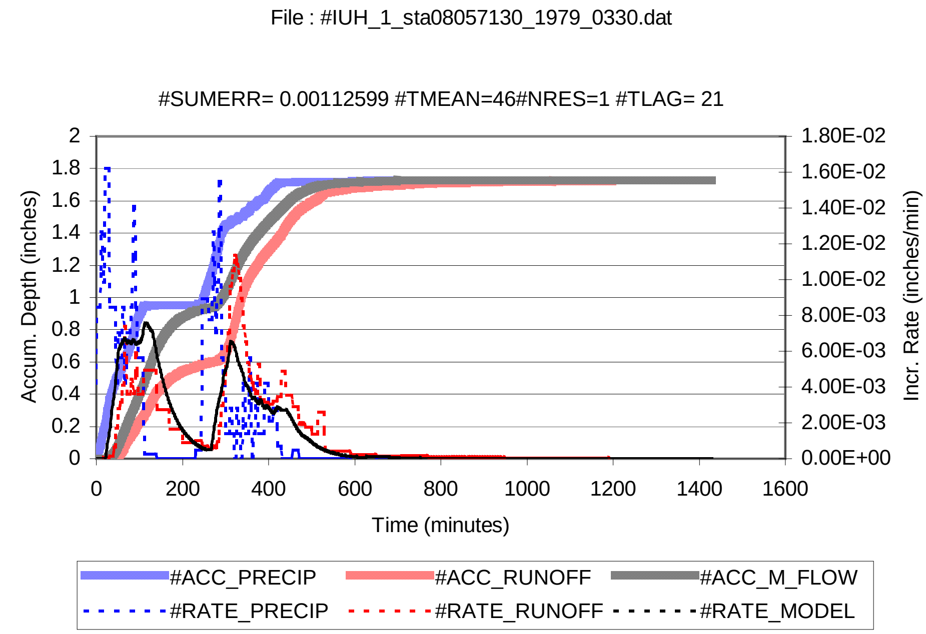

Figure 7.2. IUH parameters estimated by Power Law correlation with watershed properties Dallas Module – "small" area

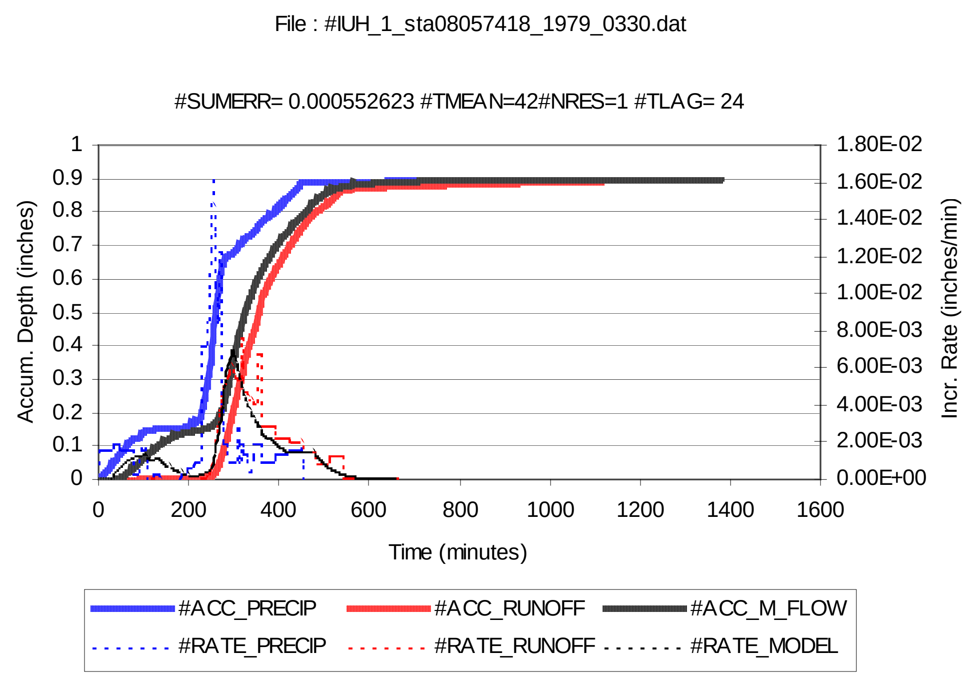

Figure 7.3. IUH parameters estimated by Linear correlation with watershed properties Dallas Module – "median" area

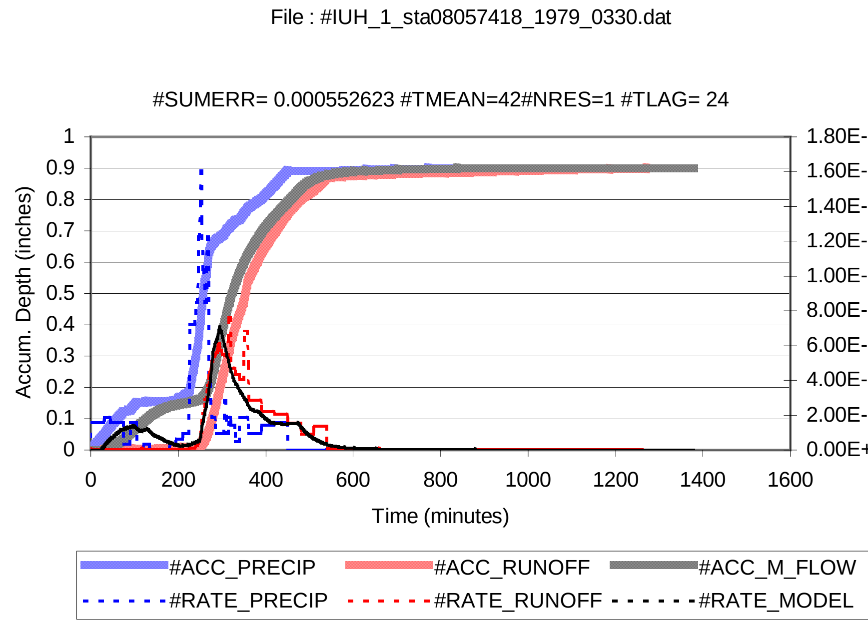

Figure 7.4. IUH parameters estimated by Power Law correlation with watershed properties Dallas Module – "median" area

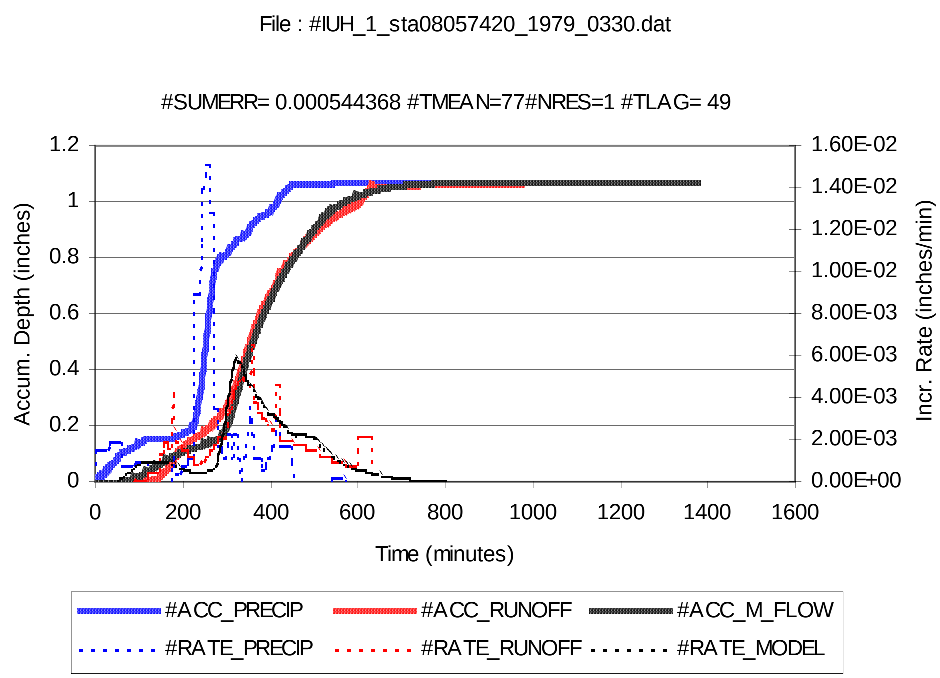

Figure 7.5. IUH parameters estimated by Linear correlation with watershed properties Dallas Module – "large" area

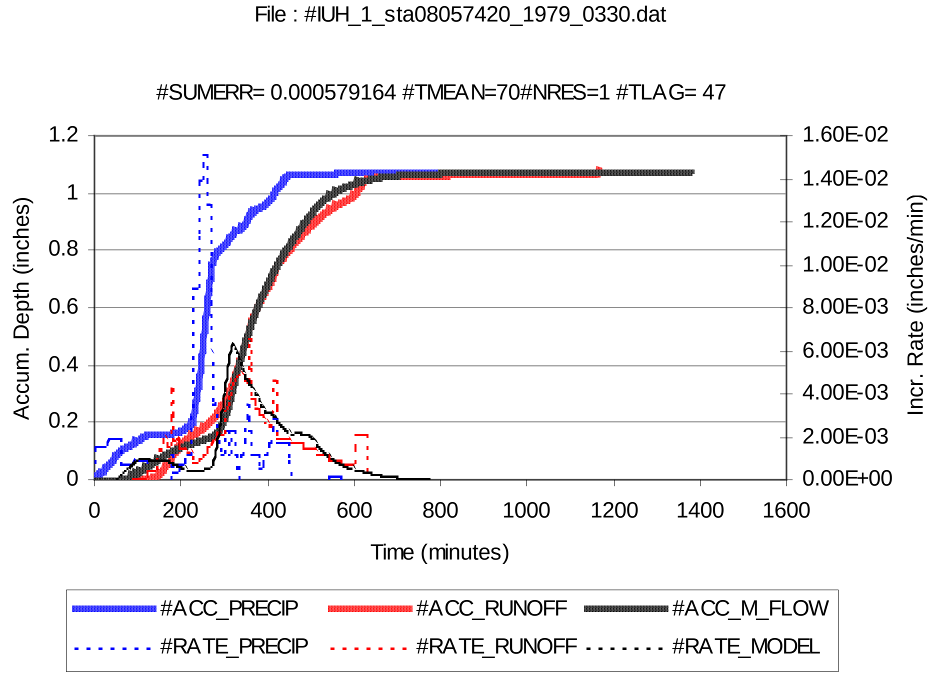

Figure 7.6. IUH parameters estimated by Power Law correlation with watershed properties Dallas Module – "large" area

As a check on the reliability of the results obtained from the application of the above-mentioned procedures, peak discharge prediction error was calculated for each storm analyzed using the following formula:

$$\text{Error} = \frac{Q_{\text{mod}\,el} - Q_{real}}{Q_{real}} x\,100$$

Table 7.2 lists the storms analyzed and the percent errors obtained for each peak.

Module	Storm No.	Model	First peak	Second peak	Third peak	Forth peak
Dallas	sta8057130_1979_0330	Model 1	4.355811	-40.4016	NA	NA
		Model 2	2.461622	-42.4908	NA	NA
	sta8057418_1979_0330	Model 1	-7.42303	NA	NA	NA
		Model 2	31.56947	NA	NA	NA
	sta8057420_1979_0330	Model 1	-9.7117	NA	NA	NA
		Model 2	-3.74954	NA	NA	NA
Fort Worth	staSSSC_1973_0603	Model 1	-7.8961	-13.3045	-9.08599	-28.1468
		Model 2	17.81126	6.11859	34.68631	-7.73122
	sta8048600_1973_0603	Model 1	3.152549	-13.3399	NA	NA
		Model 2	16.11931	-5.78203	NA	NA
	sta8048520_1973_0603	Model 1	99.1576	13.26527	-45.7219	NA
		Model 2	99.1576	13.26527	-45.7219	NA
San Antonio	sta8178690_1979_0601	Model 1	3.152549	-13.3399	NA	NA
		Model 2	16.11931	-5.78203	NA	NA
	sta8178555_1979_0601	Model 1	-7.8961	-13.3045	-9.08599	-28.1468
		Model 2	17.81126	6.11859	34.68631	-7.73122
	sta8181400_1981_0612	Model 1	-40.2108	NA	NA	NA
		Model 2	-41.4991	NA	NA	NA

NA – Not Applicable
$+^{ve}$ value - Over Predicted
$-^{ve}$ value - Under Predicted

Module	Storm No.	Model	First peak	Second peak	Third peak	Forth peak
Austin	sta8157000_1978_0511	Model 1	-22.9895	NA	NA	NA
		Model 2	35.3383	NA	NA	NA
	sta8158100_1978_0410	Model 1	-17.0885	NA	NA	NA
		Model 2	15.7075	NA	NA	NA
	sta8158800_1983_0520	Model 1	-64.016	-58.862	-80.968	NA
		Model 2	-67.105	-61.03	-81.722	NA
Small Rural Sheds	sta8050200_1961_0325	Model 1	22.421	-11.592	NA	NA
		Model 2	79.359	28.356	NA	NA
	sta8042650_1978_0409	Model 1	1.674	NA	NA	NA
		Model 2	-10.404	NA	NA	NA
	sta8052700_1976_0530	Model 1	12.630	17.544	NA	NA
		Model 2	12.832	17.652	NA	NA

NA – Not Applicable
$+^{ve}$ value - Over Predicted
$-^{ve}$ value - Under Predicted

7.2 CONCLUSIONS

This research illustrates that IUH-type models are feasible for rainfall-runoff modeling of watersheds in Texas in the size range of 0.1 – 100 mi^2. A simplistic IUH model based on a cascade of reservoirs can match peak discharge rates to within 15% of observed values, and match the arrival time of the peak within an hour or so.

7.3 FUTURE WORK

The comparison of the IUH method to NRCS methods should be performed

REFERENCES

1. American Society of Civil Engineers. (1996). "Hydrology Handbook." ASCE Manuals and Reports on Engineering Practice No. 71.
2. Todini, E. 1988. "Rainfall-runoff modeling-past, present and future. *J. Hydrology,* 100: 341-352.
3. Ponce, V. M. 1989. "Engineering Hydrology: Principles and Practices*,"* Eaglewood Cliffs, N. J., Prentice Hall.
4. Linsley, R. K., Jr., Kohler, M. A., and Paulhus, J. L. H., 1949. "Hydrology for Engineers", McGraw-Hill, New York.
5. McCuen, R. H., *Hydrologic Analysis and Design*, 2nd ed., Prentice Hall, Upper Saddle River, New Jersey, 07458, 1998.
6. Dooge, J.C.I., 1959. "A general theory of the unit hydrograph," *Journal of Geophysical Research*, 64(2), pp. 241-256.
7. Nash, J. E., 1959. "Systematic determination of unit hydrograph parameters," *Journal of Geophysical Research*, 64(1), pp 111-115.
8. Nash, J.E. (1958). "The form of the instantaneous unit hydrograph*." Intl. Assoc. Sci. Hydrology, Pub 42, Cont. Rend. 3* pp. 114-118.
9. Mays, Larry W. and Coles, Lynn, 1980. "Optimization of unit hydrograph determination," *Journal of the Hydraulics Division*, 106(HY1), ASCE, pp 85-97.
10. Snyder, W. M., 1956. "Hydrograph analysis by the method of least squares," *Proceedigns, American Society of Civil Engineers*, 81, pp 793-1—793-25.

11. Chow, V. T., 1964. "*Handbook of Applied Hydrology",* Sec 14., McGraw Hill, New York.
12. Dooge, J. C. I., 1973. "Linear theory of hydrologic systems." *U.S. Dept. of Agriculture, Technical Bulletin 1468.*
13. Linsley, R. K., Jr., Kohler, M. A., and Paulhus, J. L. H., 1949. "Applied Hydrology", McGraw-Hill, pp. 387-389.
14. Sherman, L. K., 1932. "Streamflow from rainfall by unit-graph method," *Engineering News-Record*, 108(14), pp. 501-506.
15. Shaw E. M., 1988. "*Hydrology in Practce",* Van Nostrand Reinhold (International), London.
16. Chow, V. T., Maidment, D. R., and Mays, L. W., 1988. *Applied Hydrology*, McGraw-Hill, 572pp.
17. Soil Conservation Service (now NRCS), 1972. *National Engineering Handbook*, Chapter 4, U.S.D.A., Washington D.C.
18. Clark, C.O., 1945. " Storage and the unit –hydrograph." *Amer. Soc. Civ. Eng.* vol. 110. PP 1416-1446.
19. Commons, G. G., 1942. " Flood hydrographs," *Civil Engineering,* 12(10), pp 571-572.
20. Gray, D. M., 1962. "Derivation of hydrographs for small watersheds from measurable physical characteristics" Department of agricultural engineering, Research Bulletin 506, AMES, Iowa.