Synthesis of Unit Hydrographs from a Digital Elevation Model

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Disclaimer

- The contents do not reflect the official view or policies of the Texas Department of Transportation (TxDOT).
- This paper does not constitute a standard, specification, or regulation.

Background and Significance

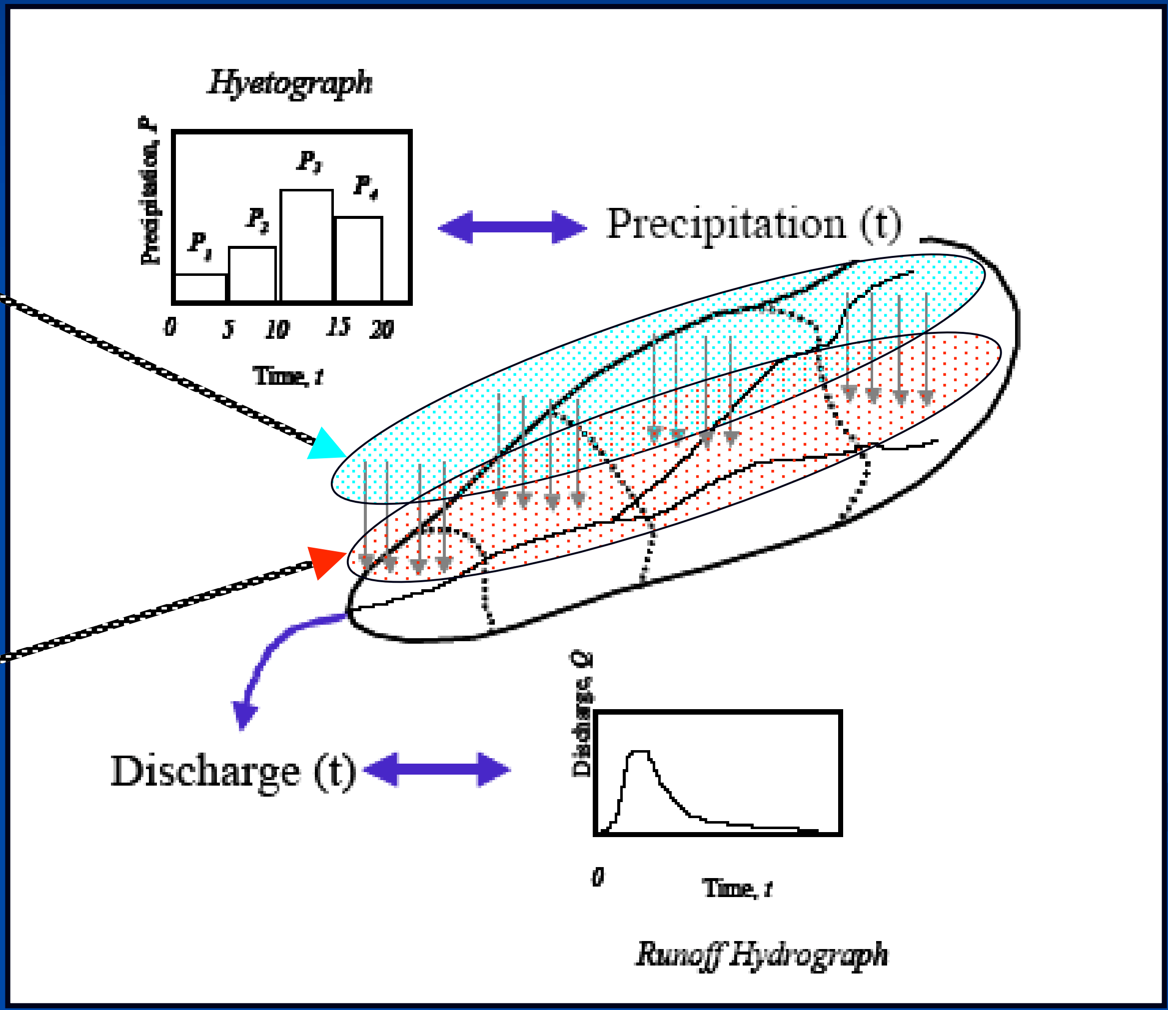
- Unit hydrograph (UH) methods are used by TxDOT (and others) to obtain peak discharge and hydrograph shape for drainage design.
  - Drainage areas too large for rational methods.
  - For drainage areas small enough for lumped-parameter model.
  - Generally on un-gaged watersheds.
- Estimating the time-response characteristics of a watershed is fundamental in rainfall-runoff modeling.

Timing Parameters

- The time-response characteristics of the watershed frequently are represented by two conceptual time parameters, time of concentration (*Tc*) and time to peak (*Tp*).
  - *Tc* is typically defined as the time required for runoff to travel from the most distant point along a pathline in the watershed to the outlet.
  - *Tp* is defined as the time from the beginning of direct runoff to the peak discharge value of a unit runoff hydrograph.

Schematic

- Loss model
  - Account for portion of rainfall that becomes available for runoff.
- UH model
  - Temporal redistribution of the available excess precipitation at the outlet.



Loss Models

- A mathematical construct that accounts for ALL rainfall losses on a watershed - the loss model does NOT redistribute the signal in time.
  - **Proportional loss model** (Univ. Houston, USGS)
  - Phi-index (Lamar)
  - Initial Abstraction - Constant Loss (USGS, TTU)
  - Infiltration capacity (UH, Lamar)

UH Models

- A mathematical construct that accounts for temporal redistribution of the excess (after loss) rainfall signal.
  - Gamma unit hydrograph (USGS, Lamar)
  - Discrete unit hydrographs (Lamar, TTU)
  - Generalized gamma and **geomorphic IUH** (UH).

Estimating Timing Parameters

- Analysis
  - Use actual rainfall and runoff data for a watershed.
- Synthesis
  - Absence of data, sub-watershed, etc.
  - Variety of formulas for timing parameters:
    - Characteristic length
    - Characteristic slope
    - Flow regimes (overland, concentrated, channel)
    - Friction, conveyance, land type, etc.
  - Loss models

Estimating Timing Parameters

- Representative formulas:
  - Overland Flow:

NRCS travel-time method (NRCS, 2004) was implemented using the following equation and Manning's equation (McCuen, 2005),

$$T_t^l = \frac{L}{60 V_t} \text{ and} \qquad (1)$$

$$V_t = \frac{1.486}{n} R_h^{0.67} S^{0.5}, \qquad (2)$$

Kerby (1959) provides a method to estimate T_t^l using the following equation

$$T_t^l = \left[\frac{0.67(L \times N)}{S^{0.5}}\right]^{0.467},$$

The KWF, suggested by Morgali and Linsley (1965) and Aron and Erborge (1973), was implemented using

$$T_t^l = \frac{0.94(L \times n)^{0.6}}{i^{0.4} S^{0.3}}, \qquad (4)$$

Estimating Timing Parameters

- Representative formulas:
  - Channel Flow

The NRCS travel-time method (NRCS, 2004) was implemented for shallow-concentrated and channel flow using eq. 1, by substituting the length of shallow-concentrated flow or the main channel length for L as appropriate. The average velocity (V_t) was computed for shallow-concentrated flow and channel flow by solving Manning's equation (eq. 5) and extracting average velocity using continuity (eq. 6). The equations are as follows

$$Q = \frac{1.486}{n} A R_h^{0.67} S^{0.5}, \text{ and} \quad (5)$$

$$Q = A V_t, \quad (6)$$

The Kirpich (1940) method was implemented using the following equation,

$$T_t^l = 0.0078 L^{0.77} S^{-0.385}, \quad (7)$$

The Haktanir and Sezen (1990) method was implemented using the following equation

$$T_L = 0.401 L_m^{0.841} \text{ and} \quad (8)$$

$$T_c = \frac{T_L}{0.6}, \quad (9)$$

Estimating Timing Parameters

- The formulas beg the questions:
  - Which "lengths, slopes, friction factors" ?
  - What is "bankful discharge" on an ungaged watershed ?
  - Which "paths" to examine ?

Statistical-Mechanical Hydrograph

- Leinhard (1964) postulated that the unit hydrograph is a raindrop arrival time distribution.

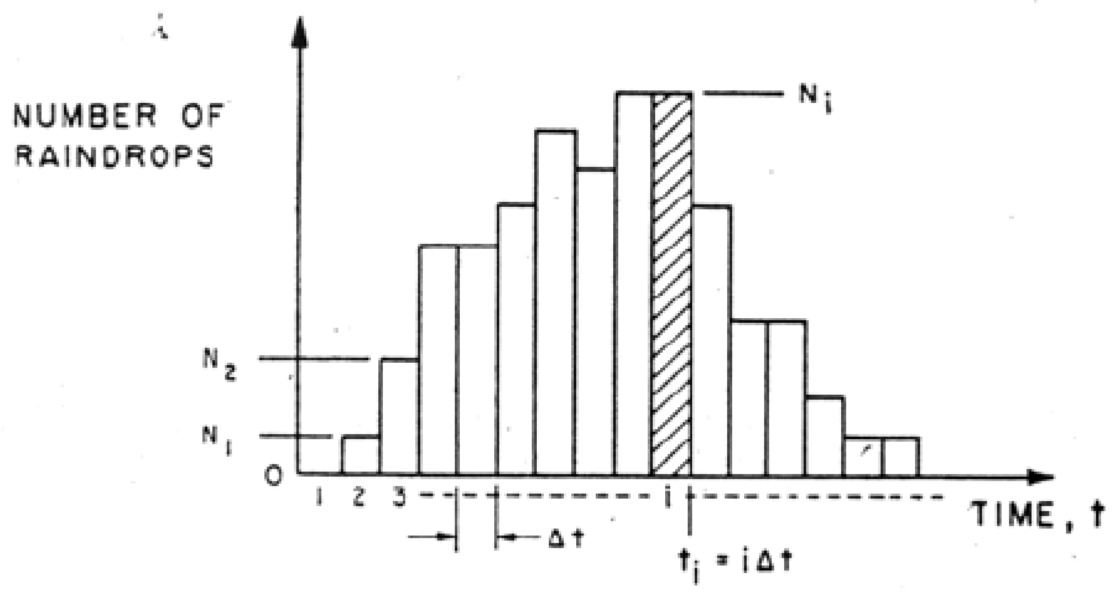


Fig. 1. The hydrograph interpreted as a raindrop histogram.

Statistical-Mechanical Hydrograph

- Further Assumed:
  - The arrival time of a raindrop is proportional to the distance it must travel, *l*.
  - The number of drops arriving at the outlet in a time interval is proportional to the square of travel time (and l^2).
  - By enumerating **all** possible arrival time histograms, and selecting the most probable from maximum likelihood arrived at a probability distribution that represents the temporal redistribution of rainfall on the watershed.

Statistical-Mechanical Hydrograph

- Resulting distribution is a generalized gamma distribution.

$$f(t) = \frac{\beta}{\Gamma(n/\beta)}\left(\frac{n}{\beta}\right)^{n/\beta}\frac{1}{t_{rm\beta}}\left(\frac{t}{t_{rm\beta}}\right)^{n-1}\exp\left[-\frac{n}{\beta}\left(\frac{t}{t_{rm\beta}}\right)^{\beta}\right]$$

- The distribution parameters have physical significance.
- $t_{rm\beta}$ is a mean residence time of a raindrop on the watershed.
- ***n***, is an accessibility number, related to the exponent on the distance-area relationship (a shape parameter).
  - β, is the degree of the moment of the residence time;
  - $\beta = 1$ is an arithmetic mean time
  - $\beta = 2$ is a root-mean-square time

Statistical-Mechanical Hydrograph

- Conventional Formulation:

$$Q_p = f(T_p) \qquad t_{rm\beta} = \left(\frac{n}{n-1}\right)^{1/\beta} T_p$$

$$\frac{Q}{Q_p} = \left(\frac{t}{T_p}\right)^{n-1} \exp\left[-\frac{n-1}{\beta}\left(\left(\frac{t}{T_p}\right)^{\beta} - 1\right)\right]$$

Estimating Timing Parameters

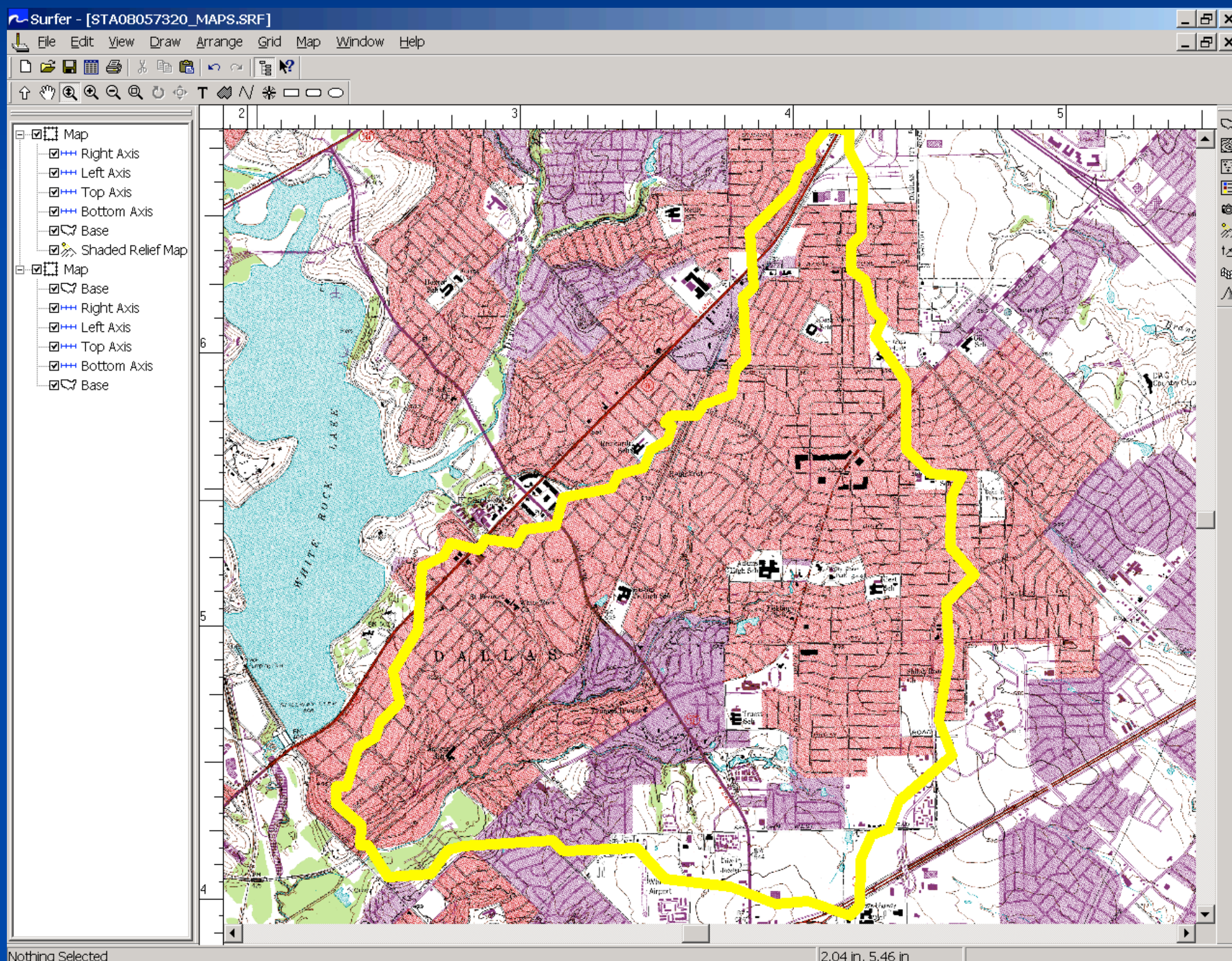
- The derivation based on enumeration suggests an algorithm to approximate watershed behavior.
  - Place many "raindrops" on the watershed.
  - Allow them to travel to the outlet based on some reasonable kinematics. (Explained later - significant variable is a "*k*" term - represents friction)
  - Record the cumulative arrival time.
  - Infer $t_{rm\beta}$ and ***n*** from the cumulative arrival time distribution.
  - The result is an instantaneous unit hydrograph.

Estimating Timing Parameters

- Illustrate with Ash Creek Watershed
  - Calibration watershed – the "*k*" term was selected by analysis of one storm on this watershed, and applied to all **developed** watersheds studied.
  - About 7 square miles. (20,000+ different "paths")

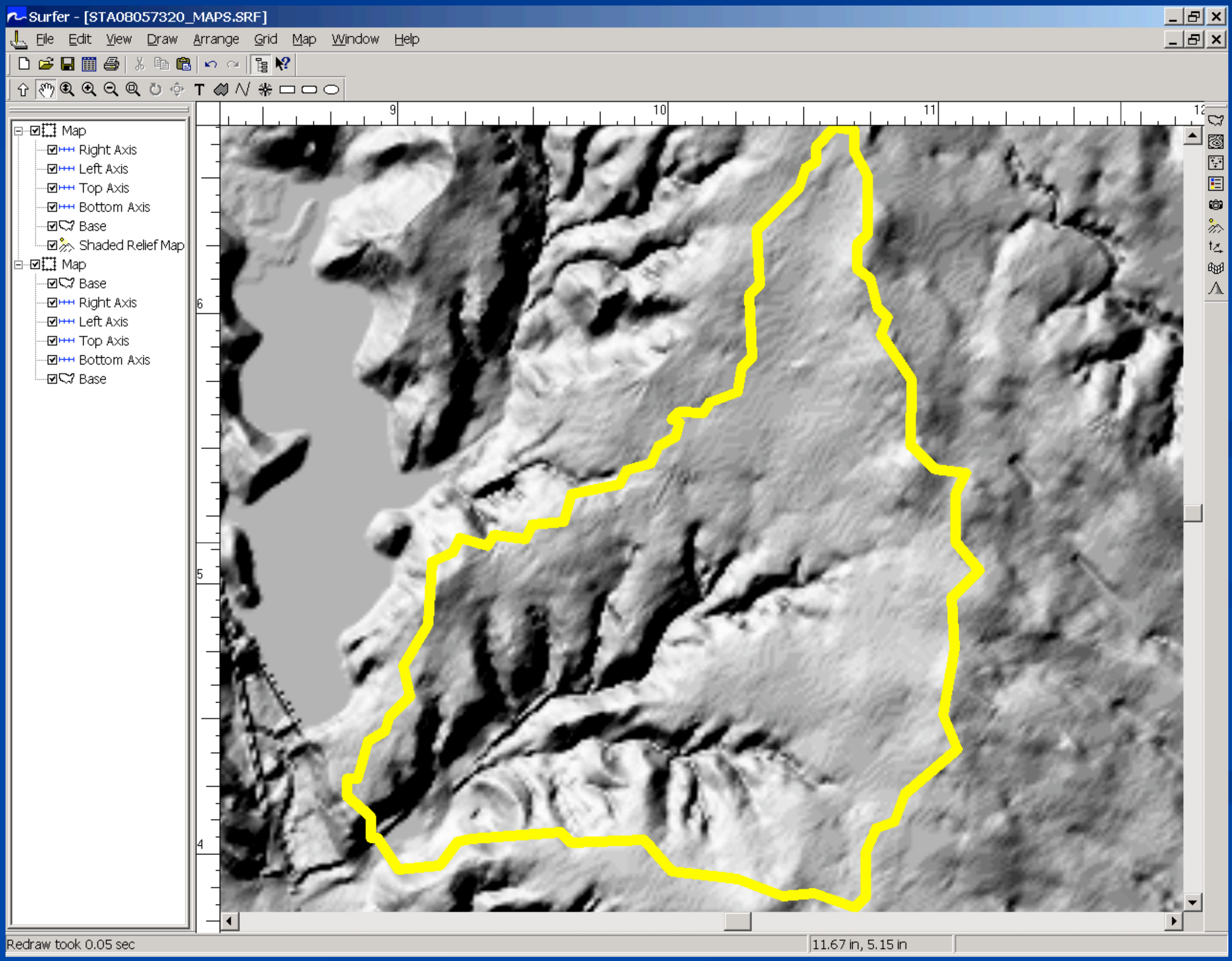
Ash Creek Watershed

(sta08057320)



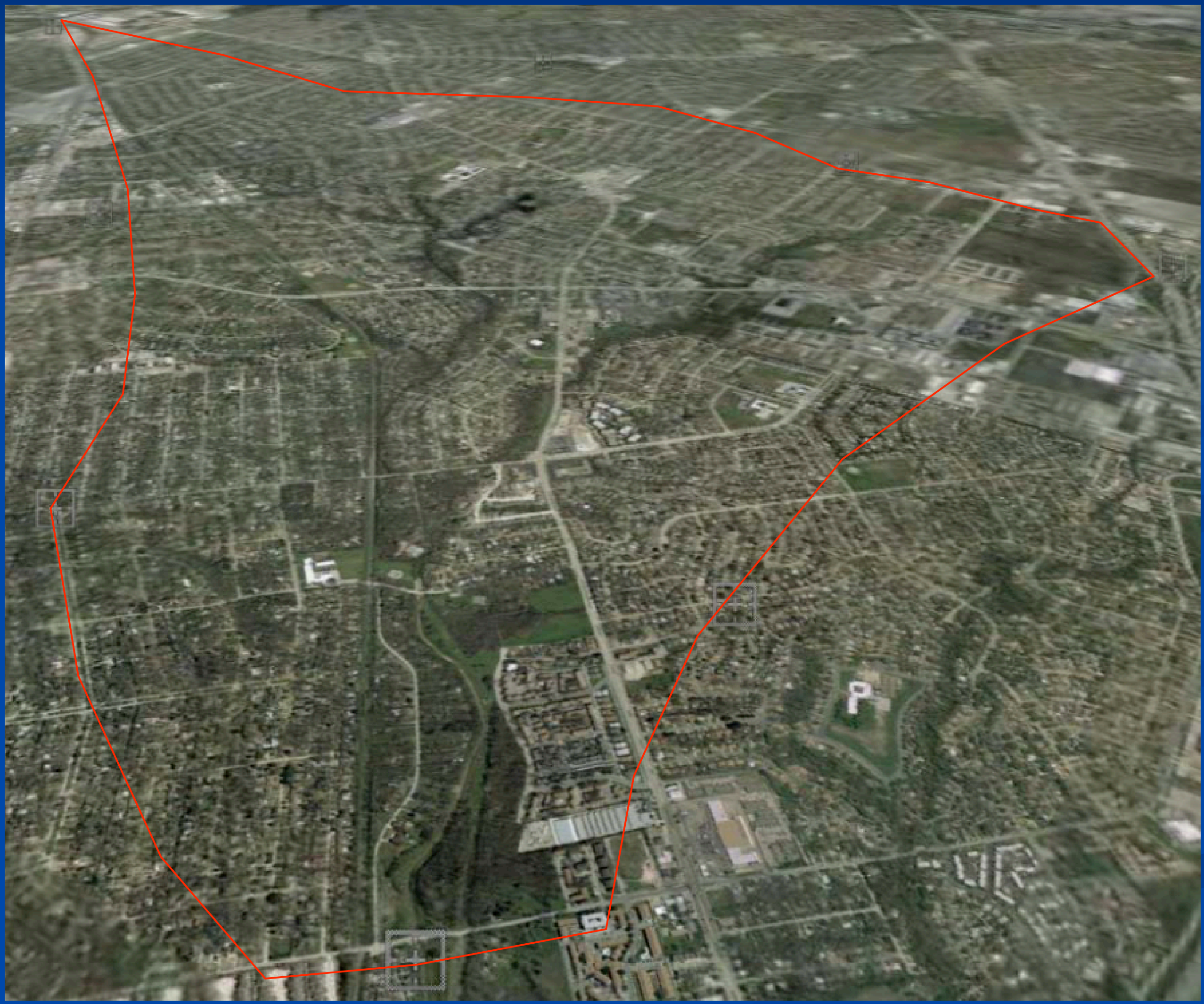
Ash Creek Watershed

(sta08057320)



Ash Creek Watershed

(sta08057320)



Estimating Timing Parameters

- Place many “raindrops” on the watershed.

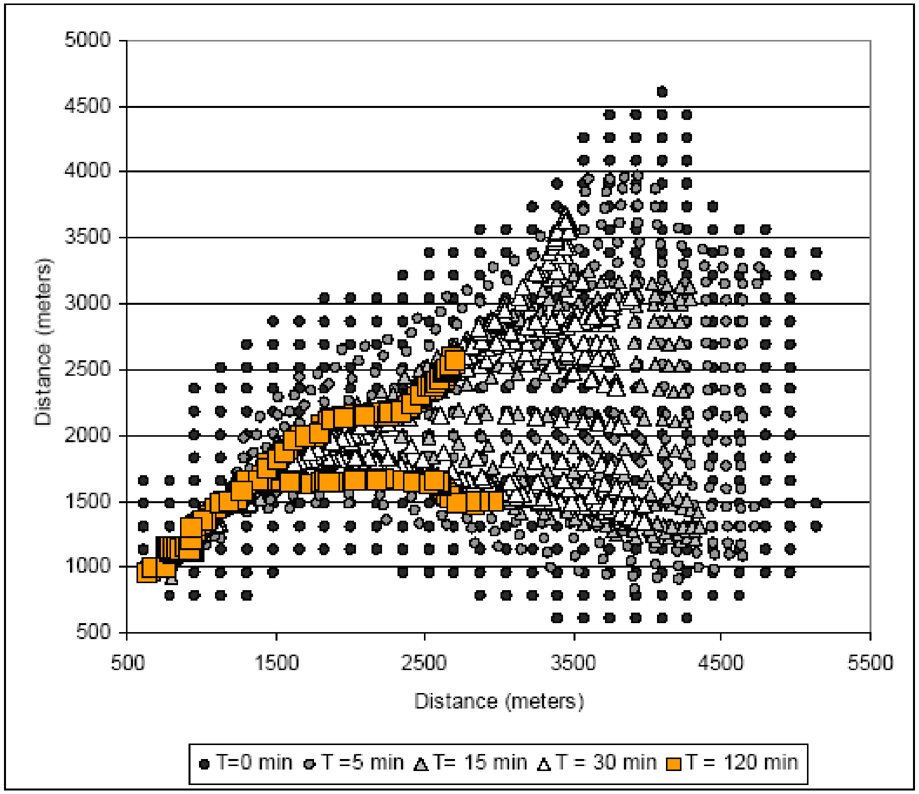


Figure 5 Particle positions at various times; Ash Creek watershed.

Estimating Timing Parameters

- Allow them to travel to the outlet based on some reasonable kinematics.
  - Path determined by 8-cell pour point model.
  - Speed from quadratic-type drag, *k* selected to "look" like a Manning's equation.

$$u(\xi) \cdot \left| u(\xi) \right| = k^2 * \left. \frac{dz}{d\xi} \right|_{(\xi)}$$

$$k = \frac{1.5}{n_f} d^{\frac{2}{3}}$$

Estimating Timing Parameters

- Allow them to travel to the outlet based on some reasonable kinematics.
  - Path determined by 8-cell pour point model.
  - Speed from local topographic slope and *k*
  - Each particle has a unique pathline.
  - Pathlines converge at outlet.

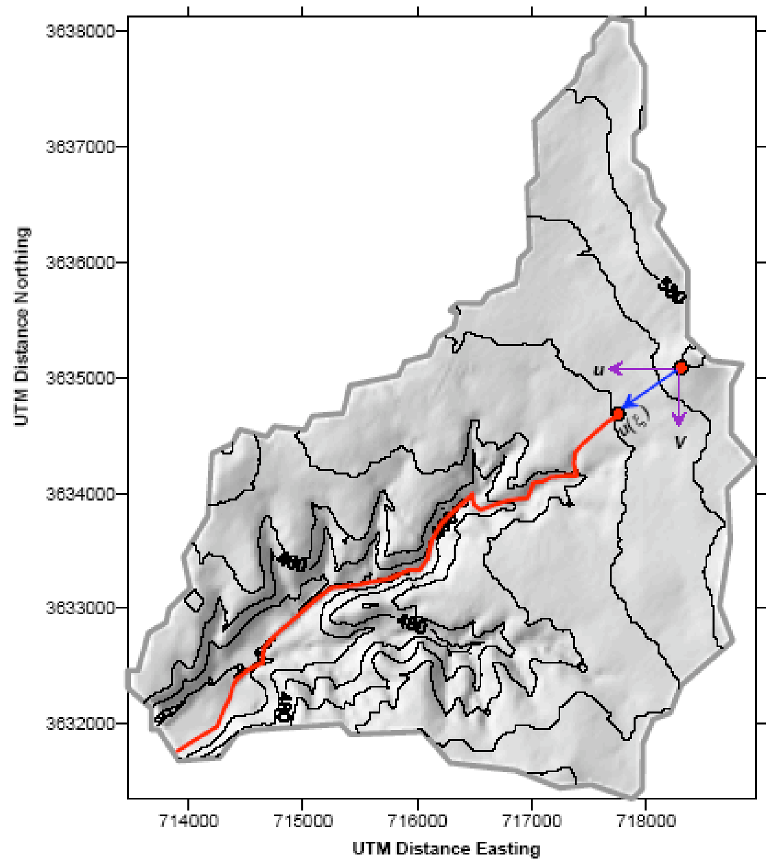


Figure 2. Shaded relief map of Ash Creek watershed.

Estimating Timing Parameters

- Record the cumulative arrival time.

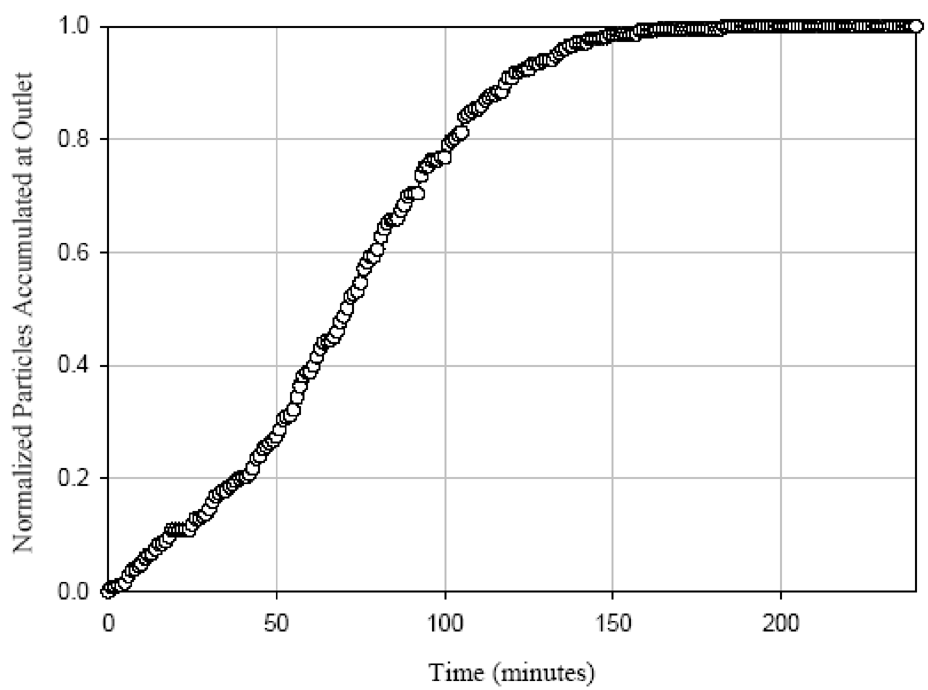
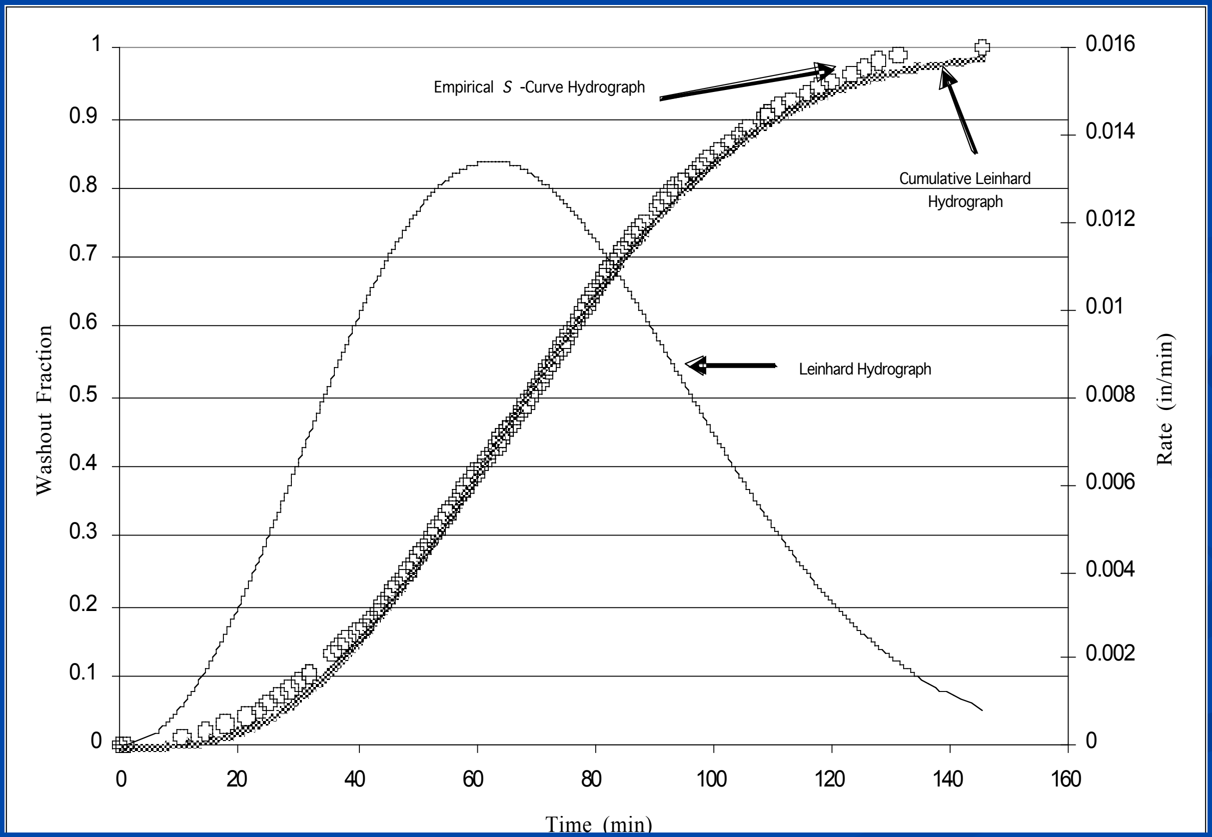


Figure 6. Empirical S-curve hydrograph.

Estimating Timing Parameters

- Infer $t_{rm\beta}$ and ***n*** from the cumulative arrival time distribution.



Estimating Timing Parameters

- The result is an instantaneous unit hydrograph (IUH).
  - IUH and observed storm to produce simulated runoff hydrograph.
  - The *k* is adjusted, particle tracking repeated until the observed and simulated hydrographs are the same.
  - This *k* value is then used for all watersheds.
  - Only change from watershed to watershed is topographic data (elevation maps)

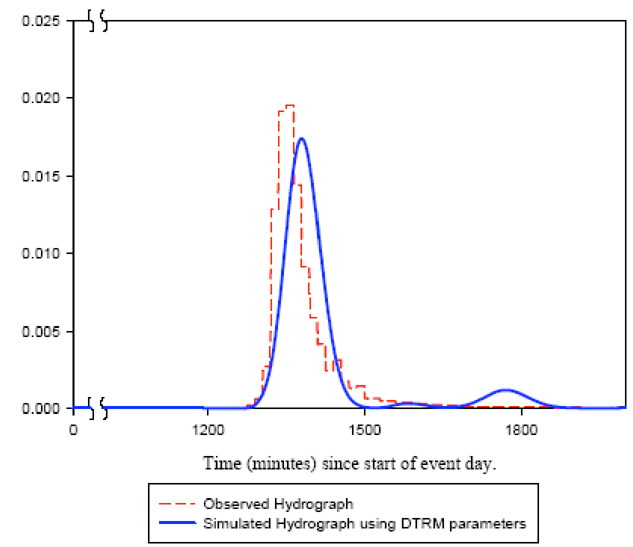
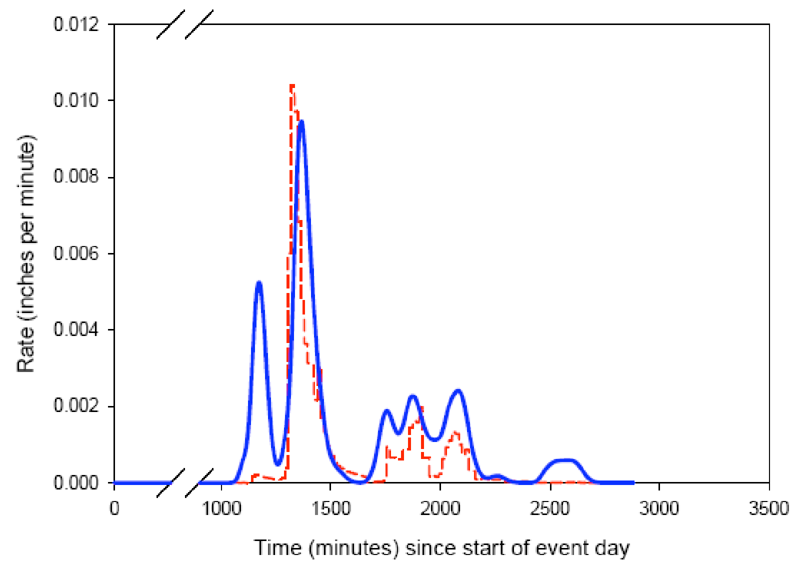


Figure 4. Observed and Modeled Runoff Hydrograph for Ash Creek Watershed, June 1973 storm.

Estimating Timing Parameters

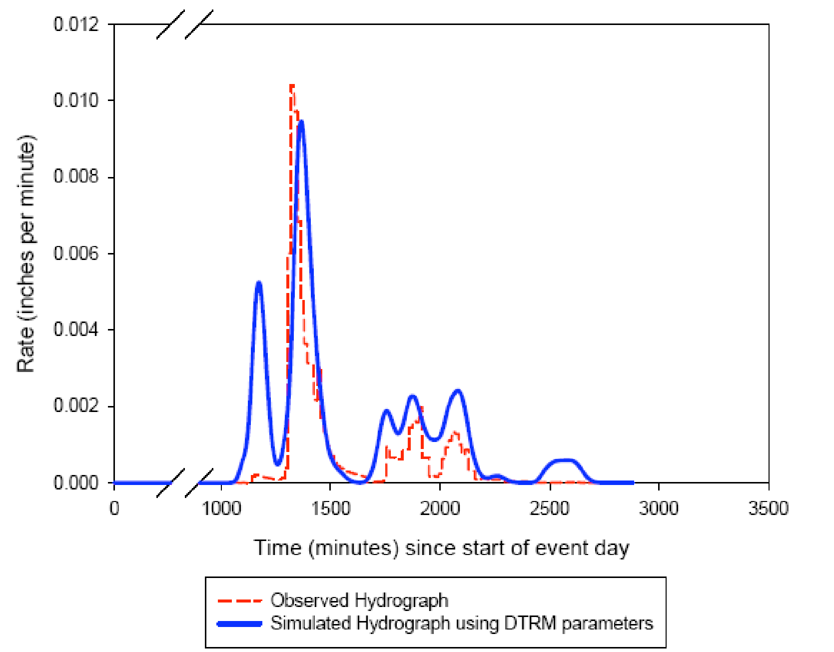
- Typical result
  - Ash Creek Watershed
    - May 1978 storm
    - IUH from the calibrated June 1973 storm.



Observed and Simulated Hydrograph for Ash Creek Watershed, May, 1978 Storm

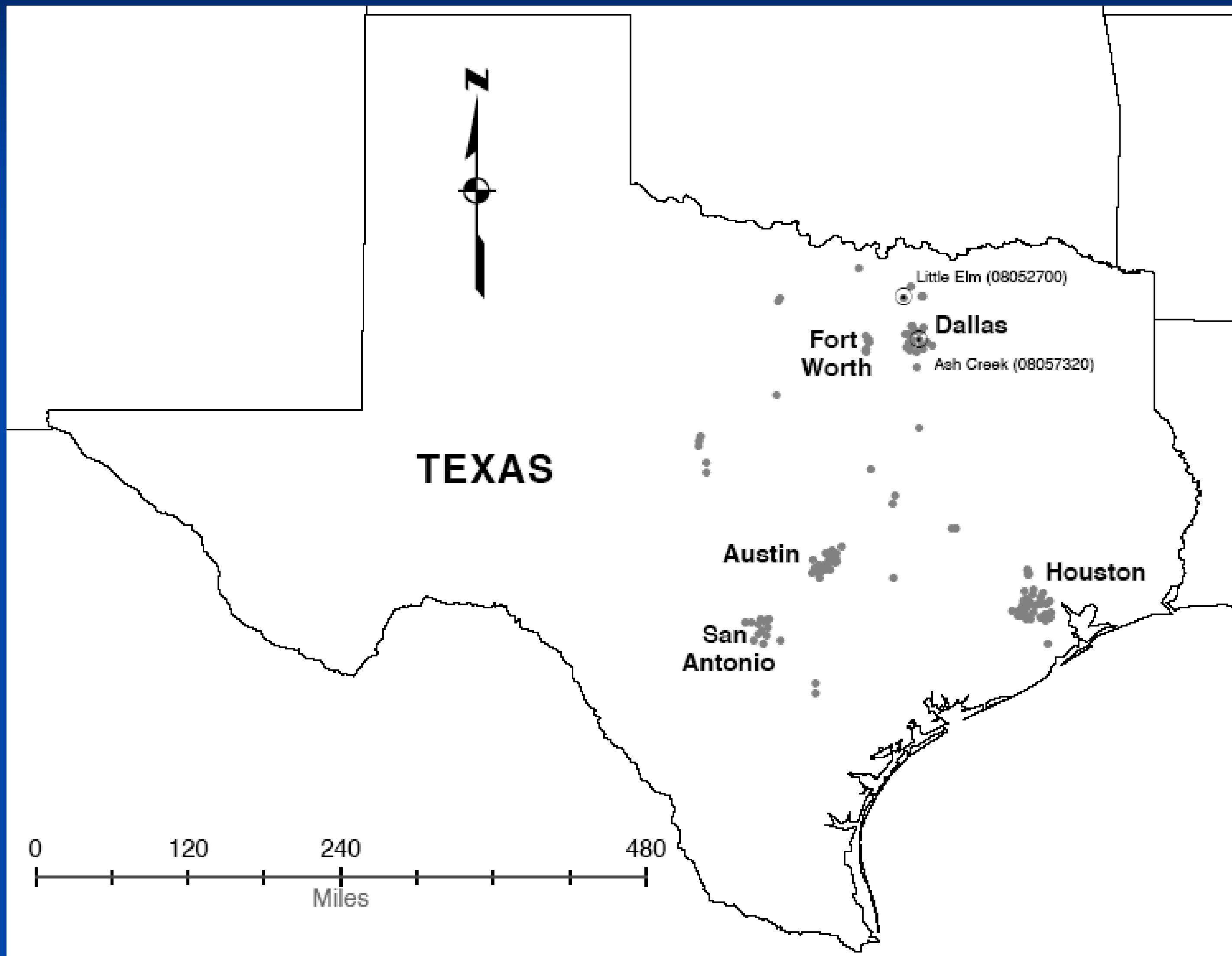
Estimating Timing Parameters

- Typical result
  - Ash Creek Watershed
    - May 1978 storm
    - IUH from the calibrated June 1973 storm.



Observed and Simulated Hydrograph for Ash Creek Watershed, May, 1978 Storm

Study Watersheds

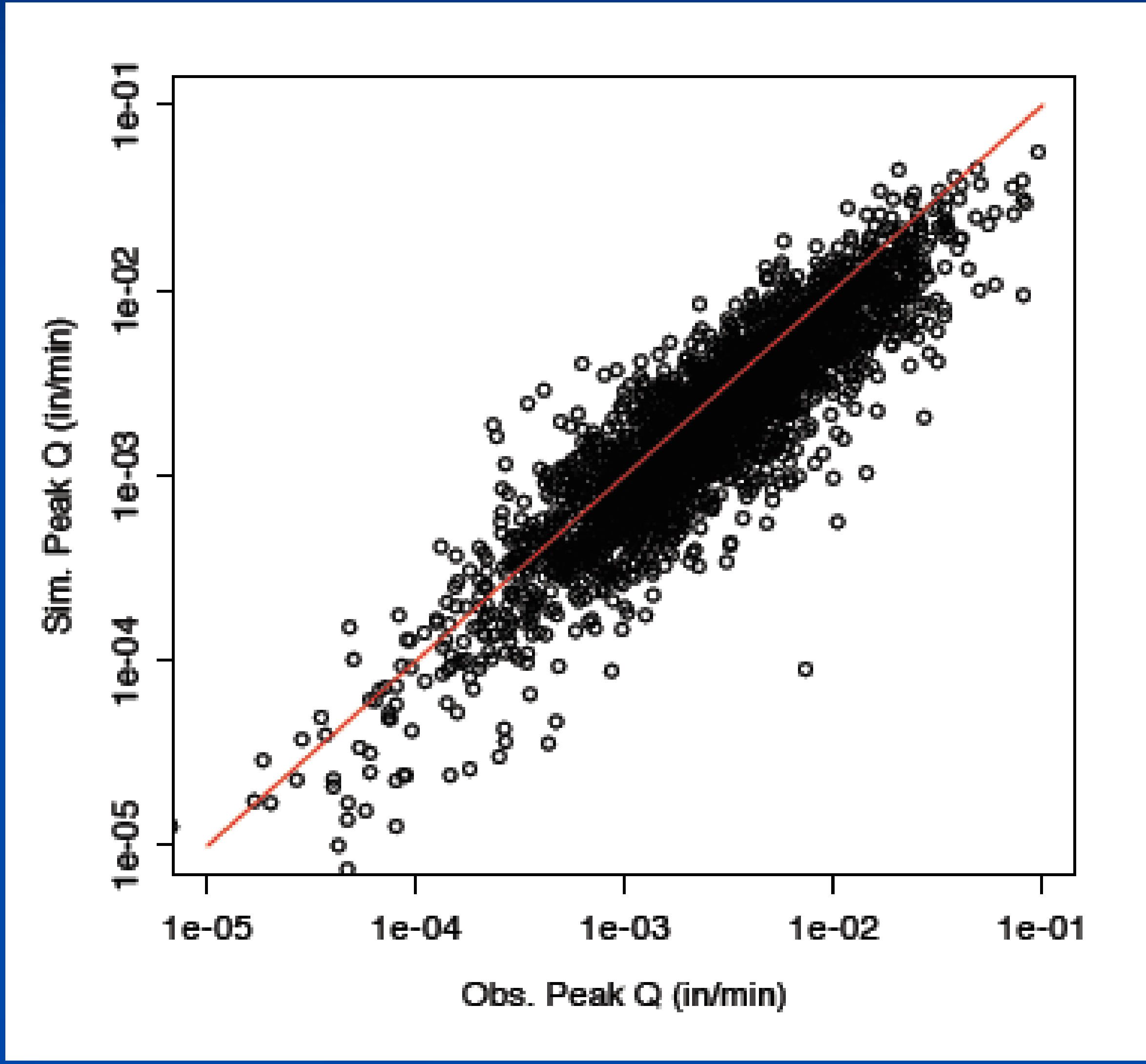


Study Watersheds

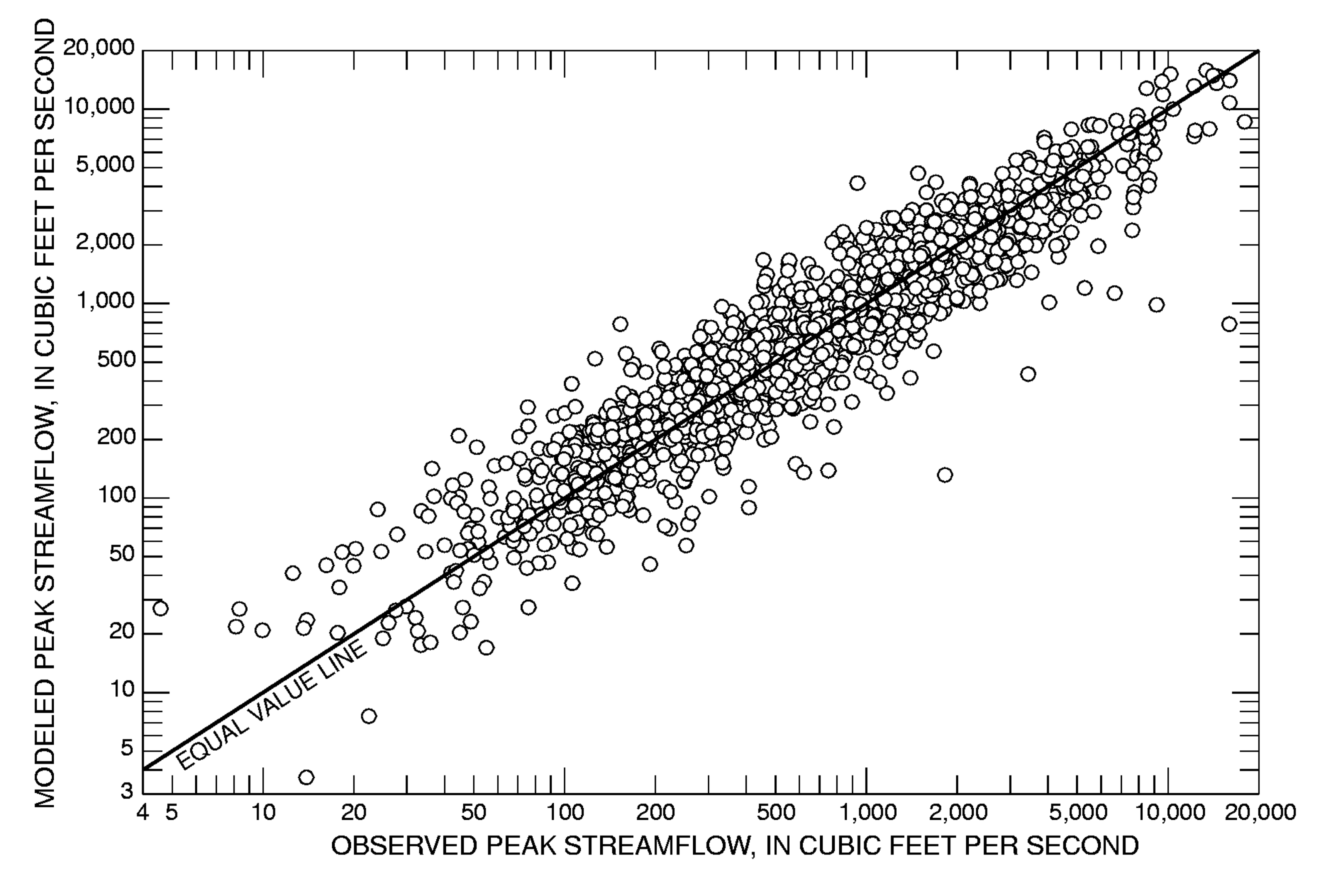
- 130+ watersheds
  - 2600 paired rainfall-runoff events studied. (Data base now has over 3,400 storms)
  - Most stations have 5 or more storms, some nearly 50 events.
  - Watershed boundaries, etc. determined by several methods.
  - Using the single value of "*k*" determined on the Ash Creek calibration event, applied the particle tracking approach to all **developed** watersheds. A second value of "*k*" for **undeveloped** watersheds is obtained from Little Elm watershed in an identical fashion.

Illustrative Results (GIUH)

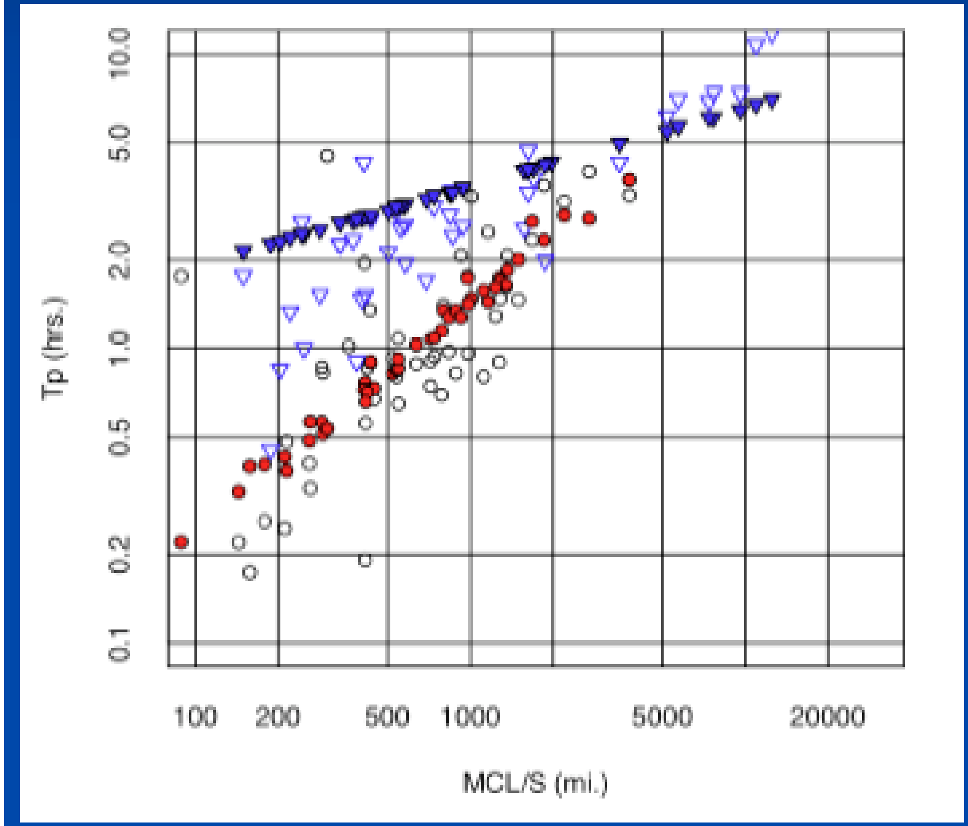
- Peak comparison.
- Bias (low)
  - "*k*" value same all developed.
  - "*k*" value same all undeveloped.

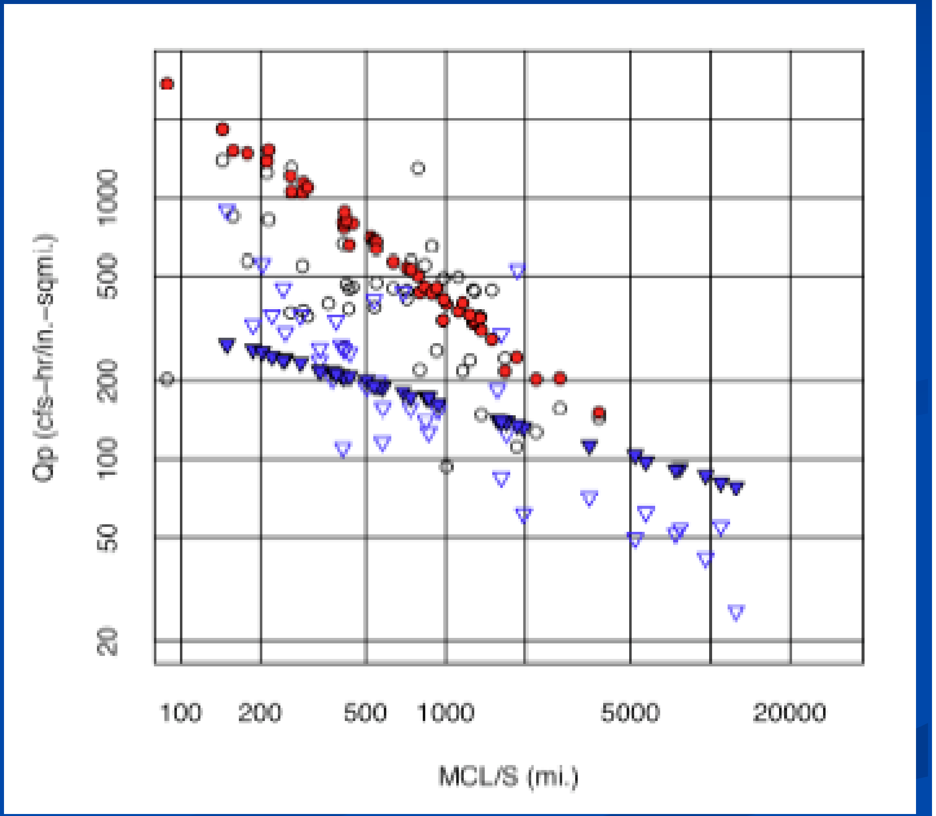


Illustrative Results (CONV)

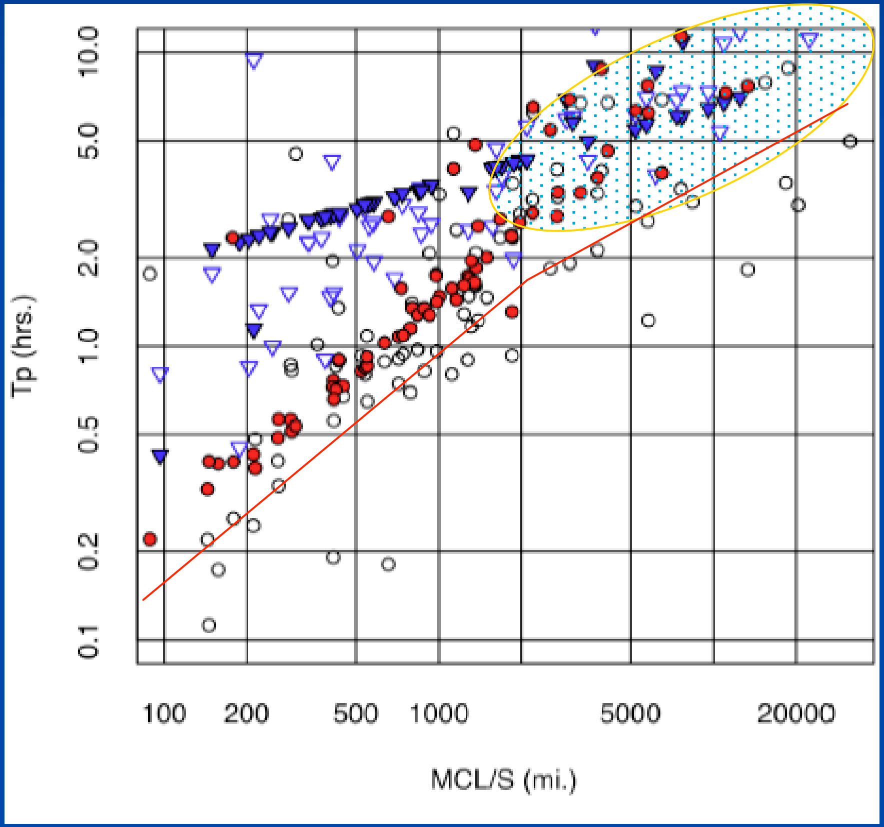


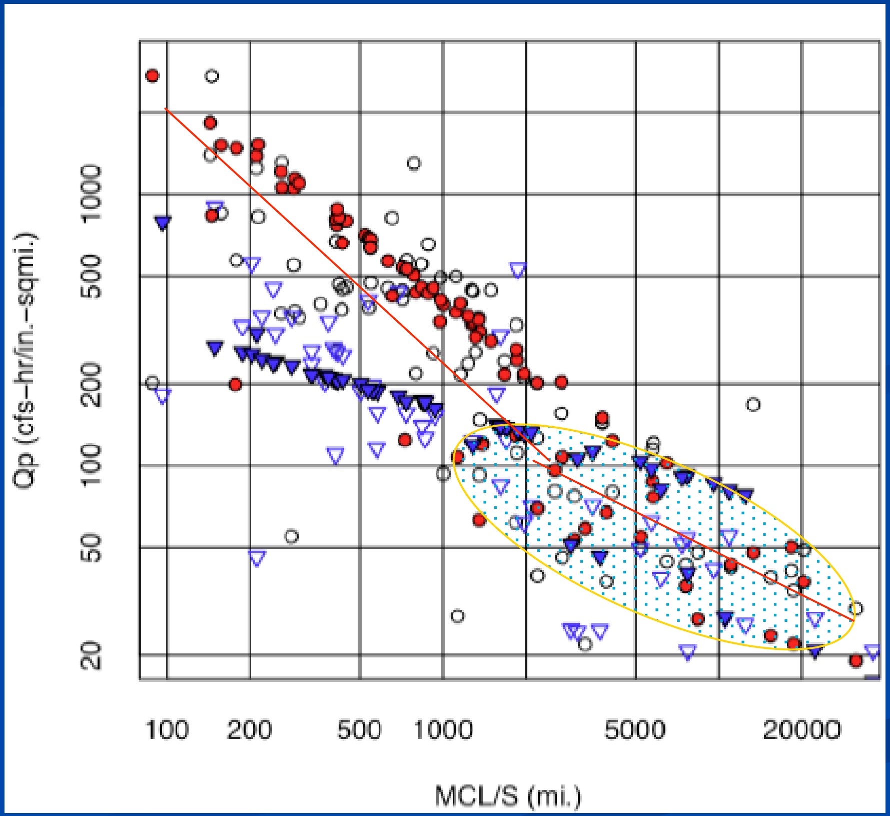
Development Distinction





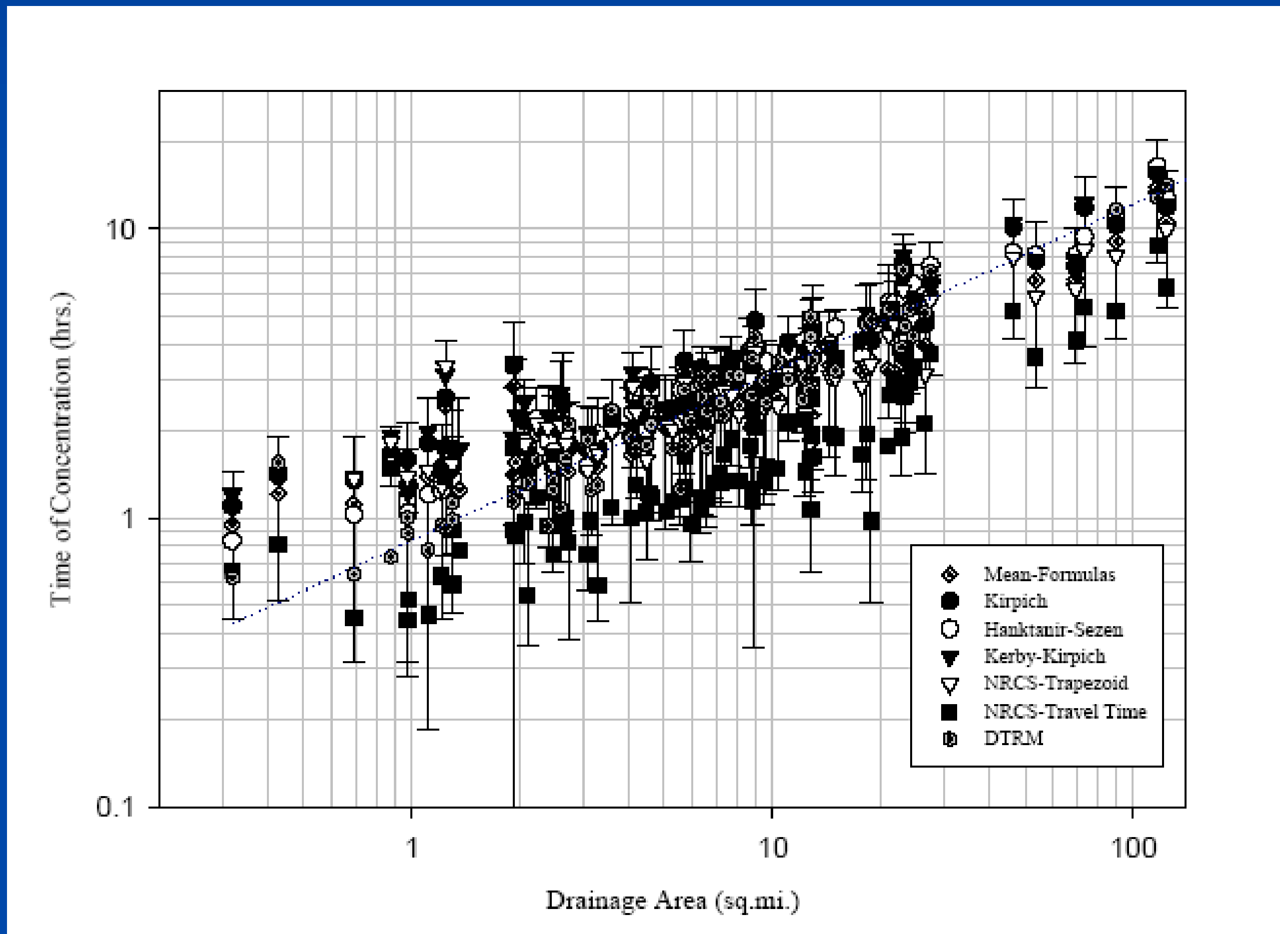
Development Distinction





Quantitative Results

- Comparison to other methods



Conclusions

- The terrain-based generates qualitatively acceptable runoff hydrographs from minimal physical detail of the watershed.
- The approach simulated episodic behavior at about the same order of magnitude as observed behavior, without any attempt to account for land use.

Conclusions

- For the watersheds studied, topography is a significant factor controlling runoff behavior and consequently the timing parameters common in all hydrologic models.

Publications

- http://library.ctr.utexas.edu/dbtw-wpd/textbase/websearchcat.htm (Search for authors: Asquith; Roussel; Thompson; Fang; or Cleveland).
- http://cleveland1.cive.uh.edu/publications (selected papers on-line).
- http://infotrek.er.usgs.gov/pubs/ (Search for author Asquith; Roussel)
- http://www.techmrt.ttu.edu/reports.php (Search for author Thompson)
- http://ceserver.lamar.edu/People/fang/research.html