Storage Coefficients and Vertical Hydraulic Conductivities in Aquitards Using Extensometer and Hydrograph Data

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Abstract

A method is presented to estimate elastic and inelastic specific storage and vertical hydraulic conductivities in aquitards in multilayered flow systems. Borehole extensometer records and ground-water hydrographs from piezometers are used to construct plots of effective stress and deformation. Elastic and inelastic specific storage are estimated from the plots of effective stress and deformation during loading and unloading cycles. The elapsed loading time is estimated from the same plots and is used to calculate vertical hydraulic conductivity using Terzaghi's consolidation theory. The method is applied to three sites in Houston, Texas.

Introduction

The analysis of ground-water flow in compressible aquifer systems is a topic of great interest particularly in coastal aquifers where land subsidence is a vital concern due to storms and flooding. The response to pumping of aquifers bounded by low permeability strata (relative to the aquifer itself), or aquitards, is highly dependent on the physical characteristics of the aquitards. An understanding of their behavior is important for the study of land subsidence and contaminant transport in compressible aquifer systems.

Water released from storage in compressible aquifer systems comes from three sources: the expansion of the water itself, the compression of the aquifer, and the compression of the semipermeable confining beds adjacent to and within the aquifer. When the pumping does not cause permanent rearrangement of the skeletal structure and the water can be replaced by increasing the pore pressure, the process is called elastic compression. However, if the pumping is severe enough to cause permanent deformation, the process is called inelastic compression. Storage lost during inelastic compression cannot be recovered.

Figure 1 is a schematic of a multilayered aquiferaquitard system. The governing equation of horizontal ground-water flow in any of the isotropic aquifer layers is (Marsily, 1986)

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$$\frac{\partial}{\partial x} \left(T_i \frac{\partial h_i}{\partial x} \right) + \frac{\partial}{\partial y} \left(T_i \frac{\partial h_i}{\partial y} \right) + q_{pi} + q'_{lji} - q'_{lki} =$$

$$S_i \frac{dh_i}{dt} \qquad j, k = 1, 2, \dots, m; j \neq k$$
(1)

where h_i is the hydraulic head in the i-th aquifer; T_i is the transmissivity in the i-th aquifer; S_i is the aquifer storativity in the i-th aquifer; q_{pi} is the source/sink term for a well in the i-th aquifer; q'_{lki} is the leakage from j-th aquitard into the i-th aquifer; q'_{lki} is the leakage term from k-th aquitard into the i-th aquifer, i = 1, 2, ..., n; n is the number of aquifer layers; and m is the number of aquitard layers. A confined flow approximation is implicitly assumed for the upper aquifer (aquifer #3 in Figure 1), which is depicted as unconfined.

The vertical flow (leakage flux) in any of the aquitards is (Marsily, 1986)

$$\frac{\partial}{\partial z} (q'_{lj.}) = \frac{\partial}{\partial z} (K'_{j} \frac{\partial \phi_{j}}{\partial z}) = S'_{j} \frac{d\phi_{j}}{dt}$$
 (2)

where q'_{ij} is the vertical leakage in aquitard j; K'_{j} is the vertical hydraulic conductivity in aquitard j; S'_{j} is the storage coefficient in aquitard j; and ϕ_{j} is the hydraulic head in aquitard j. Equations (1) and (2) are coupled through the boundary conditions:

$$\phi_{j}(x, y, z, 0) = \phi_{j0}$$
 (3)

$$\phi_i(x, y, z_u, t) = h_u(x, y, t)$$
 (4)

$$\phi_i(x, y, z_l, t) = h_l(x, y, t)$$
 (5)

where ϕ_{j0} is the initial hydraulic head in aquitard j; z_u is the location of the upper interface of the j-th aquitard with aquifer u; z_l is the location of the lower interface of j-th aquitard with the l-th aquifer; h_u is the hydraulic head in the

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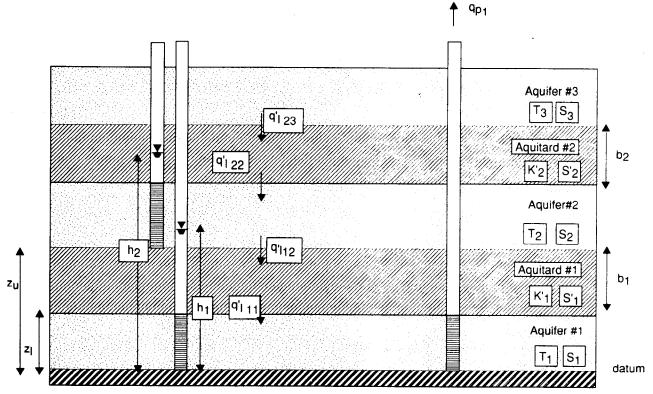


Fig. 1. Multilayered aquifer system.

aquifer above the j-th aquitard; and h_l is the hydraulic head in the aquifer below the j-th aquitard. The thickness of the aquitard is the difference between the upper and lower interface,

$$b = z_u - z_l \tag{6}$$

Hantush and Jacob (1955) presented a solution describing the response of a three-layer (aquifer-aquitard-aquifer) system where the aquitard storage is assumed to be negligible. In this case the leakage flux is proportional to the hydraulic gradient across the aquitard. Their solution is applicable to aquitards of overconsolidated sediments or where the upper aquifer receives enough recharge to balance downward leakage. Hantush (1960) extended his approach to account for aquitard storage. More general approaches for such systems have been developed by Neuman and Witherspoon (1969), Herrera and Figueroa (1969), and Frind (1979). A difficulty in applying any of these techniques is determining reasonable values of the vertical hydraulic conductivity and the storage coefficient of the aquitard.

In this paper we describe a method to determine vertical hydraulic conductivity and specific storage for a simple layered system using piezometer data, extensometer data, and principles of soil mechanics.

Specific Storage and Storage Coefficients

The specific storage of a saturated, compressible porous media is expressed as (Bear, 1972)

$$S'_{s} = \rho g(\alpha + n\beta) \tag{7}$$

where ρ is the fluid density; g is the gravitational acceleration; α is the solid matrix compressibility; n is the porosity; and β is the fluid compressibility. The matrix compressibility is defined as

$$\alpha = -\left(\frac{\Delta V_b}{\Delta \sigma}\right) \left(\frac{1}{V_b}\right) \tag{8}$$

where V_b is the bulk volume of a sample; and $\Delta \sigma$ is the incremental effective stress (loading) on the sample. The compressibility of water is usually negligible compared to the solid matrix compressibility (Domenico and Mifflin, 1965) and the specific storage is approximated using

$$S_s' = \rho g \alpha \tag{9}$$

The storage coefficient of a geologic unit that has thickness, b, is the product of the specific storage and the thickness,

$$S' = S_s'b \tag{10}$$

Loading and Deformation

Figure 2 depicts the conceptual relationship of loading and deformation used in this paper. At some effective stress (loading) value, σ , a sample has a thickness, b, and a void ratio ϵ_0 . When the effective stress is increased by an increment $\Delta\sigma$, the material deforms. In this paper we assume only vertical deformation. The thickness and void ratio become smaller, and liquid is released from the material. When the material reaches equilibrium with the new effective stress, it has thickness, $b-\Delta b$, and void ratio, $\epsilon_0-\Delta \epsilon$. The time for equilibrium to be reached is dependent on how quickly liquid is lost from the material.

Dimensionless Time and Degree of Consolidation

Frind (1979) demonstrated that the hydraulic response of an aquitard is controlled by a dimensionless time which he expressed as

$$t_{d} = \frac{K't}{S'_{s}b^{2}} \tag{11}$$

where t is the time since an instantaneous head change is imposed on one of the aquitard boundaries. This time is identical to the dimensionless time $T_v(t_d=T_v)$ in Terzaghi's consolidation theory and can be used to estimate vertical hydraulic conductivity for a compressible stratum if the degree of compression, the elapsed time, and the specific storage are known. The degree of compression, U, is defined as the ratio of compression at time t to the ultimate compression or

$$U = \frac{\Delta b_t}{\Delta b_u} \tag{12}$$

where Δb_t is the compression at time t, and Δb_u is the ultimate compression (at $t = \infty$). The relationship of T_v and U can be found by solving Terzaghi's consolidation equations. Table 1 is a set of solutions for different initial water pressure distributions. Typically, the linear pressure distribution is used (Leonards, 1962). K' is calculated from T_v using the known time, t, degree of consolidation, U, the specific storage, S's, and the layer thickness, b.

Swelling and Compression Index

Figure 3, typical for a laboratory consolidation test, depicts the relationship between the logarithm of effective stress, σ , and the void ratio, ϵ , for a loading-unloading-reloading cycle showing inelastic compression. The initial branch (ab) has a relatively flat slope. At an effective stress close to the value σ_c , the curve undergoes a significant

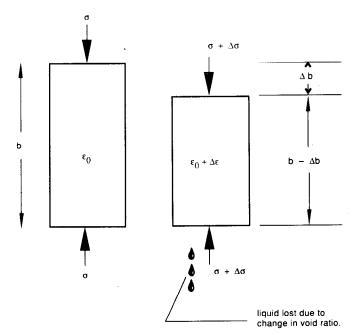


Fig. 2. Conceptual diagram of soil deformation.

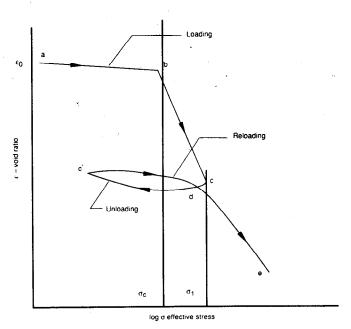


Fig. 3. Void ratio—log effective stress curve.

change. The second branch (bc) of the curve exhibits a much steeper slope. If, after the sample has been loaded to an effective stress of σ_1 , it is unloaded and then reloaded, the unloading and reloading curves form a hysteresis loop. When the effective stress again exceeds σ_1 , the reloading curve approaches (ce), which is an extension of the loading branch (bc). The effective stress, σ_c , is called the preconsolidation stress. The slope of the loading branch (bc) is called the compression index C_c. During unloading the material behaves elastically and the slope of this branch (cc') is called the swelling index C_s. The slope of the reloading branch (c'd) is called the recompression index C_{rc}. The slope of the unloading branch (cc') is normally near the value of the reloading branch (c'd) and the swelling index and recompression index are about the same for practical purposes (Schmertmann, 1953). The slope of the reloading branch as it crosses the unloading branch is near the value of the compression index.

Bravo (1990) observed that plots of deformation and effective stress for several sites in Houston showed hysteresis loops. Bravo interpreted these loops as evidence of loading and unloading cycles and suggested that these observations be used to determine the elastic and inelastic behavior of compressible aquitard layers.

Soil Compaction Indices and Storage Coefficients

Assuming only vertical deformation as depicted in Figure 2, the relationship of deformation to void ratio can be written as (Bravo, 1990)

$$\frac{\Delta b}{b} = \frac{\Delta \epsilon}{1 + \epsilon_0} \tag{13}$$

where Δb is the change in thickness of the compressible material; b is the initial thickness of the compressible material; $\Delta \epsilon$ is the incremental change in void ratio; and ϵ_0 is the initial void ratio before the stress change.

The definition of matrix compressibility, α , can then be used to express the elastic and inelastic specific storage coefficients, S_{ske} and S_{skv} respectively, as functions of effective stress, void ratio, and the swelling or compression index (Bravo, 1990).

$$S_{ske} = -\gamma_w C_s(\frac{0.434}{1 + e_0})(\frac{1}{\sigma}) \quad \text{(elastic)} \quad (14)$$

$$S_{skv} = -\gamma_w C_c \left(\frac{0.434}{1 + e_0} \right) \left(\frac{1}{\sigma} \right) \quad \text{(inelastic)} \quad (15)$$

where γ_w is the specific weight of water; and the incremental changes in stress are small.

These equations can be used to estimate the specific storage of an aquitard unit given the compression and swelling index, which can be obtained from laboratory tests. The formulation is identical, except for notation, to the nonlinear formulation explored by Rudolph and Frind (1991). Bravo (1990) concluded that for the changes in effective stress typically encountered in Houston's ground-water flow systems, a constant specific storage coefficient for each regime (elastic and inelastic) is adequate. Gambolati and Freeze (1973) made a similar conclusion when simulating land subsidence in Venice, Italy. This conclusion is significant because it allows one to estimate the values of the storage coefficients directly from piezometer and extensometer data instead of performing lab tests to determine the swelling and compression indices.

Effective Stress

The effective stress for a material a distance z from the surface can be written

$$\sigma = \gamma_s z + \gamma_w (z - h) \tag{16}$$

where γ_s is the specific weight of the porous medium above z; γ_w is the specific weight of the water; and h is the hydraulic head with reference level z = 0. The change in effective stress is directly related to the change in hydraulic head as

$$\Delta \sigma = -\gamma_{\mathbf{w}} \Delta \mathbf{h} \tag{17}$$

Procedure to Estimate Storage Coefficients and Hydraulic Conductivity

Figure 4 shows an idealized stress-deformation plot for some compressible layer. The slope of the decompression loop is used to approximate the elastic storage coefficient for the layer from

$$S_{ke} = (\frac{\Delta b}{\Delta \phi}) \tag{18}$$

where Δb is the change in deformation of the compressible layer; b is the thickness of the layer studied; and $\Delta \phi$ is the change in effective stress expressed in height of water. The permanent deformation (for the layer) is estimated by extending a line with slope equal to $1/S_{ke}$ from the point of minimum stress to an interception with the deformation axis. The permanent deformation is estimated as the offset from this point of intersection to the projection of the minimum stress point onto the cumulative consolidation

Table 1. Degree of Consolidation and Dimensionless Time, T_v, for Different Initial Pressure Distributions

U (%)	Linear	Half-sine	Sine	Triangle	
0	0.0	0.0	0.0	0.0	
5	0.0017	0.0021	0.0208	0.0247	
10	0.0077	0.0114	0.0427	0.0500	
15	0.0177	0.0238	0.0659	0.075	
20	0.0314	0.0403	0.0904	0.102	
25	0.0491	0.0608	0.117	0.128	
30	0.0707	0.0845	0.145	0.157	
35	0.0962	0.112	0.175	0.188	
40	0.126	0.143	0.207	0.221	
45	0.159	0.177	0.242	0.257	
50	0.196	0.215	0.281	0.294	
55	0.238	0.257	0.324	0.336	
60	0.286	0.304	0.371	0.384	
65	0.342	0.358	0.425	0.438	
70	0.403	0.421	0.488	0.501	
75	0.477	0.494	0.562	0.575	
80	0.567	0.586	0.652	0.665	
85	0.684	0.700	0.769	0.782	
90	0.848	0.862	0.933	0.946	
95	1.129	1.163	1.214	1.227	
100	∞	∞	∞	∞	

Adapted from: Leonards, G. A., 1962. Foundation Engineering, McGraw Hill, New York.

axis. The preconsolidation stress is estimated using the stress value at the point where the recompression loop crosses the decompression loop. The slope of the recompression loop at this point is used to estimate the inelastic specific storage (for the layer) S_{kv}. The proportion of ultimate consolidation is estimated as (Bravo, 1990)

$$U = \frac{\Delta b_p}{\Delta \phi S_{kv}} \tag{19}$$

where Δb_p is the permanent deformation for the loop studied; $\Delta \phi$ is the change in stress in the loop studied. The linear pressure distribution (Table 1) is used for estimating

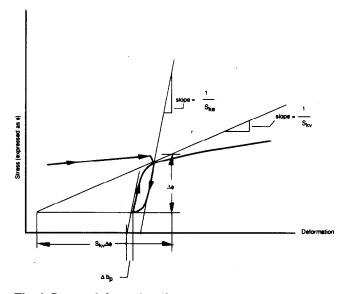


Fig. 4. Stress—deformation diagram.

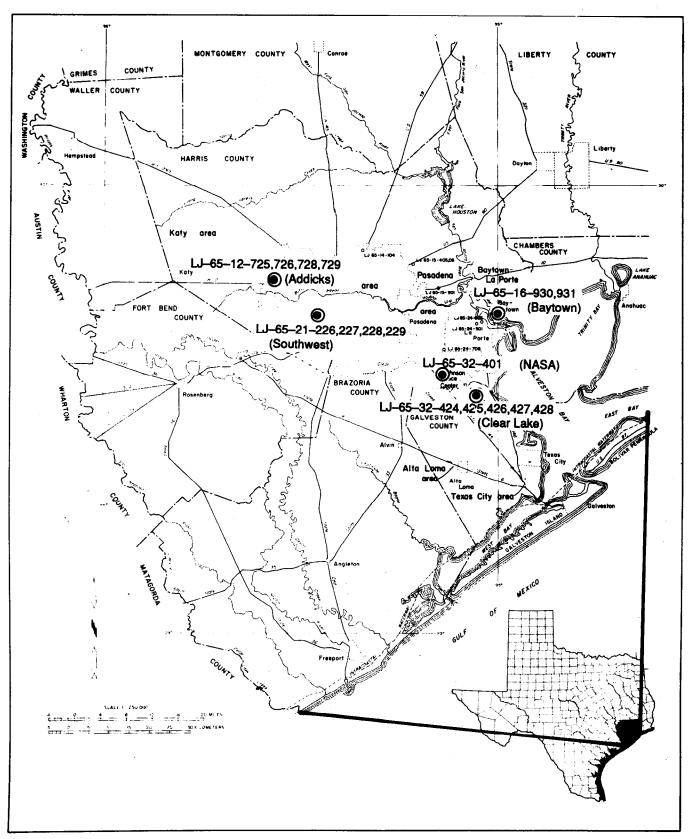


Fig. 5. Houston-Galveston region and study sites (from Jorgensen, 1975).

the time constant, T_{ν} , of consolidation and the vertical hydraulic conductivity,

$$\mathbf{K}' = \mathbf{T}_{\mathbf{v}} \, \frac{\mathbf{S}'_{\mathbf{k}\mathbf{v}} \mathbf{b}}{\mathbf{t}} \tag{20}$$

where t is the elapsed time of the reloading; and b is the thickness of the compressible layer.

The entire procedure comprises: (1) Plotting effective stress and deformation, which can be obtained from hydrograph and extensometer data. (2) Locating a suitable unloading-reloading loop. (3) Determining Ske from the slope of the unloading portion of the loop. (4) Determining S_{kv} from the slope of the reloading portion of the loop. (5) Estimating Δb_p by extending the unloading line to the deformation axis and computing the difference between the minimum stress deformation and the intersection of the unloading line with the deformation axis (Figure 4). (6) Estimating the degree of consolidation during reloading with equation (17). (7) Selecting the time factor T_v in Table 1 (using the linear pressure distribution), and computing K' from equation (20) using the elapsed time of reloading from the data set. The next section demonstrates this procedure to estimate elastic and inelastic storage coefficients and vertical hydraulic conductivity for compressible layers in Houston, Texas.

Application of Procedure to Three Sites in Houston, Texas

The Houston-Galveston area is located in southeast Texas along the Gulf of Mexico (see Figure 5). Figure 5 shows the Houston-Galveston area and the locations of the borehole extensometers and piezometers studied in this paper. Drilling logs and interpretation of geophysical logs indicate alternating sand and clay layers (Williams and Ranzau, 1987; Bebout et al., 1976). The major water-bearing units are the Chicot aquifer that overlies the Evangeline aquifer. Figure 6 shows the vertical relationship of the aquifers along a northwest to southeast transect.

Land subsidence has long been a serious problem near Houston. In 1926, a meter of subsidence (due to petroleum

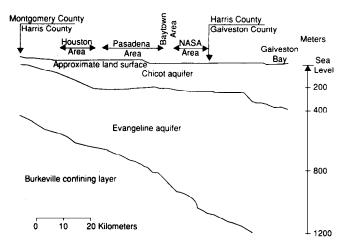


Fig. 6. North-south geologic profile (from Williams and Ranzau, 1987).

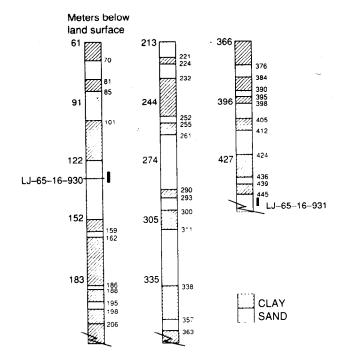


Fig. 7. Geophysical log interpretation at Baytown, Texas.

extraction) was reported at the Goose Creek oil field at the north end of Galveston Bay. Since that time, the area has experienced dramatic growth in population that was supported exclusively by withdrawal of ground water from the Chicot and Evangeline aquifers. These withdrawals have lowered water pressures in the aquifers allowing the clay layers to compress causing land subsidence up to 3 meters in some areas. Many acres of valuable land have been submerged due to subsidence, and larger areas are now subject to flooding. The areas most impacted are Baytown, Pasadena, southwest Houston, and Texas City.

In 1972, borehole extensometers and piezometers were installed at several sites throughout the region. Five of these sites are shown on Figure 5. The Baytown site is used to illustrate the procedure described in the previous sections.

Figure 7 shows a geophysical log interpretation from piezometers LJ-65-16-930 and LJ-65-16-931 that are located at the Baytown extensometer site. The interpretation is meant to indicate layers which are predominantly sand and predominantly clay. Piezometer LJ-65-16-930 is screened at 130 meters below land surface, near the bottom of the Chicot aquifer, and piezometer LJ-65-16-931 is screened at 450 meters, which is in the upper portion of the Evangeline aquifer. The total clay thickness between the two piezometers is about 150 meters. The extensometer at Baytown measures consolidation (vertical deformation) between these two piezometer depths.

Figure 8 shows a hydrograph of water levels in the two piezometers from 1976 to 1989, and Figure 9 shows the consolidation over the same period. From these figures one can observe that the rate of consolidation has decreased with increasing water levels, which supports the effective stress-deformation hypothesis. Around 1985 the extensometer plot shows an unloading-reloading cycle. The hydrographs for the same period indicate a slight decrease in effective

Table 2. Storage Coefficients and Vertical Hydraulic Conductivities for Houston

Location	$S_{\rm ske}(m^{-1})$	$S_{\rm skv}(m^{-1})$	K _v (m/day)	Aquifer units		
Baytown	9.00 × 10 ⁻⁴	5.90×10^{-3}	4.63 × 10 ⁻⁴	Undifferentiated		
Clear Lake/NASA	1.30×10^{-4}	1.70×10^{-4}	1.10×10^{-5}	Undifferentiated		
Southwest	2.95×10^{-6}	2.85×10^{-5}	0.73×10^{-5}	Undifferentiated		
Previous studies						
Meyer and Carr (1979)	1.50×10^{-5}	$4.00 \times 10^{-7} **$	3.90×10^{-4}	Chicot		
• • • •	8.70×10^{-5}	$2.34 \times 10^{-5}**$	1.40×10^{-3}	Evangeline		
Jorgensen (1975)		$7.40 \times 10^{-6}**$		Chicot		
_ , ,		$2.00 \times 10^{-5} **$		Evangeline		
Gabrysch (1984)	1.20×10^{-5} (Baytown)*			Chicot		
• • • • • • • • • • • • • • • • • • • •	6.40×10^{-5} (Baytown)*			Evangeline		
	8.10 × 10 ⁻⁶ (Clear Lake, Southwest)*			Chicot		
	2.00×10^{-4} (Clear Lake, Southwest)*			Evangeline		

^{*} Entries are for Chicot and Evangeline aquifer units.

stress, then a return to relatively steady effective stress analogous to the laboratory consolidation test described in the theory section.

Figure 10 shows the unloading-reloading loop for the time period just described. The hydraulic head change for each layer was assumed proportional to the layer thickness. The elastic storage coefficient $S_{kc} = 8.5 \times 10^{-3}$ was estimated from the slope of the unloading branch as shown on

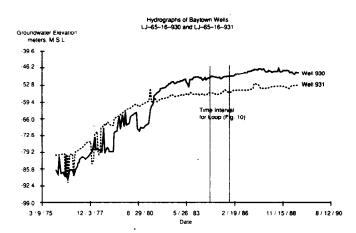


Fig. 8. Hydrographs for LJ-65-16-930, and LJ-65-16-931 at Baytown, Texas.

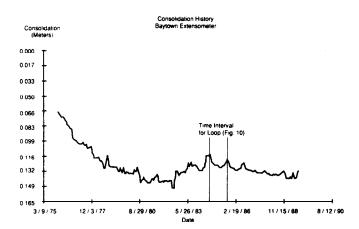


Fig. 9. Cumulative consolidation at Baytown, Texas.

the figure. The inelastic storage coefficient $S_{kv} = 6.10 \times 10^{-2}$ was estimated from the reloading branch. Δb_p was estimated using the procedure depicted on Figure 5; for this loop Δb_p was calculated to be 1.39 \times 10⁻⁴ meters. The degree of consolidation, U, was calculated using equation (19); for this loop $\Delta \phi = 0.022$ meters, and U = 0.139. The time constant T_v for the calculated degree of consolidation U = 0.139 from Table 1 is $T_v = 0.0155$. The elapsed time of reloading for this loop is t = 28 days (the data points are 28 days apart), and K' was estimated as $K' = 2.364 \times 10^{-4}$ m/d using equation (20), and a layer thickness of b = 7.0 meters. These procedures were repeated for each layer. The storage coefficients were divided by the layer thickness to give specific storage coefficients. The specific storage coefficients and hydraulic conductivities were then averaged (arithmetic mean) and these values are reported in Table 2 for each site studied.

Discussion

Table 2 shows our results along with the results of earlier studies in the region. Gabrysch (1984) reported elastic specific storage for the entire region in the Chicot and Evangeline aquifers. Meyer and Carr (1979) used computer

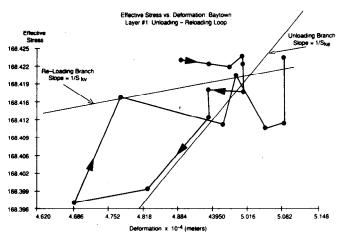


Fig. 10. Effective stress-deformation in layer #1 at Baytown, Texas (unloading-reloading loop).

^{**} These values are computed from published values assuming 250 meters (Chicot) and 500 meters (Evangeline) of compressible material.

simulation and calibration to estimate elastic and inelastic specific storage and hydraulic conductivity. Jorgensen (1975) estimated inelastic specific storage using an electric analog model of regional ground-water flow and calibration procedures.

Our results differ from the previous work in that our estimates of elastic and inelastic specific storage are larger (by several orders of magnitude in some cases). Our estimate of vertical hydraulic conductivity is smaller than the earlier estimate. One explanation for the differences is that our study focused on estimation of specific storage in the compressible layers, and assumes that aquifer units do not contribute to deformation, whereas earlier studies computed specific storage for entire units that comprised relatively compressible and incompressible portions. Additionally, the values in the earlier works are areal averages, and include behavior in areas that are less sensitive to consolidation. Lastly, we had access to 15 years of hydrograph and extensometer data that did not exist at the time of the earlier studies.

Conclusions

The results of our study, which are presented in Table 2, compare favorably with earlier works. The earlier studies used simulation or elastic material theory to determine the storage coefficients. Our estimates indicate that the specific storage in the aquitard units is larger than previously assumed, and may be significant in subsidence studies. Our technique is useful for estimating specific storage coefficients and vertical hydraulic conductivities, when extensometer and piezometric head data are available.

References

- Bear, J. 1972. Dynamics of Fluids in Porous Media. American Elsevier, New York.
- Bebout, D. G., P. E. Luttrell, and J. H. Seo. 1976. Regional tertiary cross sections—Texas Gulf Coast. Bureau of Economic Geology, Univ. of Texas at Austin. Geological Circular 76-5.
- Bravo, R. 1990. Prediction of Houston ground-water heads and

- land subsidence using three dimensional finite differences. Ph.D. dissertation, Dept. of Civil and Environmental Engineering, Univ. of Houston, TX.
- Domenico, P. A. and M. D. Mifflin. 1965. Water from low-permeability sediments and land subsidence. Water Resources Research. v. 9, no. 3.
- Frind, E. O. 1979, Exact aquitard response functions for multiple aquifer mechanics. Advances Water Resources. v. 2, pp. 77-82.
- Gabrysch, R. K. 1984. Ground water withdrawals and land surface subsidence in the Houston-Galveston region, Texas, 1906-80. U.S. Geological Survey Report 287.
- Gambolati, G. and R. A. Freeze. 1973. Mathematical simulation of the subsidence of Venice, 1, Theory. Water Resources Research. v. 9, no. 3.
- Hantush, M. S. 1960. Modification of the theory of leaky aquifers. Journal Geophysical Research. v. 65.
- Hantush, M. S. and C. E. Jacob. 1955. Non-steady radial flow in an infinite leaky aquifer. EOS Transactions American Geophysical Union. v. 36, no. 1.
- Herrera, I. and G. E. Figueroa. 1969. A correspondence principle for the theory of leaky aquifers. Water Resources Research. v. 5, no. 4.
- Jorgensen, D. G. 1975. Analog model studies of ground water hydrology in the Houston district, Texas. U.S. Geological Survey Report 190.
- Leonards, G. A. 1962. Foundation Engineering. McGraw Hill, New York.
- Marsily, G. de. 1986. Quantitative Hydrogeology. Academic Press, New York.
- Meyer, W. R. and J. E. Carr. 1979. A digital model for simulation of ground-water hydrology in the Houston area, Texas. Texas Dept. of Water Resources. Report LP-103.
- Neuman, S. P. and P. A. Witherspoon. 1969. Theory of flow in a confined two-aquifer system. Water Resources Research. v. 5, no. 4.
- Rudolph, D. L. and E. O. Frind. 1991. Hydraulic response of highly compressible aquitards during consolidation. Water Resources Research. v. 27, no. 1.
- Schmertmann, J. H. 1953. Undisturbed consolidation behavior of clay. Transactions American Society of Civil Engineers. v. 120, p. 1201.
- Williams, J. F. and C. E. Ranzau. 1987. Ground-water withdrawals and changes in ground-water levels, ground-water quality, and land-surface subsidence in the Houston District, Houston, Texas 1980-84. U.S. Geological Survey Water-Resources Investigations Report 87-4153.