Optimal Configuration and Scheduling of Ground-Water Tracer Test

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Abstract: A technique for jointly configuring and scheduling a monitoring network for an aquifer tracer test is presented. A dynamic programming (DP) algorithm is used to select among competing designs for a test to provide data for estimating aquifer model parameters. A maximal information criterion is used to evaluate competing designs that satisfy a particular budget constraint. Decision variables in DP formulation are sampling locations (configuration variables) and sampling initiation times (scheduling variables). A forward DP solution method is used, optimal configuration and scheduling are determined by maximizing information without exceeding a budget. Maximization of information is argued to be equivalent to minimizing total cost of installation as well as sampling and analysis subject to an information demand function constructed from the trace of the covariance matrix of the estimated parameters. A tracer test for a confined aquifer is simulated using a finite difference scheme. A short injection period followed by a monitoring period is simulated. The DP algorithm is applied to this model to design a monitoring network and schedule to estimate aquifer characteristics. The sampling network and schedule is designed with estimation of transmissivity, retardation, and dispersivities in mind. Resulting designs are reasonable, and the method can be extended to regional-sized problems with little modification.

Introduction

Economic, budgetary, and other practical limitations generally preclude the development of a dense monitoring network for obtaining data for the purpose of parameter estimation for contaminant transport models. Such models are used to predict the response of an aquifer system to management policies, and, thus, the model parameters used ultimately affect the management decisions that are made.

In the design of a sparse monitoring network, the configuration of the sample points in space must be simultaneously considered with the scheduling of the sampling and analysis in time. These design variables are interrelated. Given the sampling locations, a schedule is straightforward and vice versa. However, jointly specifying sampling locations and a sampling schedule is a difficult optimization problem. This paper describes a technique for jointly configuring and scheduling a monitoring network that maximizes the weighted trace of the estimate's information matrix when the observation error is assumed uncorrelated in space and time. The selection criterion is identical to that described by Cleveland and Yeh (1989), who used the selection criterion for the optimal design of sample locations for transport parameter identification under a given experimental duration. The work here is an extension of Cleveland and Yeh (1989) in that the solution algorithm considers both sample locations and scheduling.

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Parameters are estimated by an inverse procedure, which is applied after the monitoring network has been installed and a sampling schedule specified. Many inverse procedures are available. An exhaustive review of these is presented by Yeh (1986).

The problem of configuring and operating a network is called "experimental design" in other fields, such as medicine, automatic control, and physics. Applications in ground-water resources have been investigated by several researchers in the design of optimal pumping tests (Yeh and Sun 1984; Hsu and Yeh 1989; Nishikawa and Yeh 1989; McCarthy and Yeh 1990). Investigations for the design of optimal tracer tests and passive contamination monitoring have been made by Carrera et al. (1984), Knoppman and Voss (1987), Wagner and Gorelick (1987), and others. More recently optimal monitoring network design for ground-water quality has been investigated by Meyer and Brill (1988), Knoppman and Voss (1988), and Loaciga (1989).

In general, these works use a minimum-cost approach subject to a reliability requirement based on either the determinant or the trace of the estimates' covariance matrix. Alternatively, one can use the determinant or trace of the estimates' covariance matrix as an evaluator, in which case these are called D-optimal or A-optimal designs, respectively. Steinberg and Hunter (1984) reviewed the history of many experimental design schemes and described the more successful, including A-optimal and D-optimal designs. Nearly all approaches lead to combinatorial optimization problems that are further complicated by the fact that a particular design must be specified prior to evaluation. In this paper, the writers use a different evaluator, a special case of general criteria discussed by Sacks and Ylvisaker (1968). The resulting combinatorial optimization problem is solved using the DP approach, which has the advantage that a design need not be specified prior to evaluation.

Description of Problem

It is assumed that a forced gradient tracer test (Fig. 1) will be conducted to estimate the parameters of transmissivity, storage coefficient, porosity, dispersivity, and retardation factor for the following flow and transport models. Two-dimensional ground-water flow for a confined aquifer is governed by (Bear 1972, 1979)

$$S\frac{\partial h}{\partial t} = \frac{\partial}{\partial x}\left(T\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(T\frac{\partial h}{\partial y}\right) + M \quad \ldots\ldots (1)$$

subject to the following initial and boundary conditions

$$h(x,y,0) = \text{known}, \qquad (x,y) \in \Omega \quad \ldots\ldots (2a)$$

$$h(x,y,t) = \text{known}, \qquad (x,y) \in d\Omega_1 \quad \ldots\ldots (2b)$$

$$\left(T\frac{\partial h}{\partial x}\right)\frac{\partial x}{\partial n} + \left(T\frac{\partial h}{\partial y}\right)\frac{\partial y}{\partial n} = \text{known}, \qquad (x,y) \in d\Omega_2 \quad \ldots\ldots (2c)$$

where h = hydraulic head (length); T = transmissivity [(length)^2/time]; S = storage coefficient; M = net injection or extraction rate [(length)^3/time];

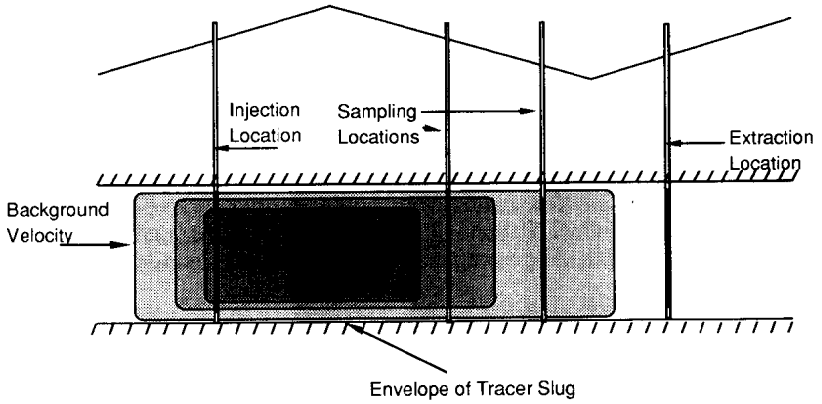

FIG. 1. Conceptual Aquifer

Ω = flow region; $d\Omega$ = boundary of flow region ($d\Omega_1 \cup d\Omega_2 = d\Omega$); and $\partial/\partial n$ = normal derivative to boundary.

The aquifer thickness is assumed to be constant and the transmissivity varies directly with hydraulic conductivity. Eq. 1 is vertically averaged. The governing equation of the solute transport process used is (Bear and Verruijt 1987)

$$R\frac{\partial C}{\partial t} = \frac{\partial}{\partial x}\left(D_{xx}\frac{\partial C}{\partial x} + D_{xy}\frac{\partial C}{\partial y}\right) + \frac{\partial}{\partial y}\left(D_{yx}\frac{\partial C}{\partial x} + D_{yy}\frac{\partial C}{\partial y}\right)$$

$$- V_x\frac{\partial C}{\partial x} - V_y\frac{\partial C}{\partial y} + M \quad \ldots\ldots (3)$$

subject to the following initial and boundary conditions:

$$C(x,y,0) = \text{known}, \qquad (x,y) \in \Omega \quad \ldots\ldots (4a)$$

$$C(x,y,t) = \text{known}, \qquad (x,y) \in d\Omega_1 \quad \ldots\ldots (4b)$$

$$\left(C\frac{V_x}{R} - \frac{D_{xx}}{R}\frac{\partial C}{\partial x} - \frac{D_{xy}}{R}\frac{\partial C}{\partial y}\right)\frac{\partial x}{\partial n} - \left(C\frac{V_y}{R} - \frac{D_{yx}}{R}\frac{\partial C}{\partial x} - \frac{D_{yy}}{R}\frac{\partial C}{\partial y}\right)\frac{\partial y}{\partial n} = \text{known},$$

$$(x,y) \in d\Omega_2 \quad \ldots\ldots (4c)$$

where C = mass of solute per volume of medium [mass/(length)^3]; D_{xx}, D_{yy} ... = components of hydrodynamic dispersion tensor; R = retardation factor; V_x = average fluid velocity in x direction (length/time); V_y = average fluid velocity in y direction; M = net mass injection or extraction rate [mass/(length)^3(time)]; Ω = flow region; and $d\Omega$ = boundary of flow region ($d\Omega_1 \cup d\Omega_2 = d\Omega$). This formulation assumes no generation or decay of solute. The hydrodynamic dispersion coefficients are computed by (Bear 1972, 1979)

$$D_{xx} = (\alpha_L - \alpha_T)\frac{v_x^2}{v} + \alpha_T V + D^* \quad \ldots\ldots (5a)$$

$$D_{xy} = D_{yx} = (\alpha_L - \alpha_T)\frac{v_x v_y}{v} \quad (5b)$$

$$D_{yy} = (\alpha_L - \alpha_T)\frac{v_y^2}{v} + \alpha_T V + D^* \quad (5c)$$

$$\bar{V} = (V_x^2 + V_y^2)^{1/2} \quad (5d)$$

where α_L = longitudinal dispersivity; α_T = transverse dispersivity; and D^* = molecular diffusion. The average fluid velocities are computed using Darcy's Law,

$$V_x = -\frac{K}{e}\frac{\partial h}{\partial x} \quad (6a)$$

$$V_y = -\frac{K}{e}\frac{\partial h}{\partial y} \quad (6b)$$

where e = effective porosity; K = hydraulic conductivity (T/b), b = aquifer thickness; and T = transmissivity. It is assumed that the solute does not significantly affect the density of the fluid, so the two model equations can be solved independently and coupled through Darcy's Law only. Additionally, molecular diffusion is ignored due to its small magnitude.

An upwind formulation is used for the transport equation to minimize oscillations associated with the numerical solution when advection dominates. The formulation may cause smearing of the concentration front. A forward Euler scheme is used to solve the two equations. Stability was determined by trial and error. Details of the numerical solution scheme can be found in Cleveland (1989) and Cleveland and Yeh (1989).

The design algorithm assumes that a least-squares inverse scheme will be used. Assuming an additive error model

$$C_{\text{observed}} = C_{\text{model}}(\boldsymbol{\theta}) + \text{error} \quad (7)$$

where $\boldsymbol{\theta}$ = vector of parameters ($T, S, e, \alpha_L, \alpha_T, R$); $E(\text{error}) = 0$; $V(\text{error}) = \sigma^2\mathbf{I}$; σ^2 is the variance; and $\mathbf{I}$ is the identity matrix. The symbols E and V represent mathematical expectation and variance-covariance operator. The linear approximation of the estimates' covariance matrix is (Yeh and Yoon 1981)

$$\mathbf{V}(\boldsymbol{\theta}) = \sigma^2(\mathbf{J}^T\mathbf{J})^{-1} \quad (8)$$

where $\mathbf{J}$ = Jacobian matrix constructed from vectors of model sensitivities to parameters.

The model sensitivities are the partial derivatives of concentration with respect to model parameters. The method used to compute these sensitivities is the influence coefficient method described by Yeh (1986). For a two-point design [i.e., sample at (x_1, t_1) and (x_2, t_2)]

$$\mathbf{J} = \begin{pmatrix} (\partial C/\partial\theta_1)(x_1,t_1,\boldsymbol{\theta}) & (\partial C/\partial\theta_2)(x_1,t_1,\boldsymbol{\theta}) & \dots & (\partial C/\partial\theta_k)(x_2,t_1,\boldsymbol{\theta}) \\ (\partial C/\partial\theta_1)(x_2,t_2,\boldsymbol{\theta}) & (\partial C/\partial\theta_2)(x_2,t_2,\boldsymbol{\theta}) & \dots & (\partial C/\partial\theta_k)(x_2,t_2,\boldsymbol{\theta}) \end{pmatrix} \quad (9)$$

The information matrix is the inverse of the covariance matrix.

$$\mathbf{I}(\boldsymbol{\theta}) = \frac{1}{\sigma^2}\mathbf{J}^T\mathbf{J} \quad (10)$$

The experimental design problem seeks to maximize some measure of the information matrix subject to a budget constraint. The measure used is the weighted trace of the information matrix at selected points in space and time for a particular design. The trace of the information matrix is used since it can be computed directly from the sensitivities without inverting a matrix (unlike covariance measures). The weights are included to account for different magnitudes of sensitivities for different parameters. The measure is written as

$$Z_{i,l} = \frac{1}{\sigma^2}\text{tr}(\mathbf{J}_{i,l}^T J_{i,l} W) = \frac{1}{\sigma^2}\sum_{j=1}^{k} W_j\left(\frac{\partial C_{i,l}}{\partial\theta_j}\right)^2 \quad (11)$$

where $Z_{i,l}$ = information at ith location lth time; $\mathbf{J}_{i,l}$ = Jacobian matrix at ith location lth time; W_j = weight on jth parameter; k = number of parameters; and tr( ) = trace of matrix. This criterion is a special case of more general criteria discussed by Sacks and Ylvisaker (1968).

Assumptions about costs are presented in the following. It is assumed that the injection and extraction site is already first installed and developed. Once a sampling site is installed, sampling will continue at that site until the end of the experiment, the duration of which is assumed to be known. With these assumptions the objective is constructed as the sum of time-integrated information numbers at each site. For instance if site x_i is chosen and sampling is begun at time t_j and continued until time t_{NT} then the information available at that site is

$$\text{inf}\,(x_i, t_j) = \sum_{l=j}^{NT} Z_{i,l} \quad (12)$$

Installation cost is a function of location and depth as well as the cost of the sampler itself. In an analogous manner to site information, site cost consists of the cost of installation and consequent sampling, where sampling includes the cost of obtaining and analyzing the sample.

$$\text{cost}\,(x_i, t_j) = \bar{\bar{C}}(x_i) + \sum_{l=j}^{NT} \bar{C}(x_i, t_l) \quad (13)$$

where $\bar{\bar{C}}(x_i)$ = installation cost at site x_i; and $\bar{C}(x_i, t_l)$ = sampling and analysis cost at site x_i, time t_l.

The experimental design problem attempts to select locations (configure) and sampling initiation times (schedule) such that the information is maximized without exceeding a budget constraint.

Methodology

The algorithm used to solve the configuration and scheduling problem is identical in spirit to the forward DP algorithm for the timing, sequencing, and sizing of water resource projects developed by Becker and Yeh (1974).

The recursion formula is

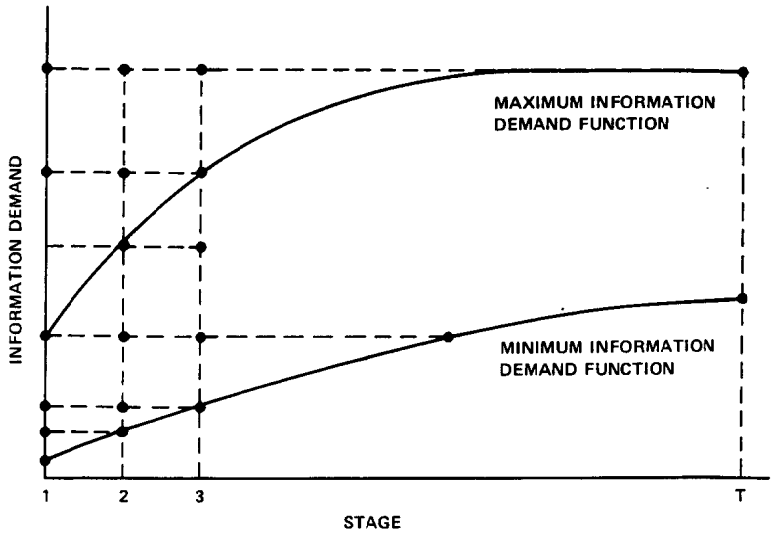

FIG. 2. Demand Envelope

$$f_l(q_l) = \min\,[\hat{C}^i(y_l^i) + f_{l-1}(q_l - x_l^i)] \quad \ldots\ldots (14)$$

subject to:

$$i \in \text{(all available locations)} \quad \ldots\ldots (15a)$$

$$q_{l-1} \le q_l \le D(t_f) \qquad x_l^i = q_l - q_{l-1} \qquad f_o = 0 \quad \ldots\ldots (15b)$$

where l = stage (sampling initiation time); $f_l(\ )$ = minimum cost function at stage l; q_l = total information available at l; x_l^i = incremental information resulting from sampling at ith location at l; $\hat{C}^i$ = cost for sampling at ith location; y_l^i = indicator of whether ith location is active at l ($y_l^i = 0$ or 1); $D(\ )$ = information demand function; and t_f = total experimental duration.

The state variable is the total information available at a given state l. The total experimental duration, t_f, is divided into a number of stages and a decision is made at the beginning of each stage. The decision variables are sampling location and sampling initiation time. Since f_l is a function of f_{l-1}, the recursive equation has to be solved forward in time. The total cost is assumed to be separable. Once a site is selected for sampling, it is no longer considered for inclusion at a later stage. Only one site at a time is considered.

The formulation is a minimum-cost approach subject to an information demand. The information demand function is constructed by computing the minimum and maximum information available. The information demand function is constructed as follows. The sum of information available at each point is computed over all time, and the location with the smallest sum is selected to define the minimum-demand function. The running sum of the information numbers at each time, Eq. 11, defines the minimum-information demand at each stage. The maximum-information demand function is the running sum of the information numbers at all possible locations at each

stage. Thus the minimum design will be a single site that samples at all times, and the maximum design will be points at all times. In the examples a single-site, single-time sample is included for illustration of information gain as budget is increased, this design is simply the best site at the last possible sample time, and is chosen externally from the algorithm.

The functions are used to define an envelope whose upper and lower limits represent the least and most acceptable amount of information. The DP algorithm is run with the information demand parametrically varied within this envelope, from the lower to upper bound. The cost increases as information demand is increased. The optimal policy is the most informative minimum-cost policy that does not violate the budget. As an added benefit, a trade-off curve is generated while obtaining the solution. Fig. 2 depicts the demand envelope as described earlier.

The heuristic has some additional assumptions embedded in the algorithm. To make the problem separable, when two sites are economically equal, the site that gives the maximum remaining integrated information at that stage is chosen as the preferred site for inclusion at that stage. Therefore, at any stage, the future potential of the additional site is considered from that stage onward.

The decision to install a site incurs a large fixed cost, the algorithm will attempt to meet the information demand with as few sites as possible. Because of the fixed-cost nature of the problem just-in-time installation is avoided, reflecting the reasonable economic objective of reducing average cost per sample.

Equivalence of Min Cost and Max Information

Equivalency of minimizing the total cost and maximizing the information is difficult to establish. If the problem could be moved into continuous space and be posed as a linear program, then, in fact, the two approaches are each other's dual. In discrete space, duality is hard to establish mathematically, and with the fixed-cost nature of the problem, it becomes harder still. Instead a heuristic argument is given.

For any given budget, several designs can be specified, each design will yield a different amount of information. Obviously, the design with the most information is the desirable design. In the minimum-cost approach, several information demand levels give designs for the same cost. Since the information available is in discrete quantities there is an information level above which the cost will change. At the levels where the changes occur, the minimum-cost solution is the maximum-information solution for that cost (or budget). Using parametric programming over a reasonably fine discretization of the information demand one can locate designs that are sufficiently dual for the equivalence statement to be made.

Application

The methodology is applied to the following problem. Fig. 3 shows the computation domain that is used to approximate the conceptual aquifer of Fig. 1, with a thickness of 10 m. The experimental duration is assumed to be 10 days (t_f), with sampling every day (stage). Water is injected and extracted during the entire experiment at a rate of 14 m^3/day, but for the first

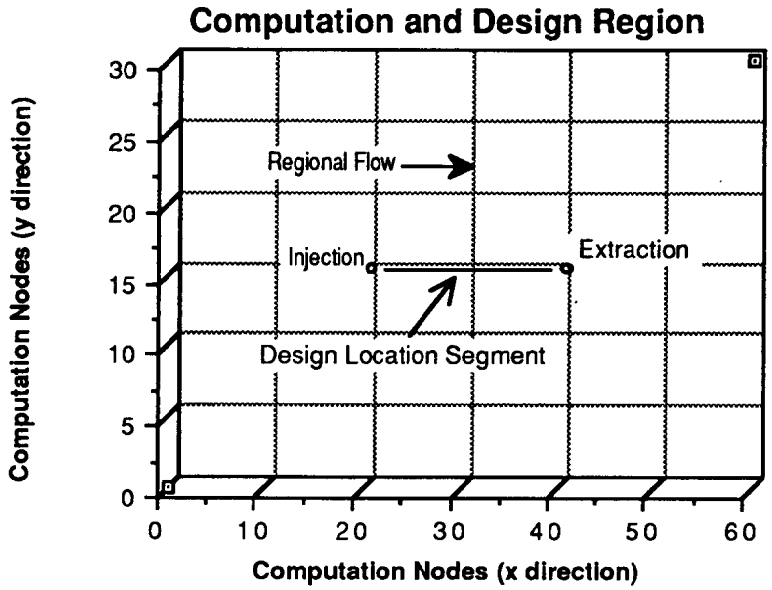

FIG. 3. Computation Domain

half day a tracer mass (700 g) is added. The aquifer is assumed to be uncontaminated prior to injection.

Table 1 shows the three parameter sets used in this application for which optimal designs will be generated. We have chosen these three parameter sets to demonstrate the proposed DP algorithm. In practice, prior information is used to select the initial set of parameters upon which the first round of experimental design is carried out. The data thus collected are used for updating the parameter estimates by solving the inverse problem of parameter identification. A second round of experimental design and data collection is then performed. The concept of sequential design and its convergence property has been reported by Hsu and Yeh (1989) and Nishikawa and Yeh (1989), storage coefficient (S), effective porosity (e), longitudinal dispersivity (α_L), transverse dispersivity (α_T), and the retardation factor (R). The second, third, and fourth columns give the assumed values of each of the parameters in the first column for three example applications.

TABLE 1. Parameter Sets for Example Application

Parameters (1)	Set 1 (2)	Set 2 (3)	Set 3 (4)
T	30.0	30.0	30.0
S	0.1	0.1	0.1
e	0.5	0.5	0.5
α_L	0.1	0.5	0.1
α_T	0.1	0.5	0.1
R	1.0	1.0	2.0

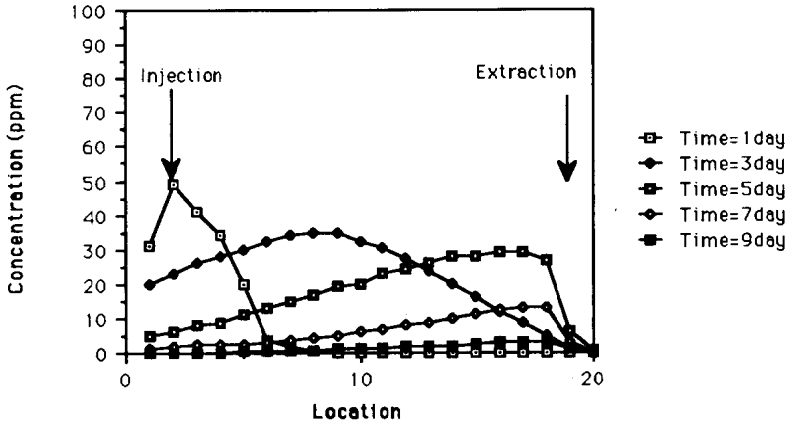

FIG. 4. Concentration Profiles for Parameter Set 1

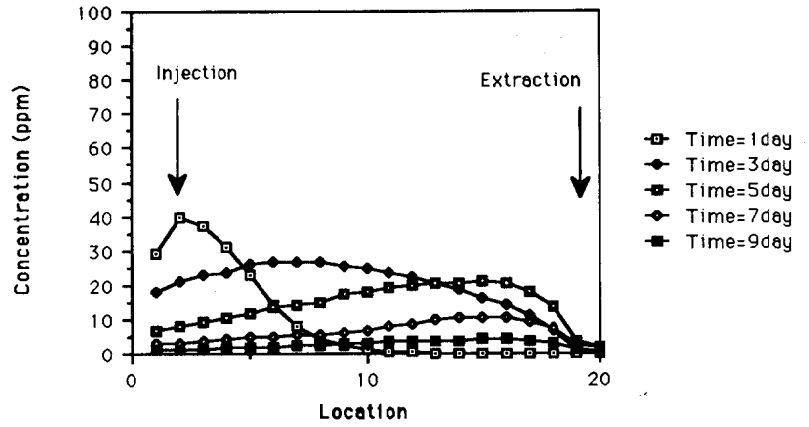

FIG. 5. Concentration Profiles for Parameter Set 2

Figs. 4, 5, and 6 show the modeled concentrations at five sample times for the three parameter sets along the line segment joining the injection and extraction site. In each case, the peak of the concentration profile is near the extraction well for the later sample times. Comparison of Figs. 4 and 5 shows that the peak concentrations for the earlier sample times occur in about the same location, but the distribution in Fig. 5 is more spread out, as would be expected with the higher dispersivity. In Fig. 6, the early sample-time peak concentration is half as far from the source than in Fig. 4, as expected for the retardation factor of 2.0 in Fig. 6 parameter set.

Figs. 7, 8, and 9 show the instantaneous information profiles for the parameter sets of columns two through four of Table 1. Each of these figures exhibits an interesting characteristic in that the information profiles tend to follow (peak after) the concentration profiles at a location, reflecting that

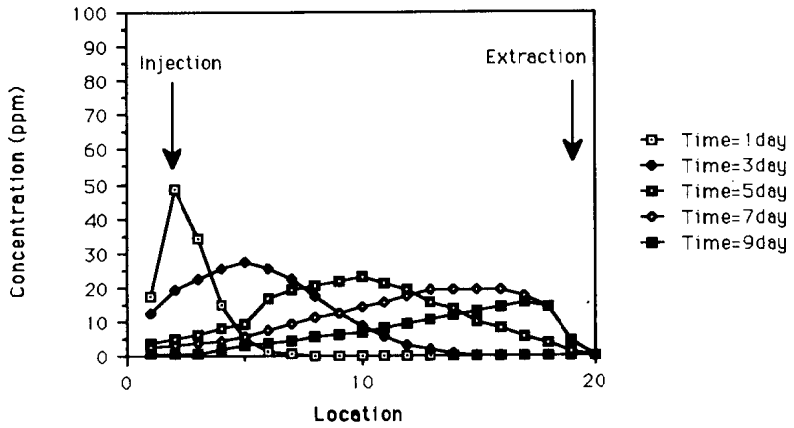

FIG. 6. Concentration Profiles for Parameter Set 3

contaminant (tracer) mass must arrive and pass by a point in sufficient quantity to obtain information on the parameters controlling the contaminant migration.

Table 2 lists the optimal designs generated using the DP algorithm for the first parameter set and various budget levels. The notation in this table is (location: sample initiation time) for column three where location two is the injection site and location 19 is the extraction site. Column one of Table 2 gives the budget available, while column two gives information returned. The information has no units and differs with each parameter set, so comparisons between tables are not made. For the first parameter set, it can be observed that the optimal locations are near the extraction site. These locations are at the same distance from the source that an advective front would be expected to pass in half the experiment duration.

Table 3 lists the optimal designs for the second parameter set. The notation

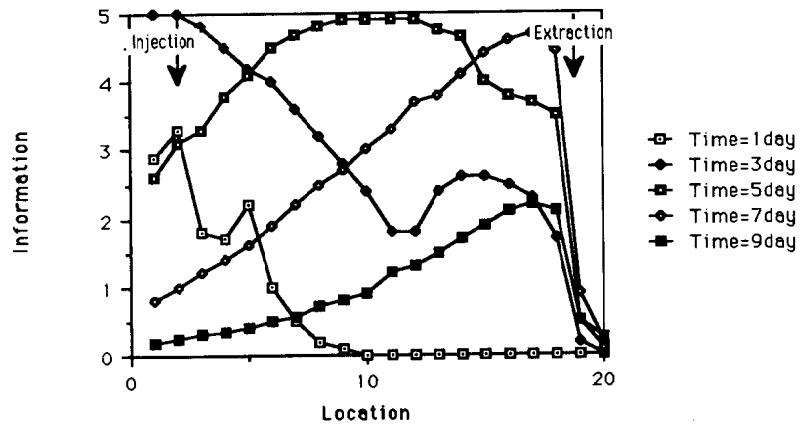

FIG. 7. Instantaneous Information for Parameter Profiles for Parameter Set 1

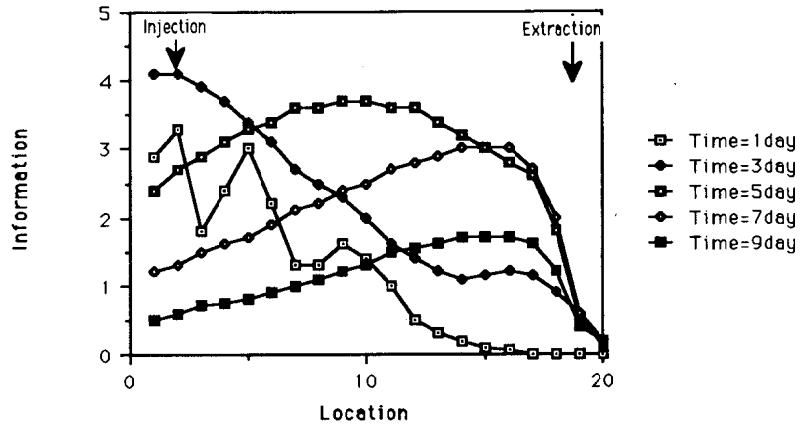

FIG. 8. Instantaneous Information for Parameter Profiles for Parameter Set 2

and meaning of each column is the same for Table 2. In this example the optimal locations are very near the source (in fact,at the source) with sampling begun immediately. The larger dispersivity values in this example spread the contaminant more quickly to where the quantity arriving and passing points far from the source is insufficient to yield as much information as a sampling point near the source. It is noted here that the parameters are assumed time-invariant and scaling effect is not considered. The first row of Table 3, where budget is sufficient for only a single sample, selects a location that is characteristic of the previous example. An implication of these results is, when dispersion is significant, near-source sampling will allways be indicated because the mass is spread far enough from the source that the concentration contrast over time becomes negligible and little information is available.

Table 4 lists the optimal designs for the third parameter set where retar-

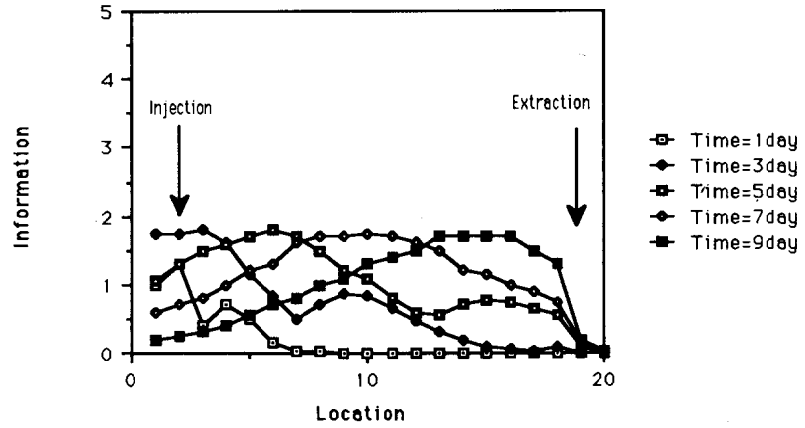

FIG. 9. Instantaneous Information for Parameter Profiles for Parameter Set 3

TABLE 2. Designs for Parameter Set 1

Budget (1)	Information (2)	Design (location:sampling start) (3)
11	1.2	17:10
21	24.4	15:1
31	31.1	15:1, 17:10
41	49.7	14:2, 15:1
51	53.0	14:2, 15:1, 17:9
61	74.4	14:2, 15:1, 13:1
71	77.7	14:2, 15:1, 13:1, 17:9
81	99.0	14:2, 15:1, 13:1, 16:1
91	102.3	14:2, 15:1, 13:1, 16:1, 17:9

dation is 2.0. Again, sampling is indicated near the source, but, in this case, retardation retains most of the information close to the source through the experiment, as can be seen in the information profiles in Fig. 9 for this parameter set. Near the source, in the early sample times the information is high and does not decay very quickly. Ignoring the sampling point at the source and looking at the other indicated points, doubling the retardation results in reduction of the distance between the source and the optimal locations by a factor of two.

TABLE 3. Designs for Parameter Set 2

Budget (1)	Information (2)	Design (location:sampling start) (3)
11	1.2	15:10
21	21.5	2:1
31	22.7	2:1, 15:10
41	42.6	2:1, 6:1
51	43.8	2:1, 6:1, 15:10
61	63.6	2:1, 6:1, 4:1
71	64.8	2:1, 6:1, 4:1, 15:10
81	84.6	2:1, 6:1, 4:1, 7:1
91	85.8	2:1, 6:1, 4:1, 7:1, 15:10

TABLE 4. Designs for Parameter Set 3

Budget (1)	Information (2)	Design (location:sampling start) (3)
11	1.6	17:10
21	10.6	3:1
31	12.2	3:1, 17:10
41	21.1	3:1, 4:1
51	22.7	3:1, 4:1, 17:10
61	31.5	3:1, 4:1, 6:1
71	33.1	3:1, 4:1, 6:1, 17:10
81	41.7	3:1, 4:1, 6:1, 5:1
91	43.3	3:1, 4:1, 6:1, 5:1, 17:10

For all three parameter sets, the specified designs are reasonable and the behavior of the information profiles can be explained in terms of mass location and spreading. The primary implication is that when mass moves as a slug with little spreading, knowledge of velocity is the most useful indicator of where and when to sample, implying that the hydraulic parameters of conductivity and storage coefficient are important. This observation is reassuring since these parameters can be estimated independently by a pumping test. In the case of retardation or quick spreading, the velocity importance is dominated by the quick loss of concentration contrast over time at points far from the source. In general, a sample point near the source in a practical case ensures some information will be available even if unknown hydraulic structures cause the tracer to be missed entirely at points further from the source.

Conclusion

A dynamic programming approach to the problem of configuring and scheduling a ground-water tracer test has been illustrated. The test is used to estimate model parameters for use in management studies. The objective is to design an experiment to maximize the weighted trace of the estimates' information matrix without exceeding a cost constraint. The methodology allows that it may not be cost-effective to sample at a site before sufficient mass has arrived. Three examples were provided to illustrate the feasibility of the method. In each example, optimal designs that were intuitively reasonable were generated. The method was illustrated for small-scale tests, but would be applicable to regional-sized problems with little modification.

Acknowledgments

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APPENDIX II. NOTATION

The following symbols are used in this paper:

b = aquifer thickness;
C = concentration;
$\hat{C}^i$ = sampling cost;
$\tilde{C}(\underline{x}, t)$ = sampling and analysis cost;
$\bar{C}(x)$ = installation cost;
Cost (x, t) = cost;
D_{xx}, D_{xy}, D_{yy}, ... = components of hydrodynamic dispersion tensor;
D^* = molecular diffusivity;
$D(\ )$ = demand function;
$d\Omega$ = boundary of flow region;
$E(\ )$ = expectation;
e = effective porosity;
$f_i(\ )$ = minimum cost function at stage i;
h = piezometric head;
$\mathbf{I}(\ )$ = information matrix;
inf (x, t) = information;
i = stage; index;
$\mathbf{J}$ = Jacobian;

K = hydraulic conductivity;
M = source/sink term;
n = normal direction;
q_i = total information available at i;
R = retardation factor;
S = storage coefficient, index set;
T = transmissivity;
t_f = experimental duration;
t_r = tracer of matrix;
V_x, V_y = average fluid velocity in the x and y direction;
$V(\ )$ = variance, covariance;
W_j = parameter weight;
x_i^k = incremental information;
y_i^k = indicator varible;
$Z_{i,l}$ = information at ith location, lth time;
α_L = longitudinal dispersivity;
α_T = transverse dispersivity;
$\partial/\partial n$ = normal derivative;
$\boldsymbol{\theta}$ = parameter vector;
σ^2 = variance; and
Ω = flow region.