Recovery Performance for Vertical and Horizontal Wells Using Semianalytical Simulation

by Theodore G. Cleveland[a]

Abstract

A semianalytical transient expression was used to evaluate the performance of a finite-length horizontal well as compared to a vertical well in a hypothetical recovery scenario. Recovery was simulated in two hypothetical confined aquifers to study the effect of aquifer thickness and well orientation on recovery performance. The results show that horizontal wells can initially mobilize larger volumes of ground water (and contaminants) but that vertical wells have the same ultimate recovery ability in a homogeneous, isotropic medium.

Introduction

The most widely used approach for remediation of ground-water contamination is the extraction of the contaminated water followed by treatment at the surface (Knox et al., 1986). Although there are some successful projects, pump-and-treat is not the simple solution originally envisioned because the complexities of the subsurface environment and the diverse nature of the contaminants combine to make the extraction of the contaminants troublesome and costly.

To solve some of these problems, enhanced recovery methods have been developed. Enhanced recovery of contaminants (and oil in petroleum engineering) relies on modifying physical and chemical characteristics of the solid and fluid phases in a porous medium, as well as improving access to the subsurface with unusual drilling techniques. One promising practical approach is the use of horizontal recovery wells.

Horizontal drilling technology has improved to the point that it is cost comparable to vertical drilling techniques and is being proposed for use in the ground-water remediation industry. Losonsky and Beljin (1991) estimate the installation of a single horizontal recovery well with 200 feet of screen compared to five vertical wells with 10 feet of screen each to be 26% more costly, while the annual operation of the horizontal well is 267% *less* costly. Karlsson (1992) presents an economic analysis comparing the total project cost of a horizontal well compared to five vertical wells in a sandy aquifer or 10 wells in a silty clay aquifer and concludes that the vertical well installations are 151% and 413% more costly for the two aquifers.

Numerical studies in reservoir engineering indicate that for steam flood oil recovery, horizontal wells exhibit the best recovery rates, although vertical wells achieve nearly the same ultimate recovery (Hasamuit and Abou-Kassem, 1991). Langseth (1990) performed a numerical study for a ground-water system and produced similar results although the simulations were terminated before equivalent asymptotic performance could be compared. These numerical experiments suggest that horizontal wells can achieve faster removal rates than vertical wells in ground-water systems. This paper describes semianalytical simulations aimed at establishing the relative efficiency of horizontal recovery wells in a hypothetical pump-and-treat system as compared to vertical wells.

Theory

The governing equation for head in an infinite three-dimensional homogeneous and isotropic aquifer in spherical coordinates is (Marsily, 1986)

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial h}{\partial r}\right) = \frac{S_s}{K}\frac{\partial h}{\partial t} \qquad (1)$$

where r is the radial distance from the origin to a field point, h is the hydraulic head (dimensions L) at that point, S_s is the specific storage (dimensions L^3/L^4) of the aquifer, and K is the hydraulic conductivity (dimensions L/t).

This equation is analogous to linear heat flow theory with hydraulic conductivity analogous to thermal conductivity, and specific storage analogous to the product of specific heat and material density. From these analogs it then follows that thermal diffusivity is analagous to the ratio of hydraulic conductivity to specific storage. Using these heat flow analogs, the elementary solution for a pulse injection of liquid at the origin, with an initial head of zero

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everywhere, and head at $r = \infty$ equal to zero is expressed as (Carslaw and Jaeger, 1959)

$$h(r, t) = \frac{q}{\sqrt{64\pi^3 (K^3/S_s^3) t^3}} \exp\left(-\frac{r^2 S_s}{4Kt}\right) \quad (2)$$

where q is the strength of the injection source (dimensions L^4), and represents the energy per unit weight of water added to a unit volume of aquifer during an infinitesimally short interval of time. This solution may be alternatively interpreted as the added head (energy per unit weight of water) in an infinite aquifer due to a volume of fluid $S_s q$ *instantaneously* introduced at $t = 0$ at the origin.

When water is added at the volumetric *rate* $Q = \phi(t) S_s$ per unit time $[\phi(t)\,dt = q]$, from $t = 0$ to $t = t$, then the effect of continuous injection at the origin is obtained by convolution of the elementary solution over subsequent time intervals:

$$h(r, t) = \int_0^t \frac{\phi(\tau)}{\sqrt{64\pi^3 (K^3/S_s^3)(t-\tau)^3}} \exp\left(-\frac{r^2 S_s}{4K(t-\tau)}\right) d\tau \quad (3)$$

This integral expresses the distribution of head due to a continuous point source of strength $\phi(t)$ (dimensions L^4/t) from time $t = 0$ onwards. The source strength, $\phi(t)$, represents the energy per unit weight of water added to a unit volume of aquifer per unit time. Evaluation of 3 with $\phi(t) = q''$ (constant) gives,

$$h(r, t) = \frac{q'' S_s}{4\pi K r} \operatorname{erfc}\left(\sqrt{(r^2 S_s)/(4Kt)}\right)$$
$$= \frac{Q}{4\pi K r} \operatorname{erfc}\left(\sqrt{(r^2 S_s)/(4Kt)}\right) \quad (4)$$

where $Q = q'' S_s$ is the volumetric rate of water injected (L^3/T), and erfc$(\cdot)$ is the complementary error function. As $t \to \infty$, the solution reduces to $h(r, t) = (Q/4\pi K r)$, which is identical to the elementary steady-state solution for injection at a point in an infinite porous medium (Phillip and Walter, 1992).

Figure 1 depicts an infinite region with such a point source bounded above and below by impermeable planes. The presence of these upper and lower impermeable boundaries can be included by using image well theory (Bear,

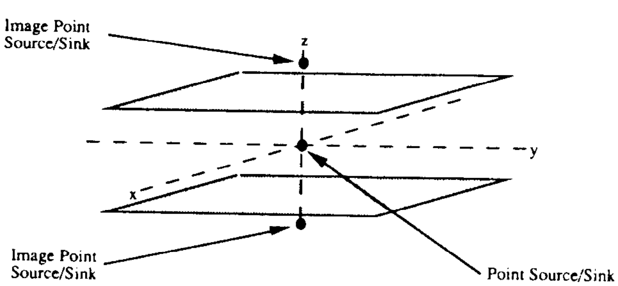

Fig. 1. Conceptual aquifer system and point source/sink.

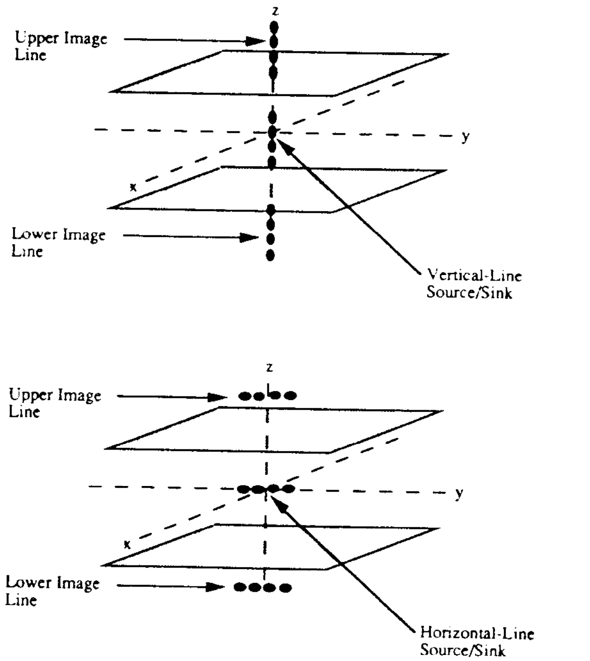

Fig. 2. Conceptual system — (a) vertical well orientation; (b) horizontal well orientation.

1979). The image approach was used because it is relatively easy to apply for the highly symmetric cases studied. Because the domain of interest contains two boundaries, an infinite number of image wells is required. However, accurate simulation with a finite set of wells is possible because the influence of image wells distant from the area of interest is negligible. For transient simulations, the number of image wells required is dependent on the simulation time—in general, the closer to steady state the system becomes, the greater number of wells required. For this paper the adaptive method described by Philip and Walter (1992) was used where successive pairs of image wells are added until the percent change in calculated head falls below some tolerance.

To simulate vertical and horizontal wells the point source solution can be integrated along the length of the well in the plane of interest, and then image well theory is applied. Figure 2(a) depicts the geometric relationship between a vertical well and its image sources, and Figure 2(b) depicts the same relationship for a horizontal well. Mathematically, the finite-line sources in an infinite region are represented as

$$h(r, t) = \int_{r'(\tau = -L/2)}^{r'(\tau = L/2)} \frac{Q}{4\pi K r'} \operatorname{erfc}\left(\sqrt{(r'^2 S_s)/(4Kt)}\right) dr' \quad \ldots (5)$$

where L is the length of the screened interval of the well, $r'(\tau) = \sqrt{x^2 + y^2 + (z - \tau)^2}$ for a vertical well (Figure 2), and $r'(\tau) = \sqrt{x^2 + (y - \tau)^2 + z^2}$ for a horizontal well aligned with the y-axis. An infinite series, closed-form analytical solution for equation (4) (Hantush and Papadopulos, 1962) can be determined by repeated integration by parts;

however, numerical quadrature using Legendre polynomials yields relatively accurate solutions. The quadrature scheme described by Press et al. (1986) was used to integrate the point source terms along finite lengths in the plane(s) of interest.

The asymptotic in time solution to equation (5) for a vertically oriented line source is

$$h(r, t \rightarrow \infty) = h(r) = \frac{Q}{8\pi KL} \ln\left(\frac{z + L + r'(L)}{z - L + r'(-L)}\right) \quad \ldots (6)$$

Numerical quadrature for large time transient solutions ($t > 10{,}000$ days) were indistinguishable from this asymptotic solution except for field points extremely close to the origin. This difference occurs because the singularity at the origin cannot be numerically integrated using a small number of quadrature points. The difference vanishes when an impractically large number of quadrature points are used. In this study, a solution that would be inaccurate for very small distances from the origin (for instance, $r \leq 1.0 \times 10^{-3}$ meters) was accepted so that the simulation could use relatively few (less than 20) quadrature points.

Incorporating image wells for the upper and lower bounding surfaces and changing to Cartesian coordinates with the wells centered at the origin gives

$$s(x, y, z, t) = h(x, y, z, t) + \sum_{i=1}^{N} [h(x, y, z + iB, t) + h(x, y, z - iB, t)] \quad (7)$$

where $s(x, y, z, t)$ is the total head at a field point, $h(x, y, z, t)$ is evaluated using equation (4) with $r = \sqrt{x^2 + y^2 + z^2}$, N is the finite number of image wells used, and B is the thickness of the aquifer. Multiple wells can be simulated using superposition because the governing equation is linear in head.

The discharge at any point is found by differentiation of the head field. Due to linearity of the governing equation the discharge field can also be computed using image theory and superposition. The discharge in the radial direction from a single line sink is

$$q_r = -K \frac{\partial h}{\partial r} \quad (8)$$

The radial discharge field is used for computational convenience. To convert this radial discharge field into a Cartesian discharge field, the following coordinate transformation is used:

$$q_\tau = q_r \frac{\tau}{\sqrt{x^2 + y^2 + z^2}}; \quad \tau = x, y, \text{or } z \quad (9)$$

The discharge field is easily converted into a fluid velocity field by dividing the discharge by the medium's porosity,

$$V_\tau = \frac{q_\tau}{n} \quad (10)$$

where n is the effective porosity of the aquifer. Anisotropy in hydraulic conductivity can be incorporated by scaling of the coordinates (Bear, 1979).

The purpose of this study was to evaluate performance between vertical and horizontal wells. One possible measure of performance for comparison is the volume of aquifer that has a discharge greater than some lower bound after a certain period of pumping. This measure serves to compare the volume of contaminated aquifer that could be potentially restored by either the vertical or horizontal well field. Mathematically, this performance measure can be computed from

$$J(t) = \int_x \int_y \int_z I(x, y, z, t)\, dxdydz \quad (11)$$

where $I(x, y, z, t) = 1$ when $q_r \geq (n/t) \cdot \sqrt{x^2 + y^2 + z^2}$, and zero otherwise. The lower bound for specific discharge represents a minimum discharge required at a point so that a water particle starting from that point would reach the recovery well in the time interval simulated. The efficiency of horizontal to vertical wells is the ratio of this measure for each well.

Application

These semianalytical solutions were implemented to calculate the potential and discharge field in a hypothetical confined aquifer with the geometries shown in Figures 2(a) and 2(b). To test the validity of the simulator, the vertical well solution was compared to the Theis solution for a fully penetrating vertical well. The vertical solution and Theis solution should be close, except possibly very near the well. Figure 3 shows the drawdown predicted by the Theis solution and the vertical finite-line sink as a function of distance from the well. The average relative error between the finite-sink solutions and the Theis solution depicted in Figure 3 is 0% for the early time solution and 6% for the late time solution. The late time finite-sink solutions deviate from the Theis solution because the program was instructed to limit the maximum number of image wells. In general, the agreement is excellent and the solution methodology is appropriate for this study.

Simulations were conducted for a "thin" confined aquifer using the simulation parameters shown in Table 1a, and

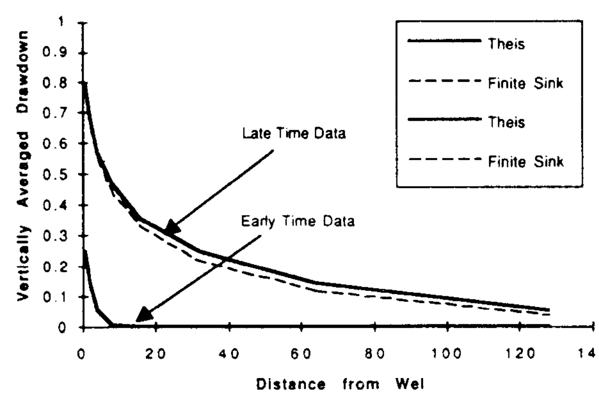

Fig. 3. Comparison of finite-sink simulation and Theis solution for vertically oriented finite-length sink.

Table 1a. Simulation Parameters for Thin Aquifer

Hydraulic conductivity	0.000001 cm/sec
Specific storage	0.001/meter
Aquifer thickness	10.0 meters
Aquifer porosity	0.30
Well discharge	6.0 cu. meters/day

Table 1b. Simulation Parameters for Thick Aquifer

Hydraulic conductivity	0.000001 cm/sec
Specific storage	0.001/meter
Aquifer thickness	100.0 meters
Aquifer porosity	0.30
Well discharge	6.0 cu. meters/day

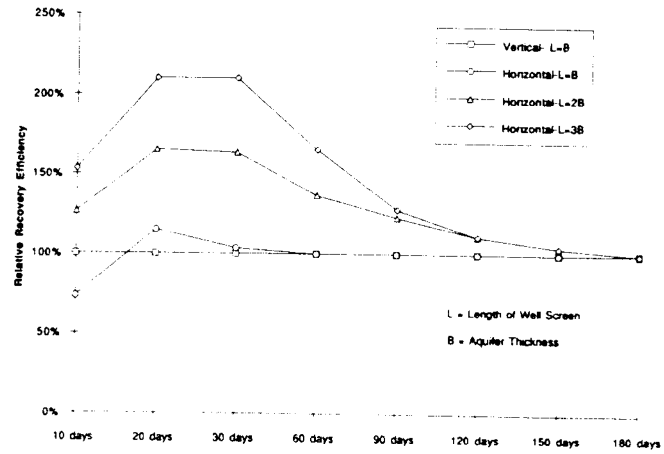

Fig. 5. Comparison of transient recovery efficiencies for horizontal wells compared to a vertical well for thin aquifer.

for a "thick" confined aquifer using the simulation parameters shown in Table 1b. The vertical well simulation assumes that the well is fully penetrating, and the horizontal well simulation assumes that the well is centered in the aquifer. An arbitrary aquifer volume of 40 × 40 × 10 (thin aquifer) cubic meters and 40 × 40 × 100 (thick aquifer) cubic meters centered on the well was chosen as the arbitrary recovery volume of interest. The threshold velocity for computing transient recovery efficiency was chosen to be the smallest velocity at a field point so that a water particle starting at that point would reach the recovery well during the simulation interval. For instance, at 10 days of simulation a particle starting 10 meters from the recovery well would need a distance-averaged velocity of at least one meter per day to reach the well. Since the velocity field is variable in space and time, numerical integration was used to determine the distance-averaged velocity at a field point.

Figure 4 depicts the proportion of the arbitrary volume swept by the flow field in the thin aquifer for different times as a function of horizontal well length. The horizontal well lengths are expressed as multiples of the aquifer's thickness. As the length of the horizontal well is increased, the short time horizon recovery proportion increases dramatically. As the system reaches asymptotic in time conditions, the ultimate volume swept is the same. Figure 5 shows the transient recovery efficiency for horizontal and vertical wells for the thin aquifer. Observe that the vertical well always has 100% efficiency as it is the basis for the efficiency computation.

Simulations for the thick aquifer yielded somewhat different results although the trends are similar. The vertical well outperforms the horizontal well in ultimate recovery for the arbitrary volume selected for horizontal well lengths of less than eight times the aquifer thickness. Figure 6 shows the proportion of the volume swept by the vertical and horizontal well configurations, and Figure 7 shows the corresponding transient recovery efficiency. Observe that horizontal wells are nearly as efficient as a vertical well for early line recovery, but their ultimate performance is inferior until the well length exceeds the aquifer thickness by a factor of eight. Although this result seems to be a limitation, horizontal wells are usually quite long in comparison to the formation thickness and for these cases the study indicates that a horizontal well will perform adequately.

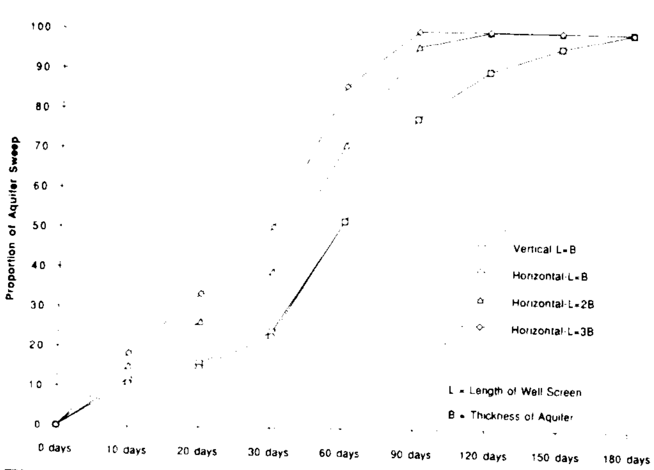

Fig. 4. Comparison of horizontal and vertical well sweep proportions for identical formation constants and total discharge rates for thin aquifer.

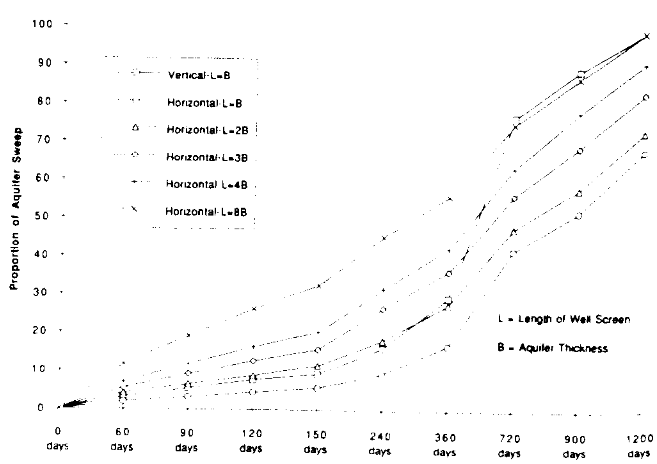

Fig. 6. Comparison of horizontal and vertical well sweep proportions for identical formation constants and total discharge rates for thick aquifer.

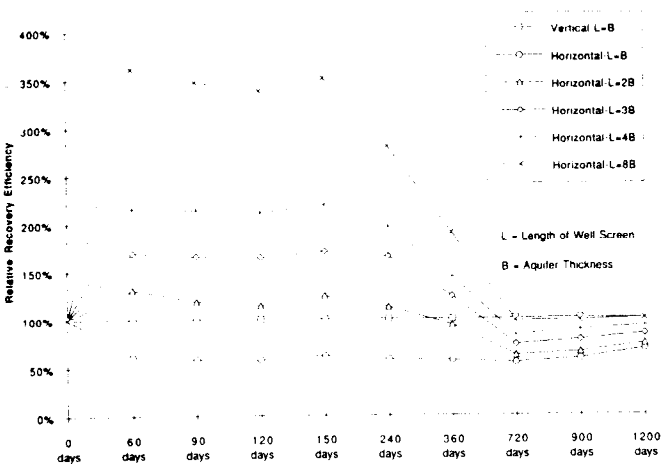

Fig. 7. Comparison of transient recovery efficiencies for horizontal wells compared to a vertical well for thick aquifer.

Conclusion

A semianalytical transient expression for a finite-length horizontal well in a confined aquifer was used to evaluate the performance of a horizontal well as compared to a vertical well in a hypothetical recovery scenario. The measure of performance was the ratio of volume of aquifer that had a specific discharge larger than some lower bound and an arbitrary reference volume. The lower bound for specific discharge was determined as the minimum discharge required at a point so that a water particle starting from that point would reach the recovery well in the time interval simulated. Thin and thick aquifers were simulated using the semianalytical model. The results show that horizontal wells can initially mobilize larger volumes of aquifer liquid but that vertical wells have the same ultimate recovery ability. The results in this study imply that hydraulics alone cannot justify a short horizontal well. However, in the case where a long plume of contaminated water must be recovered quickly, this study suggests that a single horizontal well may be both hydraulically and economically superior to an array of vertical wells.

This paper assumed that the aquifer was confined, isotropic, and homogeneous. Horizontal wells have been proposed to enhance recovery in the less permeable portions of heterogeneous systems and in water-table aquifers. Future study should determine the behavior of a horizontal well in a water-table aquifer, and study the effects of heterogeneity on the relative performance of the two well orientations.

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