Water Management

Modified Rational Unit Hydrograph Method and Applications

--Manuscript Draft--

Manuscript Number:	WATER-D-13-00032R3
Full Title:	Modified Rational Unit Hydrograph Method and Applications
Article Type:	Paper
Corresponding Author:	Xing Fang, Ph.D. Auburn University Auburn, Alabama UNITED STATES
Corresponding Author's Institution:	Auburn University
First Author:	Nirajan Dhakal, Ph.D.
Order of Authors:	Nirajan Dhakal, Ph.D.
	Xing Fang, Ph.D.
	David B. Thompson, Ph.D.
	Theodore G. Cleveland, Ph.D.
Abstract:	The modified rational method (MRM) is an extension of the rational method to develop triangular and trapezoidal runoff hydrographs. A trapezoidal unit hydrograph (UH) was developed from the MRM for the case when the duration of rainfall is less than the time of concentration of the watershed and is called the modified rational unit hydrograph (MRUH). The MRUH method was applied to 1,400 rainfall-runoff events at 80 watersheds in Texas. Application of the MRUH method involved three steps: (1) determination of rainfall excess using the runoff coefficient, (2) determination of the MRUH using drainage area and time of concentration, and (3) simulating event runoff hydrographs. The MRUH performed as well as the Gamma function UH, Clark-HEC-1 UH, and NRCS curvilinear UH methods when the same rainfall loss model was used. The MRUH method can be applied to time-variable rainfall distributions and at watersheds with drainage areas greater than typically used for the rational method (a few hundred acres).
Additional Information:	
Question	**Response**
Please enter the total number of words in your abstract, main text, references and figure captions.	145 words for abstract, 3626 words for main text, 611 words for references, and 323 words for figure captions
Please enter the number of figures, tables and photographs in your submission.	Nine figuresFour tables

Powered by Editorial Manager® and ProduXion Manager® from Aries Systems Corporation

*Main Text
Click here to download Main Text: Paper-MRUH-Sept.doc

- **Article type: paper**
- **Date of text revised: September 25, 2013**
- **Number of words in your main text and tables: 5800**
- **Number of figures: 9**

Modified Rational Unit Hydrograph Method and Applications

Nirajan Dhakal, PhD[1], Xing Fang, PhD, PE, DWRE[2], David B. Thompson, PhD, PE, DWRE[3], and Theodore G. Cleveland, PhD, PE[4]

[1] Post-Doctoral Fellow, Department of Civil Engineering, University of Maine, Orono, ME, USA

[2] Professor, Department of Civil Engineering, Auburn University, Auburn, AL, USA

[3] Director of Engineering, R.O. Anderson Engineering, Inc., Minden, Nevada, USA

[4] Associate Professor, Department of Civil and Environmental Engineering, Texas Tech University, Lubbock, TX, USA

Contact address, telephone and e-mail address of the submitting author:

238 Harbert Engineering Center, Department of Civil Engineering, Auburn University, Auburn, AL, USA 36849

(334) 844-8778

xing.fang@auburn.edu

Abstract

The modified rational method (MRM) is an extension of the rational method to develop triangular and trapezoidal runoff hydrographs. A trapezoidal unit hydrograph (UH) was developed from the MRM for the case when the duration of rainfall is less than the time of concentration of the watershed and is called the modified rational unit hydrograph (MRUH). The MRUH method was applied to 1,400 rainfall-runoff events at 80 watersheds in Texas. Application of the MRUH method involved three steps: (1) determination of rainfall excess using the runoff coefficient, (2) determination of the MRUH using drainage area and time of concentration, and (3) simulating event runoff hydrographs. The MRUH performed as well as the Gamma function UH, Clark-HEC-1 UH, and NRCS curvilinear UH methods when the same rainfall loss model was used. The MRUH method can be applied to time-variable rainfall distributions and at watersheds with drainage areas greater than typically used for the rational method (a few hundred acres).

Notation

α = shape parameter of gamma unit hydrograph (GUH)

A = drainage area in hectares or acres or km^2 or mile^2

AI = cumulative area as a fraction of watershed area (dimensionless)

C = runoff coefficient (dimensionless)

C_{lit} = composite literature-based runoff coefficient

C_{vbc} = back-computed volumetric runoff coefficient

D = storm duration in min. or hr.

D_w = watershed equivalent diameter in km

EF = Nash-Sutcliffe efficiency (dimensionless)

I = average rainfall intensity (mm/hr or in./hr) with the duration equal to time of concentration

i = gross rainfall intensity in mm/hr. or in./hr.

$i_e = Ci =$ effective rainfall intensity in mm/hr. or in./hr.

L = main channel length in mile

m_o = the dimensional correction factor (1.008 in English units, 1/360 = 0.00278 in SI units)

$Q(t)$ = direct runoff hydrograph (DRH) ordinates derived by discrete convolution in m^3/s or ft^3/s

Q_p = peak discharge of DRH in m^3/s or ft^3/s

QB = relative error in observed and simulated DRH peak discharges (dimensionless)

$\overline{Q_{pm}}$ = mean of the modeled DRH peak discharges (subscript m stands for modeled)

$\overline{Q_{po}}$ = mean of the observed DRH peak discharges (subscript o stands for observed)

Q_{pD} = peak discharge of the MRM's DRH for the case when $D < T_c$

Q_{pR} = peak discharge of the rational method in m^3/s or ft^3/s

Q_{pUG} = peak discharge of the GUH in m^3/s or ft^3/s

Q_{pUM} = peak discharge of the MRUH in m^3/s or ft^3/s

Q_{pUN} = peak discharge of the NRCS UH in m^3/s or ft^3/s

$Q_{uG}(t)$ = GUH ordinates in m^3/s or ft^3/s

$Q_{uI}(t)$ = IUH ordinates in m^3/s or ft^3/s

$Q_{uM}(t)$ = MRUH ordinates in m^3/s or ft^3/s

R^2 = coefficient of determination (dimensionless)

$RRMSE$= the root mean squared error of the DRH ordinates normalized by observed Q_p

S = main channel slope (ft/mile)

S_o = channel slope (m/m or ft/ft) for equations in Appendix B

TB = relative error in observed and simulated DRH times to peaks

T_c = time of concentration in min. or hr.

TI = fraction of time of concentration (dimensionless)

T_p = time to peak of DRH in min. or hr.

T_{pU} = time to peak of UH in min. or hr.

T_{pUG} = time to peak of the GUH in min. or hr.

T_{pUN} = time to peak of the NRCS UH in min. or hr.

W = watershed width in km

Key words:

Hydrology; Design methods; Models

1. Introduction

The rational method was originally developed for estimating peak discharge Q_{pR} for sizing drainage structures, such as storm drains and culverts (Kuichling, 1889). The Q_{pR} (in m^3/s or ft^3/s) is computed using:

1. $$Q_{pR} = m_0 CIA$$

where C is the runoff coefficient (dimensionless), I is the average rainfall intensity (mm/hr or in./hr) over a critical period of storm duration (i.e., time of concentration T_c), A is the drainage area (hectares or acres), and m_o is the dimensional correction factor (1/360 = 0.00278 in SI units, 1.008 in English units). Kuichling (1889) and Lloyd–Davies (1906) are credited with independent development of the rational method (Singh and Cruise, 1992).

Incorporation of detention basins to mitigate effects of urbanization on peak flows requires design methods to include the volume of runoff as well as the peak discharge (Rossmiller, 1980). Poertner (1974) developed the modified rational method (MRM) to use when designing hydraulic structures involving storage. The MRM approximates a direct runoff hydrograph (DRH) resulting from a design storm as being either triangular or trapezoidal in shape (Smith and Lee, 1984; Walesh, 1989; Viessman and Lewis, 2003) depending on the relation between the storm duration D and time of concentration T_c. Smith and Lee (1984) revisited the rational method that implied a rectangular response function, which is an instantaneous unit hydrograph (IUH), and developed DRHs using IUH for both constant and variable rainfall intensity events. Singh and Cruise (1992) analyzed the rational formula using a systems approach and concluded that watershed's IUH is a rectangular distribution with the base time equal to T_c of the watershed if a watershed can be represented as a linear, time-invariant system. They used the convolution to derive the S-hydrograph and *D*-hour unit hydrograph (UH) from application of the rational method. Guo (2000, 2001) developed a rational hydrograph method (RHM) for continuous, time-variable rainfall events. Bennis and Crobeddu (2007) developed an improved RHM for small urban catchments using a rectangular impulse response function. However, except Smith and Lee (1984) and Bennis and Crobeddu (2007), all studies related to MRM consider MRM producing DRHs from constant rainfall distributions (Rossmiller, 1980; Viessman and Lewis, 2003). All of the methods were developed and tested for small watersheds with limited data. Similarly, none of the studies has tested the sensitivity of the proposed methods to C and T_c.

In this study, MRM was applied to develop a trapezoidal UH that is termed the modified rational unit hydrograph (MRUH). The purposes of the study were (1) to evaluate the applicability of the method to watersheds of size greater than typically used with either the rational method or the MRM (that is, a few hundred acres), and (2) to study the effects of the runoff coefficient and the time of concentration on prediction of DRHs when the MRUH method is used. We used the MRUH method to compute DRHs for 1,400 rainfall-runoff events at 80 watersheds in Texas, USA. DRHs obtained from the MRUH were compared with those obtained from three other UH models—Clark UH developed for HEC–1's generalized basin (Clark, 1945; USACE, 1981), Gamma function UH for Texas watersheds (Pradhan, 2007), and Natural Resources Conservation Service (NRCS) curvilinear UH (NRCS, 1972).

2. Modified rational unit hydrograph (MRUH)

First, let us revisit the MRM. If $D = T_c$, the resulting DRH from the MRM is triangular with a peak discharge $Q_p = Q_{pR} = CIA$ at time $t = T_c$; that is Case (A) in Figure 1. If $D > T_c$, the resulting DRH is trapezoidal with a constant maximum discharge $Q_p = CIA$ from time D to T_c; that is Case (B) in Figure 1. The linear rising and falling limbs have duration of T_c, as shown in Figure 1 (e.g., from Walesh, 1989; Viessman and Lewis, 2003). If $D < T_c$, then the resulting DRH is trapezoidal with a constant maximum discharge of Q_{pD} (Eq. 2) from the end of the storm duration D to T_c as reported by Smith and Lee (1984) and Walesh (1989):

2. $$Q_{pD} = CIA\left({}^{D}\!/_{T_c} \right) = Q_{pR}\left({}^{D}\!/_{T_c} \right)$$

Smith and Lee (1984) and Singh and Cruise (1992) noted that if the rate of change of the contributing area is constant so that the accumulated tributary area increases and decreases linearly and symmetrically with the time, then the IUH or impulse response function (Chow et al., 1988) $Q_{uI}(t)$ is of rectangular shape given by:

3. $$Q_{uI}(t) = \frac{dA}{dt} = \frac{A}{T_c} \qquad (0 < t < T_c)$$

Using the rectangular response function (Eq. 3), Smith and Lee (1984) and Singh and Cruise (1992) derived the resulting DRH ordinates $Q(t)$ by convolution as:

4. $$Q(t) = \int_0^t i_e(\tau)\, Q_{uI}(t-\tau)\, d\tau$$

where τ is the time with respect to which the integration is carried out and $i_e(\tau) = Ci$ is the effective rainfall intensity with i as gross rainfall intensity. Two types of DRHs, triangular and trapezoidal shape (Figure 1), were obtained from Eq. (4) for constant rainfall intensity, depending on the storm duration.

Using MRM's DRH (Case C in Figure 1) for a *D*-hr rainfall event, the modified rational unit hydrograph or MRUH can be developed if one divides DRH's ordinates by the effective rainfall depth (i.e., $C\ I\ D$) based on the UH derivation method (Viessman and Lewis, 2003). The MRUH is trapezoidal in shape with constant peak discharge $Q_{pUM} = Q_{pD}/(C\ I\ D) = A/T_c$ from D to T_c. The time base for the MURH is $D + T_c$ and MRUH ordinates can be computed from Eq. 5:

$$\begin{aligned} &Q_{uM}(t) = \frac{A}{T_c}\frac{t}{D} && 0 \le t \le D \\ &Q_{uM}(t) = \frac{A}{T_c} && D \le t \le T_c \\ &Q_{uM}(t) = \frac{A}{T_c}\frac{T_c + D - t}{D} && T_c \le t \le T_c + D \end{aligned} \tag{5}$$

The *D*-hr MRUH results from a constant excess rainfall intensity of $1/D$ in./hr over D hrs and has a peak discharge of A/T_c in ft^3/s when drainage area A is in acres and T_c is in hours for 1 inch of rainfall excess (taking into account that one-acre inch per hour is nearly equal to one cubic foot per second). If SI units are used (drainage area A in hectare and rainfall intensity in mm/hr), the peak discharge from the MRUH should be equal to $A/(360T_c)$ in m^3/s for 1 mm of rainfall excess. Three examples of the MRUH developed for three watersheds used in this paper are shown in Figure 2. It is worth to mention that Cases (A), (B), and (C) of the MRM in Figure 1 are DRHs and none is an UH, although Cases (B) and (C) have the same shape as MRUH in Figure 2.

The assumption and restriction for the application of the rational method and original MRM include constant rainfall intensity throughout the storm duration (Rossmiller, 1980) and for small catchments, i.e., drainage area less than 0.8 km^2 or 200 acres (TxDOT, 2002). Application of the MRUH method involves three steps as stated in the abstract. Because the MRUH method is an UH method, then the approach establishes a continuity of hydrograph-development methods from very small watersheds to relatively large watersheds. The UH for a watershed can be used to predict the DRH for any given rainfall excess hyetograph (constant or time-variable rainfall distribution) using the UH discrete convolution (Chow et al., 1988; Viessman and Lewis, 2003). In summary, application of the MRUH method is straightforward and similar to application of other UH methods using discrete convolution; the assumption and restriction for the MRM are no longer necessary, which will be demonstrated through this study.

The MRUH method was first tested using rainfall-runoff data obtained for concrete surfaces from Yu and McNown (1964). The first dataset was based on a test bed with an area of 152.4 m by 0.3 m (500 ft by 1 ft), surface slope of 0.02 (dimensionless), and a constant rainfall intensity. The second dataset was based on a test bed with an area of 76.8 m by 0.3 m (250 ft by 1 ft), surface slope of 0.005, and a variable rainfall intensity. The T_c of about 5 minutes was computed using the Kirpich method (Kirpich, 1940) for both experiments. A trapezoidal 1-minute MRUH was developed for each experiment (Figure 2A). The runoff coefficient was taken to be unity. For both cases, the modeled DRHs using MRUH match the observed DRHs well (Figure 3).

The Nash-Sutcliffe efficiency *EF* (Equation A.3) was 0.93 and 0.80 for the experiments using the constant (Figure 3A) and time-variable rainfall intensity (Figure 3B), respectively. According to Bennis and Crobeddu (2007), a good agreement between the simulated and the measured data is reached when *EF* is higher than 0.7 for hydrograph simulation; therefore, above large *EF* values indicated a good fit between modeled and observed DRHs for both experiments.

3. Applications of the MRUH method in Texas watersheds

3.1 Watersheds studied and rainfall-runoff database

Watershed data taken from a larger dataset (Asquith et al., 2004) accumulated by researchers from the United States Geological Survey (USGS) Texas Water Science Center, Texas Tech University, University of Houston, and Lamar University were used for this study. Location and geographic distribution of the stations are shown in Figure 4. The drainage areas of 80 study watersheds ranged from approximately 0.8 to 65.0 km^2 (0.3 to 25 mi^2), with a median value of 15.8 km^2 (6.1 mi^2); 50 watersheds (62.5% of the 80 watersheds) have the drainage area less than 20 km^2 (7.7 mi^2). The stream slope of study watersheds ranged from 0.0026 to 0.0196 (dimensionless), with a median value of 0.0079. The main channel lengths estimated are approximately 2–80 km (1.2–49.7 miles). The percentage of impervious area (*IMP*) of 80 study watersheds ranged from 0.0 to 74.0%, with a median value of 26.0%. About 40% of the watersheds are rural watersheds with *IMP* less than 5%, and about 29% of the watersheds are urbanized with *IMP* greater than 60%.

The rainfall-runoff dataset comprised about 1,400 rainfall-runoff events recorded during 1959–1986. Event rainfall depths ranged from 3.56 mm (0.14 in.) to 489.20 mm (19.26 in.), with a median value of 57.66 mm (2.27 in.). About 41% and 86% of the events had a storm depth less than 50.8 mm (2 inches) and 101.6 mm (4 inches), respectively. The base flow separations for observed runoff hydrographs were not done. This is because majority of the gauging stations are on a small ephemeral streams; base flow represents a small component of the total flow at the station. The streamflow for the watershed frequently was zero at the beginning of the storms (Asquith et al., 2004).

3.2 Time of concentration and runoff coefficients

Time of concentration, T_c, and the runoff coefficient, C, are the required parameters for the MRUH method. The T_c were estimated by Fang et al. (2008) using four empirical equations (see Appendix B): (1) Williams equation (1922), (2) Kirpich equation (1940), (3) Johnstone–Cross equation (1949), and (4) Haktanir–Sezen equation (1990).

The excess rainfall or the net rainfall is obtained from the product of the incremental rainfall and C (the volumetric interpretation, Dhakal et al., 2012), similar to Smith and Lee (1984). Two estimates of C were examined for the application of the MRUH method. The first C is a watershed composite, literature-based coefficient (C_{lit}) derived from land-use information for the watershed and published C_{lit} values for appropriate land-uses (Dhakal et al., 2012). The second C is a back-computed, volumetric runoff coefficient (C_{vbc}) determined by preserving the runoff volume using observed rainfall and runoff data. C_{vbc} was estimated by the ratio of total runoff depth to total rainfall depth for individual observed storm event. The determination and comparison of C_{lit} and C_{vbc} for the study watersheds was documented by Dhakal et al. (2012).

3.3 DRHs derived using the MRUH method

For the 80 Texas watersheds, observed rainfall hyetograph and runoff hydrograph data were tabulated using a time interval of five minutes. Therefore, a five-minute MRUH was developed for each of the 80 study watersheds. The five-minute MRUH duration is less than T_c for all study watersheds.

The observed and simulated DRHs for the event on 07/08/1973 at the USGS streamflow-gauging station 08157000 Waller Creek, Austin, Texas are presented in Figure 5 as an illustrative example. The watershed drainage area is 5.72 km^2 (2.21 mi^2). The C_{vbc} is 0.29. The T_c values estimated using the Kirpich, Haktanir-Sezen, Johnstone-Cross, and Williams equations are 1.7, 2.2, 1.4, and 3.4 hours, respectively. Peak discharges Q_{pUM} of the 5-minute MRUH using 1 inch (or 25.4 mm) rainfall excess for the watershed are 23.7 m^3/s , 18.3 m^3/s, 28.8 m^3/s , and 11.9 m^3/s using T_c values estimated from the Kirpich, Haktanir-Sezen, Johnstone-Cross, and Williams equations, respectively. Figure 2B shows an example MRUH for the watershed developed using T_c estimated from the Kirpich method (Equation B.2); and other three MRUHs developed from other T_c methods are trapezoids with different peaks and time bases ($D + T_c$) but the area under each trapezoid is the same because MRUH is a UH. Duration of the rainfall event was 19 hours. Three distinct rainfall episodes resulted in three distinct peaks. These were reasonably represented by the DRHs derived from the MRUH using T_c estimated by the Kirpich, Haktanir-Sezen, and Johnson-Cross equations. The DRH developed from the MRUH using the Williams equation appears to over-estimate T_c for the watershed and discharge peaks of the DRH were then underestimated (Figure 5). When the MRUHs were developed using T_c values estimated from the Kirpich, Haktanir-Sezen, Johnstone-Cross, and Williams equations, the *EF* (Equation

A.3) values derived between observed DRH and modeled DRHs using above corresponding MRUHs are 0.83, 0.86, 0.70, and 0.63, respectively. Simulated times to peak (T_p) agree reasonably well with observed values (Figure 5) when using T_c estimated by Kirpich, Haktanir-Sezen, and Johnson-Cross equations for the MRUHs. However, using T_c estimated by the Williams equation for the MRUH resulted in the computed T_p exceeding the observed T_p.

Different combinations of T_c and C were used for applications of the MRUH method to predict the DRHs and to determine the sensitivity of the DRH peak discharges (Q_p) to different T_c and C values. Five combinations of T_c and C were used:

(A) T_c estimated using Haktanir-Sezen equation and C_{vbc},

(B) T_c estimated using Johnstone-Cross equation and C_{vbc},

(C) T_c estimated using Williams equation and C_{vbc},

(D) T_c estimated using Kirpich equation and C_{vbc}, and

(E) T_c estimated using Kirpich equation and C_{lit}.

Figure 6 is a plot of the observed and computed DRH peaks using C_{vbc} and T_c values calculated using the four different empirical equations. In comparison to observed Q_p modeled Q_p using T_c estimated from the Haktanir-Sezen, Johnstone-Cross and Kirpich equations not only graphically look alike (Figure 6) but also are similar with respect to three statistical parameters (Table 1): coefficient of determination R^2; Nash-Sutcliffe efficiency EF; and relative error in peak QB (defined in Appendix A). The results for EF using the Williams equation are inferior to others. The fraction of modeled Q_p results that are within 1/3 of a log-cycle from the 1:1 line are summarized in Table 1 and ranged from 67.5% (Williams equation) to 88.7% (Johnstone-Cross equation) of total events. Fractions of storms with QB less than ±50% (Cleveland et al., 2006) are listed in Table 1 for applications of the MRUH method with four combinations of T_c and C. Using T_c estimated from the Kirpich equation and C_{vbc} resulted in 75% of storms with QB less than ±50%. C_{vbc} (back-computed from rainfall and runoff data) results in preservation of event runoff volume, and Kirpich equation provides reliable estimations on watershed T_c values (Fang et al., 2008). Ideally, computed and observed peaks should plot precisely along the equal value line (black line in Figure 6). However, the UH is a mathematical model that is an incomplete description of the complexity of the combination of the rainfall-runoff process and runoff dynamics. Therefore, the relatively simple approach cannot fully capture the nuances of watershed dynamics and deviations from this ideal (the equal-value line) are expected. For example, Asquith and Roussel (2009) computed mean residual standard error about 1/3 of a log cycle for annual peak discharges at 638 streamflow gauging stations in Texas.

The observed T_p and computed T_p values of DRHs predicted using C_{vbc} and T_c values calculated using the four different empirical equations were compared using three error parameters R^2, EF and relative error in time to peak TB (Equation A.5). T_c estimated from the Haktanir-Sezen, Johnstone-Cross, and Kirpich equations produces the similar values of the quantitative measures: R^2, EF, median value of TB and fraction of storms with TB less than ± 50% (Table 1). The T_p results using the Williams equation seem to be slightly inferior to others with respect to median value of TB and % of storms within ± 1/3 of a log cycle (Table 1). In summary, for predicting Q_p and T_p, use of T_c estimated from Williams equation for the MRUH produces less accurate results than those computed using the Kirpich, Haktanir-Sezen and Johnstone-Cross equations.

Simulated Q_p results obtained from the MRUH method using the forward-computed (literature-based) runoff coefficient C_{lit} are compared against the Q_p results obtained using the back-computed runoff coefficient C_{vbc} (Figure 7). For both the cases, T_c values were estimated using the Kirpich equation. For the peak discharges predicted using C_{lit}, most of the values are above the equal value line (1:1 line). Q_p results computed using C_{vbc} are superior to those using C_{lit} with respect to all statistical measures used to assess goodness of fit (Table 2). Use of C_{lit} tends to generate estimates of Q_p that exceed expected values (observations) when the C_{lit} values are interpreted as volumetric coefficients. In contrast, there is no difference in five quantitative measures between the observed and predicted T_p values (Table 2), regardless of which runoff coefficient is used. Hence, simulation results of Q_p are more sensitive to the choice of C or rainfall loss model than to the choice of T_c. Furthermore, the T_p results are not related to C when the MRUH method was used and controlled by the time-variable rainfall distribution.

A sensitivity analysis was performed to evaluate the sensitivity of the DRH derived from the MRUH method to T_c and C. A rainfall event on 05/07/1972 for the USGS streamflow-gauging station 08178600 Salado Creek San Antonio (24.88 km^2 or 9.61 $mile^2$) was selected for the analysis. The T_c used for the MRUH was varied from -50% to +50% of T_c estimated from the Kirpich equation. Similarly, the C used for rainfall loss was varied from -50% to +50% of C_{vbc}. The EF computed between the observed DRH and modeled DRH derived from the MRUH method using C_{vbc} and T_c estimated from the Kirpich equation was 0.89. The change in EF values due to the change on T_c and C for the sensitivity analysis are presented in Table 3. Change in EF ranged from 0.01 to -0.22 for ±50% change in Tc. Similarly, the change in EF ranged from 0.02 to -0.66 for ±50% change in C. This analysis further supports the above conclusion that

DRH derived using the MRUH method are more sensitive to the choice of C than to the choice of $T_{c.}$

4. Comparison of DRHs from different UH methods

In addition to the MRUH, three other UH models—UH developed using the Clark IUH method (Clark, 1945) with HEC–1's generalized basin shape (USACE, 1981), the NRCS UH (NRCS, 1972), and the Gamma function UH (GUH) for Texas watersheds (Pradhan, 2007)—were used to develop the DRH for each rainfall-runoff event in the database for the comparison.

Regression equations were developed for five-minute GUH parameters: Q_{pUG} (in ft^3/s) and T_{pUG} (in hours) for Texas watersheds (Pradhan, 2007),

6. $$T_{pUG} = 0.55075\,A^{0.26998}\,L^{0.42612}\,S^{-0.06032}$$

7. $$Q_{pUG} = 93.22352\,A^{0.83576}\,L^{-0.326}\,S^{0.5}$$

where A is drainage area in square miles, L is main channel length in miles, and S is main channel slope (ft/mile, elevation difference in feet divided by main channel length in miles). The ordinates of the GUH can be obtained from (Viessman and Lewis, 2003):

8. $$Q_{uG}(t) = Q_{pUG}\left(t/T_{pUG} \right)^{\alpha} e^{[1-(t/T_{pUG})]^{\alpha}}$$

where α is the shape parameter of GUH, which is determined from Q_{pUG} and T_{pUG} (Aron and White, 1982).

Clark's (1945) IUH method is based on the time-area curve method (Bedient and Huber, 2002). A synthetic time-area curve derived from a generalized basin shape was used to implement Clark's IUH in HEC-1 (USACE, 1981). The equations for the time-area curve are

9. $AI = 1.414\, TI^{1.5}, \qquad 0 \le TI \le 0.5$

10. $1 - AI = 1.414\,(1-TI)^{1.5}, \qquad 0.5 < TI < 1$

where AI is the cumulative area as a fraction of watershed area and TI is fraction of T_c.

The NRCS curvilinear UH was developed in the late 1940s (NRCS, 1972). The Q_{pUN} for the NRCS UH is computed by approximating the UH with a triangular shape having base time of $8/3T_{pUN}$ and unit area (Viessman and Lewis, 2003):

11. $Q_{pUN} = \frac{484A}{T_{pUN}}$

where Q_{pUN} is ft^3/s and A is the drainage area in mi^2.

UHs developed using all four models, including the MRUH, for the watershed associated with the USGS streamflow-gauging station 08048520 Sycamore Creek in Fort Worth are shown in Figure 8A. The shape of the MRUH is trapezoidal, while UHs from the Clark-HEC-1, the Gamma, and the NRCS methods are curvilinear. The UH peak discharge from each model is different (Figure 8A). However, the area under the UH curves is the same. This is because each UH corresponds to 1 inch of a uniform excess rainfall over 5-minute duration (one impulse).

Gamma, Clark-HEC-1, and NRCS UHs developed for each watershed were applied to the 1,400 rainfall-runoff events in the database to generate DRHs using discrete UH convolution (Chow et al., 1988). C_{vbc} determined for each event was used. T_c determined using the Kirpich method (1940) was used for those methods that require T_c. As an illustrative example, observed and simulated DRHs for the rainfall event on 07/28/1973 at the USGS streamflow-gauging station 08048520 (Sycamore Creek in Fort Worth, Texas) by the four models (base flow was assumed to be zero) is presented in Figure 8B. The watershed area is of 45.66 km^2 (17.63 mi^2), T_c is 3.96 hours from the Kirpich method, and C_{vbc} is 0.20. Simulated peak discharges from the four UH methods are different, but comparable. For the particular example shown in Figure 8B, the MRUH and the Clark-HEC-1 model appear to perform better than the other UH models with regard to prediction of Q_p. For the T_p, simulated values using the four methods agree reasonably well with the observed value (Figure 8B). Additionally, the area under the four simulated DRHs matches that of observed curve because event C_{vbc} was used.

Simulated DRH ordinates derived from all the four UH models were compared with observed DRH ordinates for each rainfall event, and the root mean squared error of the DRH ordinates normalized by observed Q_p (*RRMSE*, Equation A.1) was calculated for each event and then averaged for all the events in the same watershed. A statistic summary of averaged normalized root mean squared errors for 80 study watersheds is presented in Table 4. All the four UH models behave similarly to predict DRHs based on statistical parameters in Table 4, and Figure 8B shows one example to illustrate the similarity of DRHs derived from these UH models.

The observed and modeled Q_p results from all four UH models developed using C_{vbc} and T_c from the Kirpich method are presented in Figure 9 for all 1,400 events. Modeled Q_p results from all the four UH models are similar (Figure 9). Based on the three statistical measures (*RRMSE*, R^2, and *EF*) we concluded that all the four UH models perform similarly in predicting DRH Q_p and T_p (Table 5) after considering possible errors in DRH prediction. Fractions and percentages of storms for each model meeting the tolerances of *QB* and *TB* are also listed in Table 5 and show that all the models perform similarly. However, the GUH developed for Texas watersheds perform slightly worse than the other three UH models (Table 5) in predicting DRH Q_p.

5. Summary and Conclusions

The MRM is an extension of the rational method to produce simple triangular and trapezoidal DRHs that have been used in some engineering applications. MRM's DRH for $D < T_c$ was used to derive a trapezoidal UH termed the modified rational UH or MRUH. The MRUH method was applied at 80 watersheds in Texas to determine the DRHs for 1,400 rainfall-runoff events. The purposes were (1) to evaluate the applicability of the MRUH method when applied to watersheds of larger size (0.8–65.0 km^2 or 0.3–25 mi^2), and (2) to study the effects of C and T_c on prediction accuracy of the MRUH method on DRH ordinates, DRH Q_p, and DRH T_p. Three other UH models; the Clark (using HEC–1's generalized basin equations), the Gamma, and the NRCS UHs were used to compute the DRH for each rainfall-runoff event in the same database. Simulated peak discharges of DRHs from MRUH and other three UHs agree reasonably well with observed values. The drainage area of the study watersheds (0.8–65.0 km^2 or 0.3–25 mi^2) is greater than that usually accepted for rational method application (0.8 km^2 or 0.3 mi^2).

Three general conclusions for the study are: (1) Being a UH, the MURH method can be applied to time-variable rainfall events and for watersheds with drainage areas greater than typically used with either the rational method or the MRM (a few hundred acres). (2) The MRUH performs about as well as other UH methods used in this study for predicting Q_p and T_p of the DRH, so long as the same rainfall loss model is used. (3) Modeled peak discharges from application of the MRUH method are more sensitive to the selection of C and less sensitive to T_c. In predicting peak discharges and DRHs for engineering design, rainfall loss estimation results in greater uncertainty and contributes more model errors than variations of UH methods and model parameters for UH.

Acknowledgments

The authors thank TxDOT project director Mr. Chuck Stead, P.E., and project monitoring advisor members for their guidance and assistance. They also express their thanks to anonymous reviewers. This study was partially supported by TxDOT Research Projects 0–6070, 0–4696, 0–4193, and 0–4194.

Appendix A: ***Statistical measures to evaluate model performance***

Five statistical measures were used to analyze modeled DRH results against observed ones. They are the root mean squared error (RMSE) of the DRH ordinates normalized by observed DRH Q_p, i.e. relative RMSE or *RRMSE*, the coefficient of determination R^2, the Nash-Sutcliffe efficiency

EF, the relative error in peak *QB*, and the relative error in time to peak *TB* (Loague and Green, 1991; Cleveland et al., 2006; Zhao and Tung, 1994):

A.1 $$RRMSE = \frac{\left[\sum_{j=1}^{N} (Q(t)_{mj} - Q(t)_{oj})^2 \Big/ N \right]^{0.5}}{Q_{po}} ,$$

A.2 $$R^2 = \left(\frac{\sum_{i=1}^{n} (Q_{poi} - \overline{Q_{po}})(Q_{pmi} - \overline{Q_{pm}})}{\sqrt{\sum_{i=1}^{n} (Q_{poi} - \overline{Q_{po}})^2} \sqrt{\sum_{i=1}^{n} (Q_{pmi} - \overline{Q_{pm}})^2}} \right)^2 ,$$

A.3 $$EF = \frac{\left(\sum_{i=1}^{n} (Q_{poi} - \overline{Q_{po}})^2 - \sum_{i=1}^{n} (Q_{pmi} - Q_{poi})^2 \right)}{\sum_{i=1}^{n} (Q_{poi} - \overline{Q_{po}})^2} ,$$

A.4 $$QB = \frac{Q_{pmi} - Q_{poi}}{Q_{poi}} \text{, and}$$

A.5 $$TB = \frac{T_{pmi} - T_{poi}}{T_{poi}}$$

where $Q(t)_{mj}$ is the modeled DRH ordinate (subscript *m* stands for modeled), $Q(t)_{oj}$ is the observed the DRH ordinate (subscript *o* stands for observed), *N* is the number of DRH ordinates for an event, Q_{pmi} is the modeled Q_p for the event *i*, Q_{poi} is the observed Q_p, *n* is the number of observations, $\overline{Q_{pm}}$ and $\overline{Q_{po}}$ are the mean values of the modeled and observed peak discharges, T_{pmi} is the modeled T_p, and T_{poi} is the observed T_p.

Appendix B: *Empirical equations used to estimate T_c*

Four empirical equations Williams (1922), Kirpich (1940), Johnstone-Cross (1949) and Haktanir-Sezen (1990) used to estimate T_c (in minutes) by Fang et al. (2008) are given respectively below:

B.1 $T_c = 16.32 L A^{0.4} / (D_w S_o^{0.2})$

B.2 $T_c = 3.978 L^{0.77} S_o^{-0.385}$

B.3 $T_c = 3.258 (L / S_o)^{0.5}$

B.4 $T_c = 26.85 L^{0.841}$

where L is the channel length in km, D_w is the watershed equivalent diameter in km, W is the watershed width in km, A is the area in km^2, and S_o is the channel slope in m/m or ft/ft (dimensionless).

REFERENCES

Aron G and White EL (1982) Fitting a gamma distribution over a synthetic unit hydrograph. *Water resources Bulletin* **18(1):** 95-98.

Asquith WH, Thompson DB, Cleveland TG and Fang X (2004) *Synthesis of rainfall and runoff data used for Texas department of transportation research projects 0-4193 and 0-4194.* Open-File Rep. 2004-1035, USGS, Austin, Texas, USA.

Asquith WH and Roussel MC (2009) *Regression equations for estimation of annual peak-streamflow frequency for undeveloped watersheds in Texas using an L-moment-based,* *PRESS-Minimized, residual-adjusted approach*. U.S. Geological Survey Scientific Investigations Report 2009-5087, Austin, Texas, USA.

Bedient PB and Huber WC (2002) *Hydrology and Floodplain Analysis*. Prentice Hall, Upper Saddle River, NJ, USA.

Bennis S and Crobeddu E (2007) New runoff simulation model for small urban catchments. *Journal of Hydrologic Engineering* **12(5):** 540-544.

Chow V T, Maidment DR, and Mays LW (1988) *Applied hydrology*. McGraw Hill, New York, USA.

Clark CO (1945) Storage and the unit hydrograph. *Transactions of American Society of Civil Engineering* **110:** 1419-1446.

Cleveland TG, He X, Asquith WH, Fang X and Thompson DB (2006) Instantaneous unit hydrograph evaluation for rainfall-runoff modeling of small watersheds in north and south central Texas. *Journal of Irrigation and Drainage Engineering* **132(5):** 479-485.

Dhakal N, Fang X, Cleveland TG, Thompson DB, Asquith WH and Marzen LJ (2012) Estimation of volumetric runoff coefficients for Texas watersheds using land-use and rainfall-runoff data. *Journal of Irrigation and Drainage Engineering* **138(1):** 43-54.

Fang X, Thompson DB, Cleveland TG, Pradhan P and Malla R (2008) Time of concentration estimated using watershed parameters determined by automated and manual methods. *Journal of Irrigation and Drainage Engineering* **134(2):** 202-211.

Guo CYJ (2000) Storm hydrograph from small urban catchments. *Water International* **25(3)**: 481-487.

Guo CYJ (2001) Rational hydrograph method for small urban watersheds. *Journal of Hydrologic Engineering* **6(4):** 352-357.

Haktanir T and Sezen N (1990) Suitability of two - parameter gamma and three - parameter beta distributions as synthetic unit hydrographs in Anatolia. *Hydrological Sciences* **35(2):** 167-184.

Johnstone D and Cross WP (1949) *Elements of applied hydrology*. Ronald Press, New York, USA.

Kirpich ZP (1940) Time of concentration of small agricultural watersheds. *Civil Engineering* **10(6):** 362.

Kuichling E (1889) The relation between the rainfall and the discharge of sewers in populous areas. *Transactions, American Society of Civil Engineers* **20:** 1–56.

Lloyd-Davies DE (1906) The elimination of storm water from sewerage systems. *Minutes of Proceedings, Institution of Civil Engineers,* Great Britain, **164:** 41.

Loague K and Green RE (1991) Statistical and graphical methods for evaluating solute transport models: overview and application. *Journal of Contaminant Hydrology* **7(1-2):** 51-73.

NRCS (1972) *Hydrology*. National Resource Conservation Service (NRCS), U.S. Dept. of Agriculture, Washington D.C, USA.

Poertner HG (1974) *Practices in detention of urban stormwater runoff: An investigation of concepts, techniques, applications, costs, problems, legislation, legal aspects and opinions*. No.43. American Public Works Association, Chicago, IL, USA.

Pradhan P (2007) *Rainfall loss and unit hydrograph estimation by nonlinear programming for the Texas watersheds*. Dissertation, Lamar University, Beaumont, Texas.

Rossmiller RL (1980) Rational formula revisited. *International Symposium on Urban Storm Runoff*, University of Kentucky, Lexington, KY., 1-12.

Singh VP and Cruise JF (1992) Analysis of the rational formula using a system approach. In *Catchment runoff and rational formula* (BC Yen (ed.)). Water Resources Publication, Littleton, Colo. , 39-51.

Smith AA and Lee K (1984) The rational method revisited. *Canadian Journal of Civil Engineering* **11:** 854-862.

TxDOT (2002) *Hydraulic design manual*. The bridge division of the Texas Department of Transportation (TxDOT), Austin, Texas, USA.

USACE (1981) HEC-1 flood hydrograph package, user's manual (Revision in 1987). U.S. Army Corps of Engineers (USACE), Hydrologic Engineering Center (HEC), Davis, CA.

Viessman W and Lewis GL (2003) *Introduction to hydrology*. Pearson Education, Upper Saddle River, NJ, USA.

Walesh SG (1989) *Urban Water Management*. Wiley, New York.

Williams GB (1922) Flood discharges and the dimensions of spillways in India. *Engineering (London)* **134:** 321.

Yu YS and McNown JS (1964) Runoff from impervious surfaces. *Journal of Hydraulic Research* **2(1):** 3-24.

Zhao B and Tung YK (1994) Determination of optimal unit hydrographs by linear programming. *Water Resources Management* **8(2):** 101-119.

List of Figure captions

Figure 1. The modified rational hydrographs or DRHs for three different cases: (A) $D = T_c$, (B) $D > T_c$, and (C) $D < T_c$.

Figure 2. The MRUHs developed for: (A) two lab settings from Yu and McNown (1964) and (B) for the watershed associated with USGS streamflow-gauging station 08157000 Waller Creek, Austin, Texas. T_c values used for MRUHs were computed using Kirpich method (Equation B.2)

Figure 3. Incremental rainfall hyetograph and observed and modeled DRHs using the MRUHs for the two lab tests on concrete surfaces: (A) 152.4 m × 0.3 m with 2% slope and (B) 76.8 m × 0.3 m with 0.5% slope reported by Yu and McNown (1964).

Figure 4. Map showing the USGS streamflow-gauging stations (dots) associated with the watershed locations in Texas, USA.

Figure 5. Incremental rainfall hyetograph for the event on 07/08/1973 and observed and modeled DRHs using the MRUHs with T_c estimated by four empirical equations for the watershed associated with the USGS streamflow-gauging station 08157000 Waller Creek, Austin, Texas.

Figure 6. Modeled versus observed DRH peak discharges Q_p for 1,400 rainfall-runoff events in 80 Texas watersheds. Modeled DRH peaks were developed using event C_{vbc} and MRUHs with T_c estimated using four different methods: (A) Haktanir-Sezen equation, (B) Johnstone-Cross equation, (C) Williams equation, and (D) Kirpich equation.

Figure 7. Observed and modeled DRH peak discharges developed using C_{vbc} (triangles) and C_{lit} (circles) and MRUHs with T_c estimated using the Kirpich equation for 80 Texas watersheds.

Figure 8. (A) Modified rational, Gamma, Clark-HEC-1, and NRCS UHs developed for the watershed associated with USGS streamflow-gauging station 08048520 Sycamore, Fort Worth, Texas; and (B) Rainfall hyetograph, observed and modeled DRHs using the four different UHs for the rainfall event on 07/28/1973 for the same watershed.

Figure 9. Observed and Modeled DRH peak discharges using: (A) MRUH, (B) Gamma UH, (C) Clark-HEC-1 UH, and (4) NRCS UH for 1,400 rainfall-runoff events in 80 Texas watersheds.

AUBURN
UNIVERSITY

SAMUEL GINN COLLEGE OF ENGINEERING
DEPARTMENT OF CIVIL ENGINEERING

CONSTRUCTION

ENVIRONMENTAL

GEOTECHNICAL

HYDRAULICS

PAVEMENTS & MATERIALS

STRUCTURAL

TRANSPORTATION

238 HARBERT ENGINEERING CENTER
AUBURN, AL 36849-5337

TELEPHONE:
334-844-4320

FAX:
334-844-6290

www.auburn.edu

Chief Editor
Water Management
Proceedings of the Institution of Civil Engineers

September 25, 2013

Dear Editor:

Enclosed is a finalized manuscript titled as "Modified Rational Unit Hydrograph Method and Applications" by Nirajan Dhakal, Xing Fang, David B. Thompson, and Theodore G. Cleveland. We have considered all of minor editorial comments from the reviewer to finalize the manuscript. The reply document summarized the details how we addressed reviewer comments. The manuscript is submitted for the publication in the **ICE Water Management**.

Sincerely yours,

Xing Fang

Xing Fang, PhD, PE, DWRE
Professor

Figure 1

Click here to download high resolution image

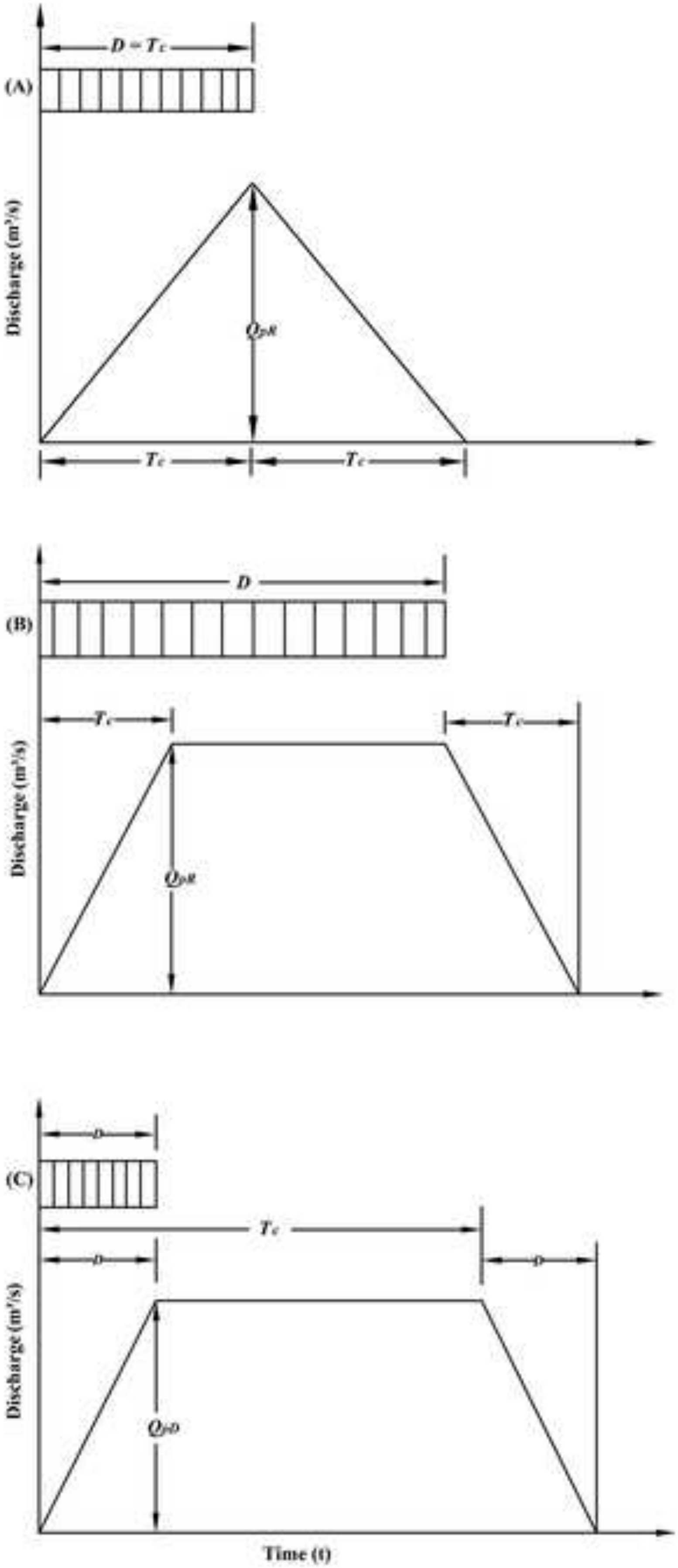


Figure 2

Click here to download high resolution image

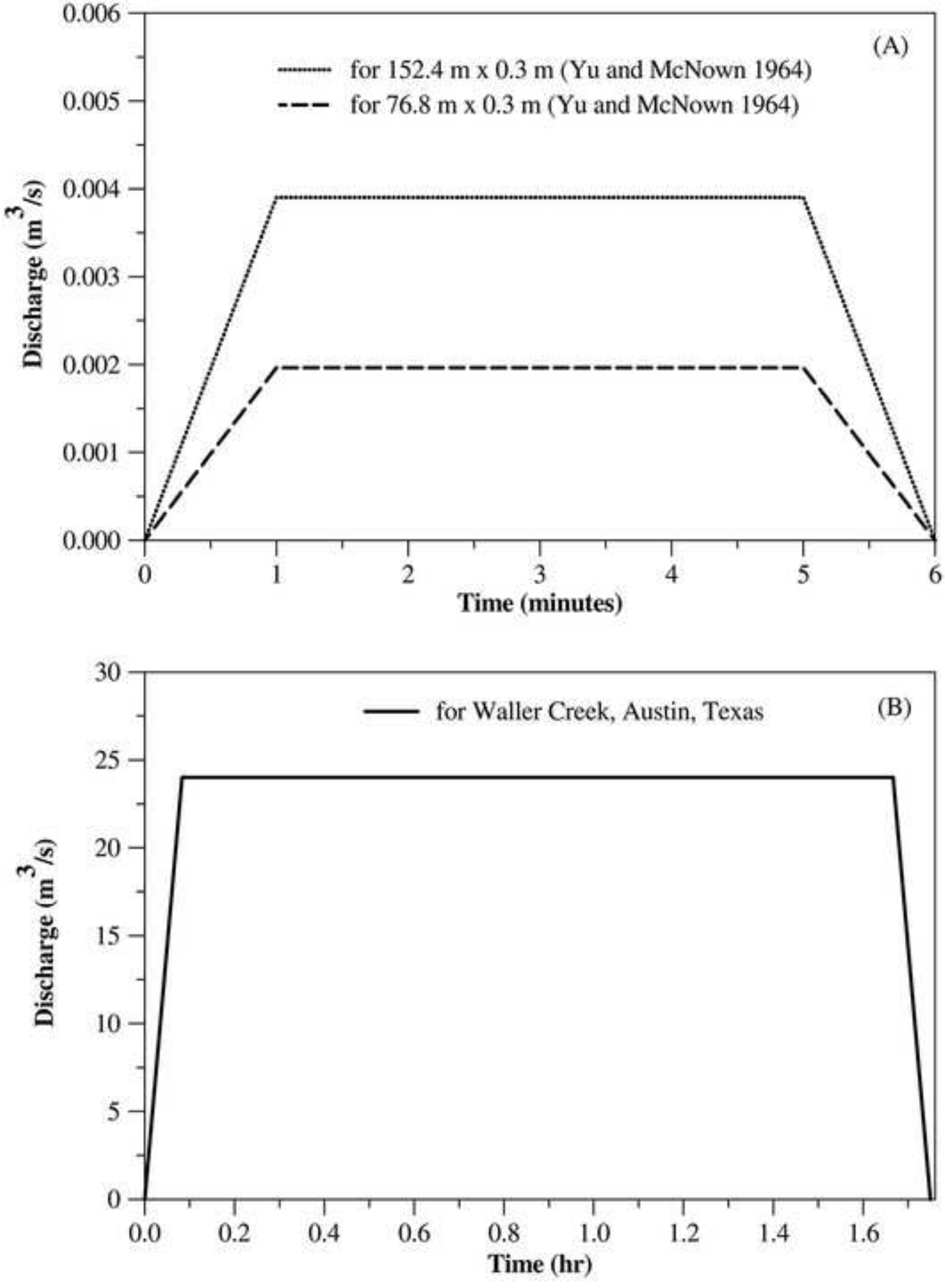


Figure 3

Click here to download high resolution image

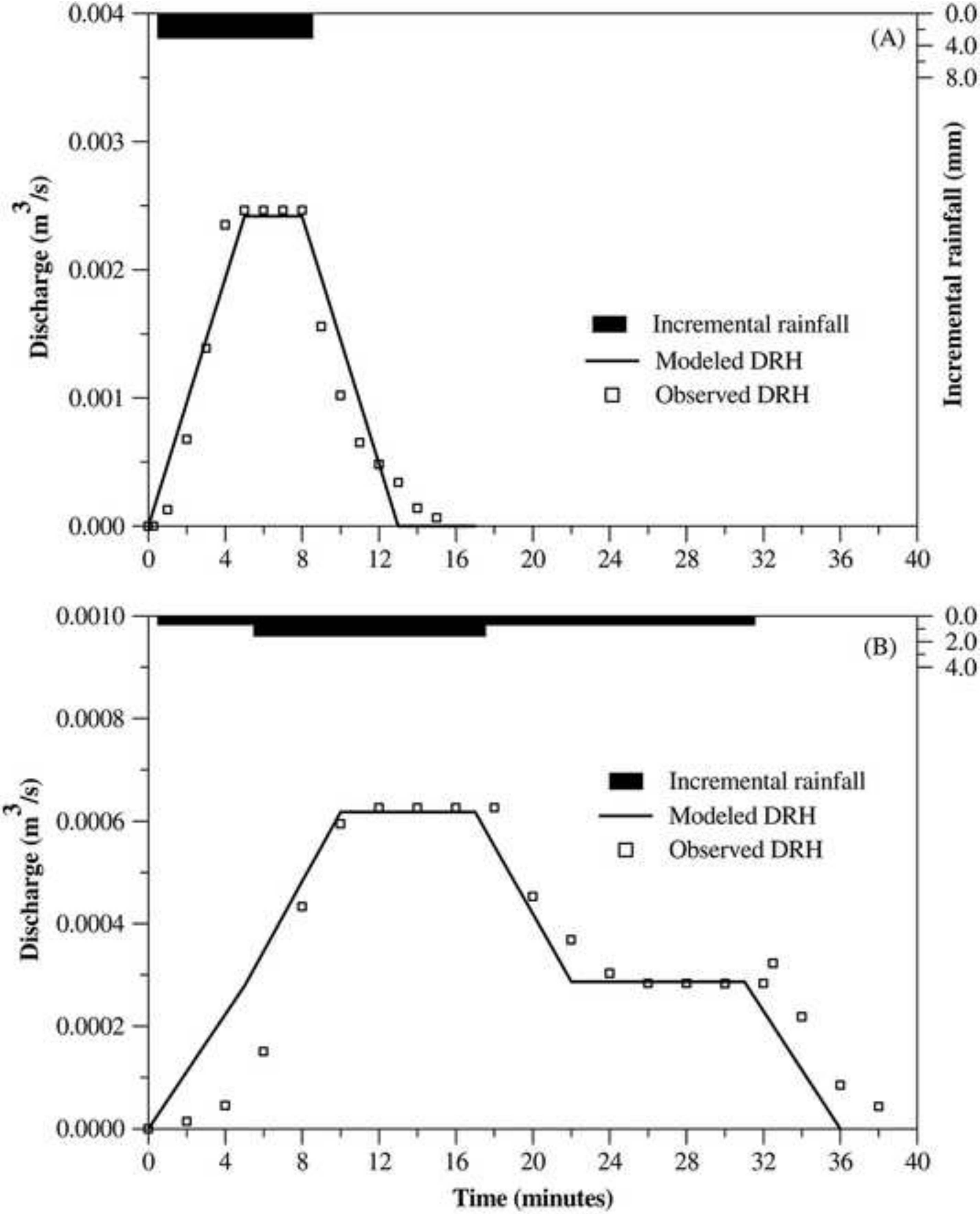


Figure 4

Click here to download high resolution image

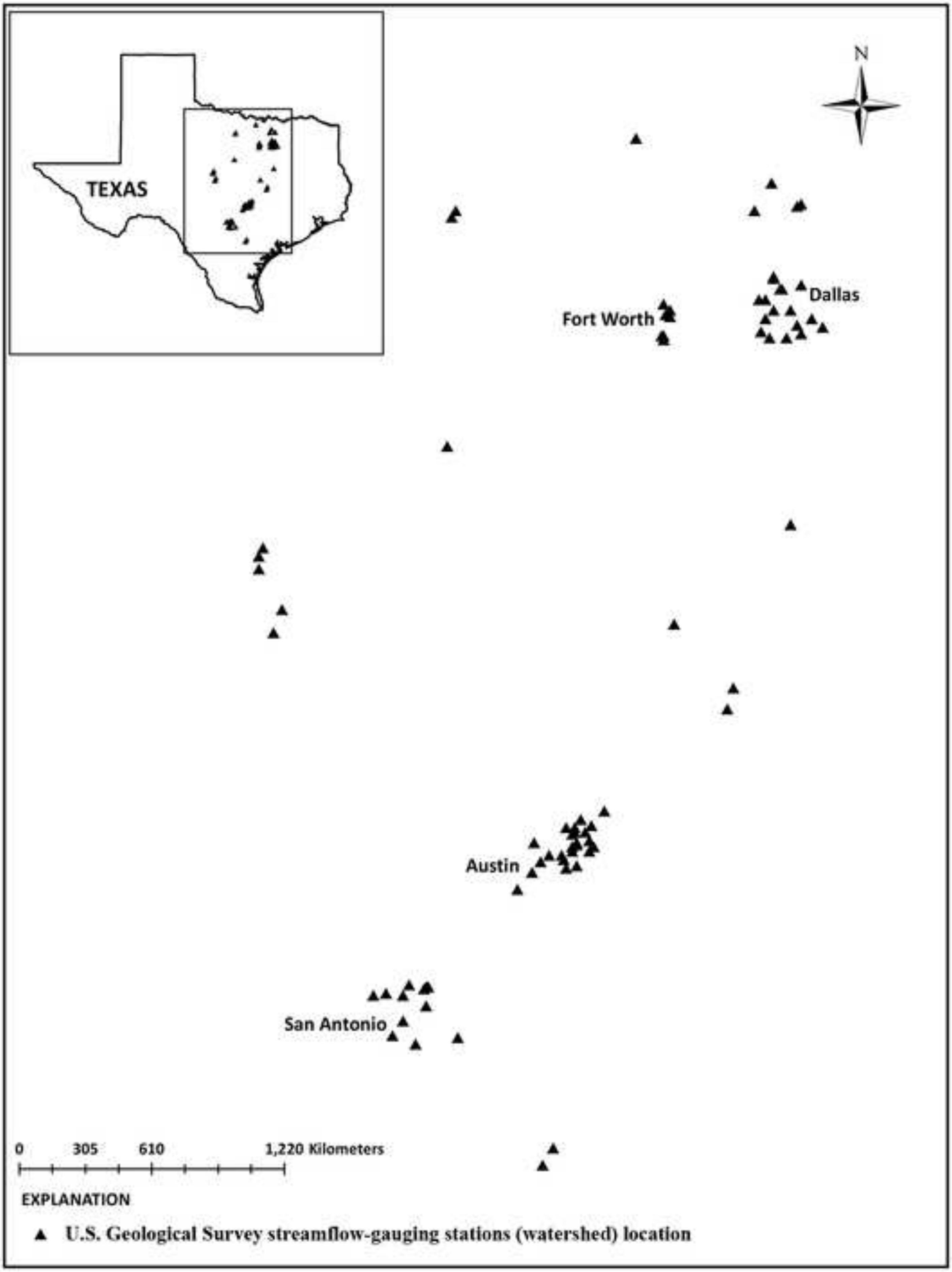


Figure 5

Click here to download high resolution image

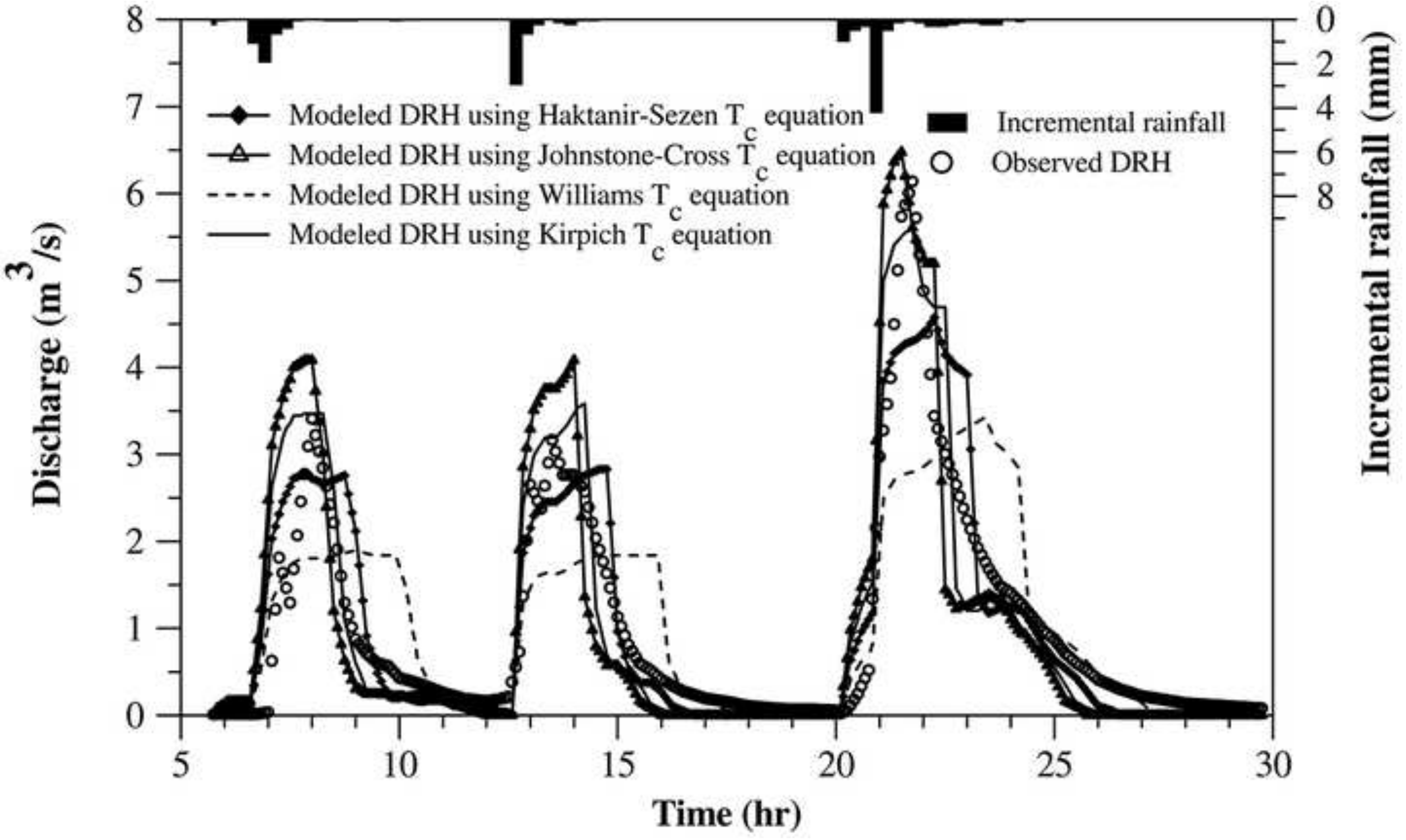


Figure 6

Click here to download high resolution image

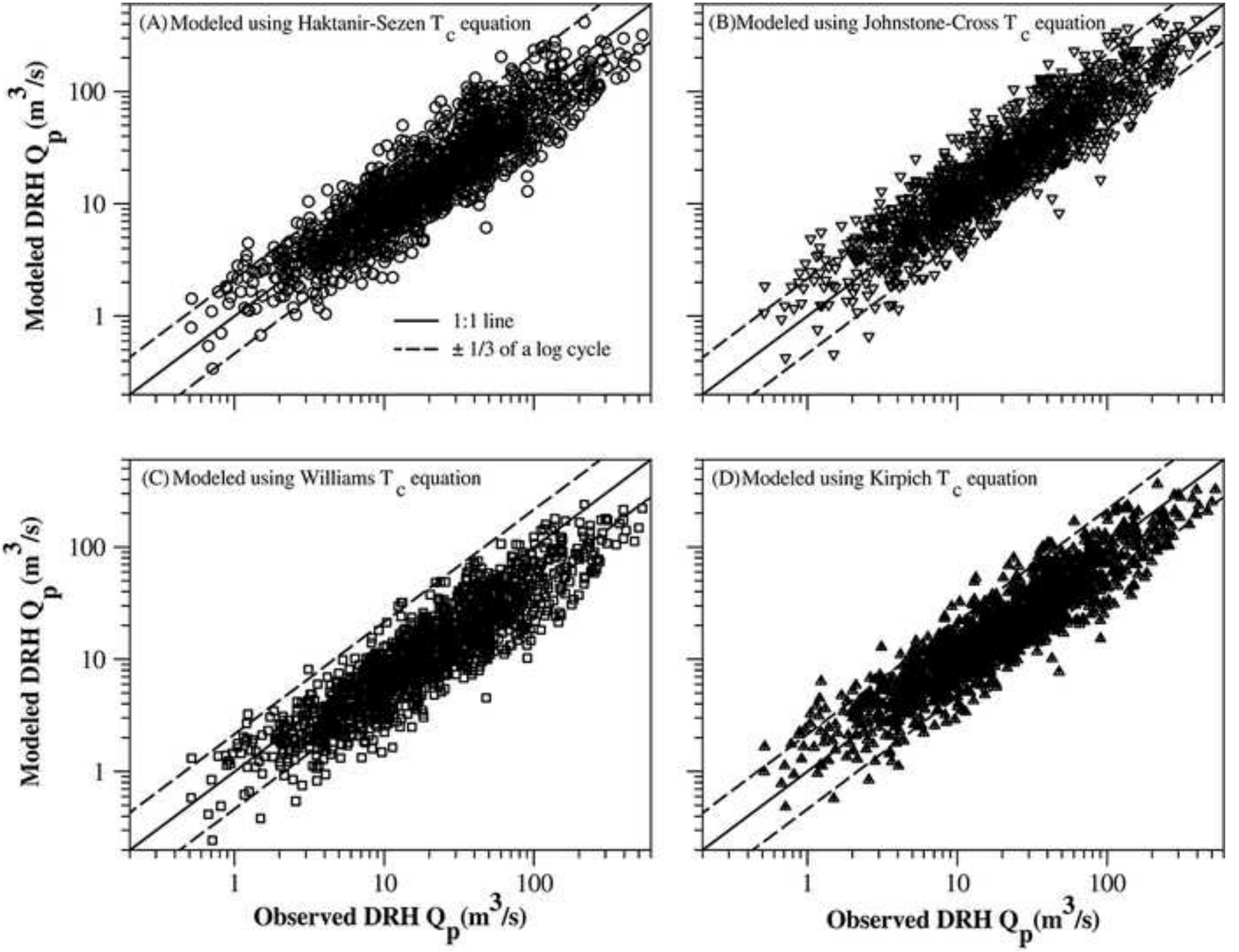


Figure 7

Click here to download high resolution image

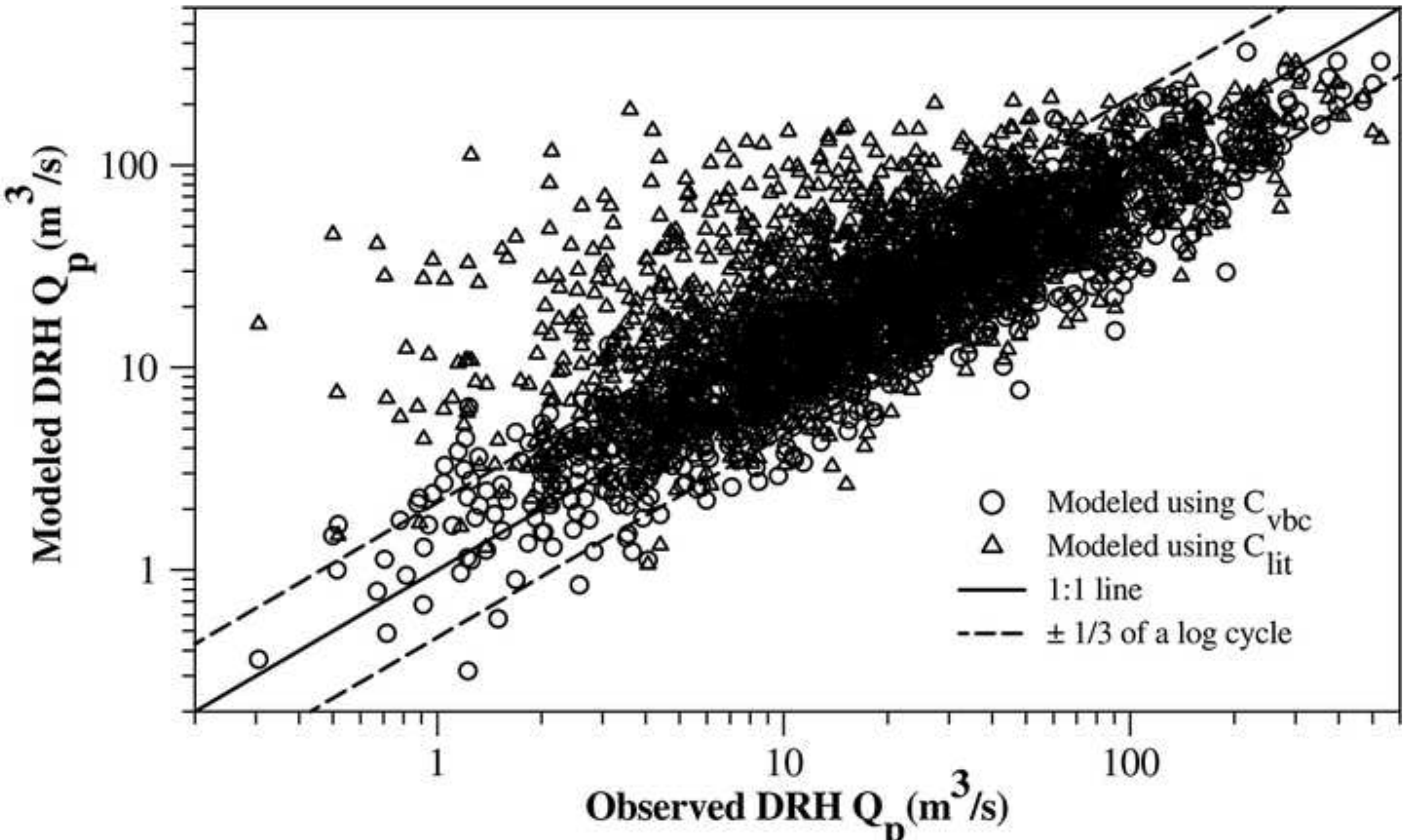


Figure 8

Click here to download high resolution image

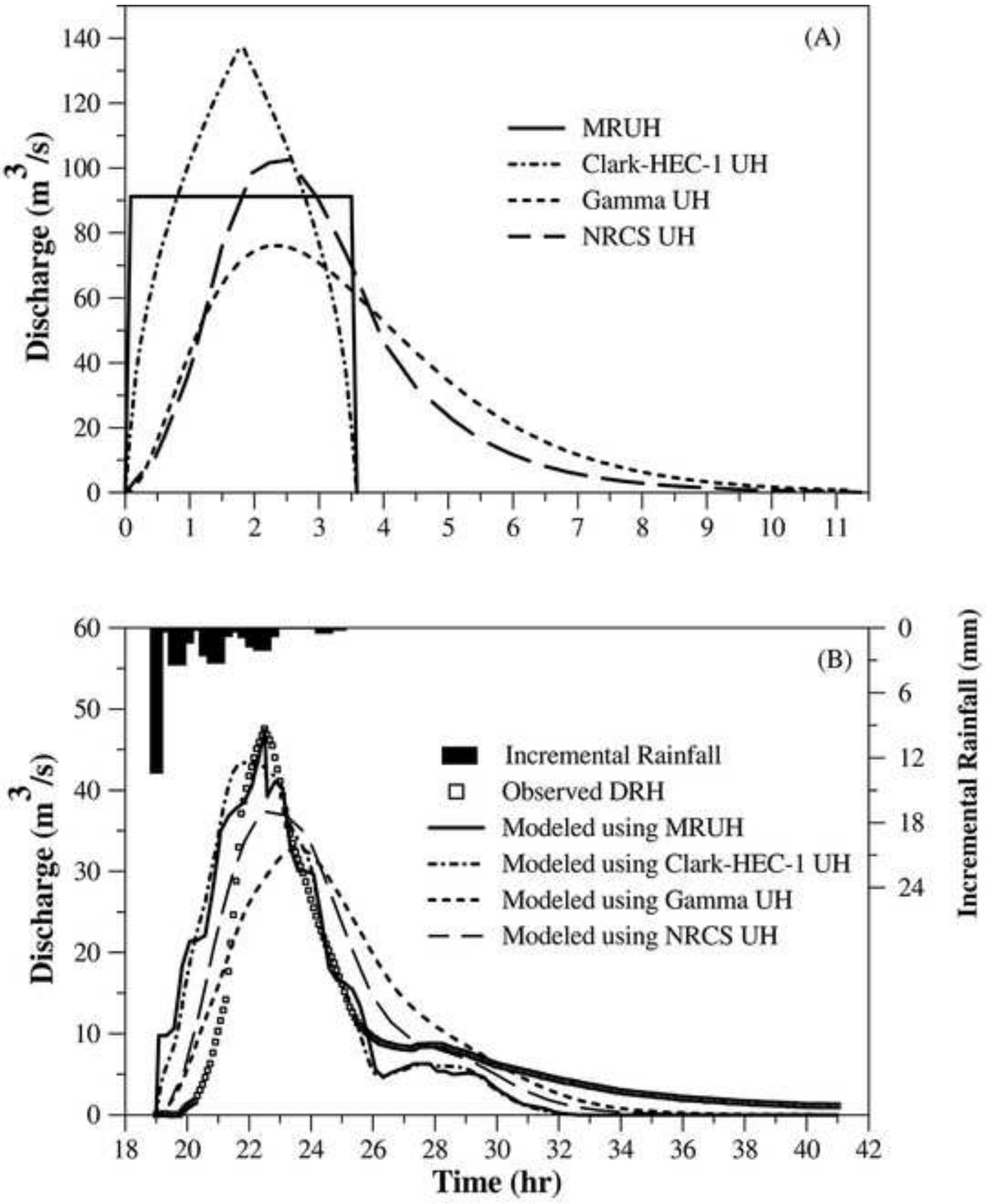
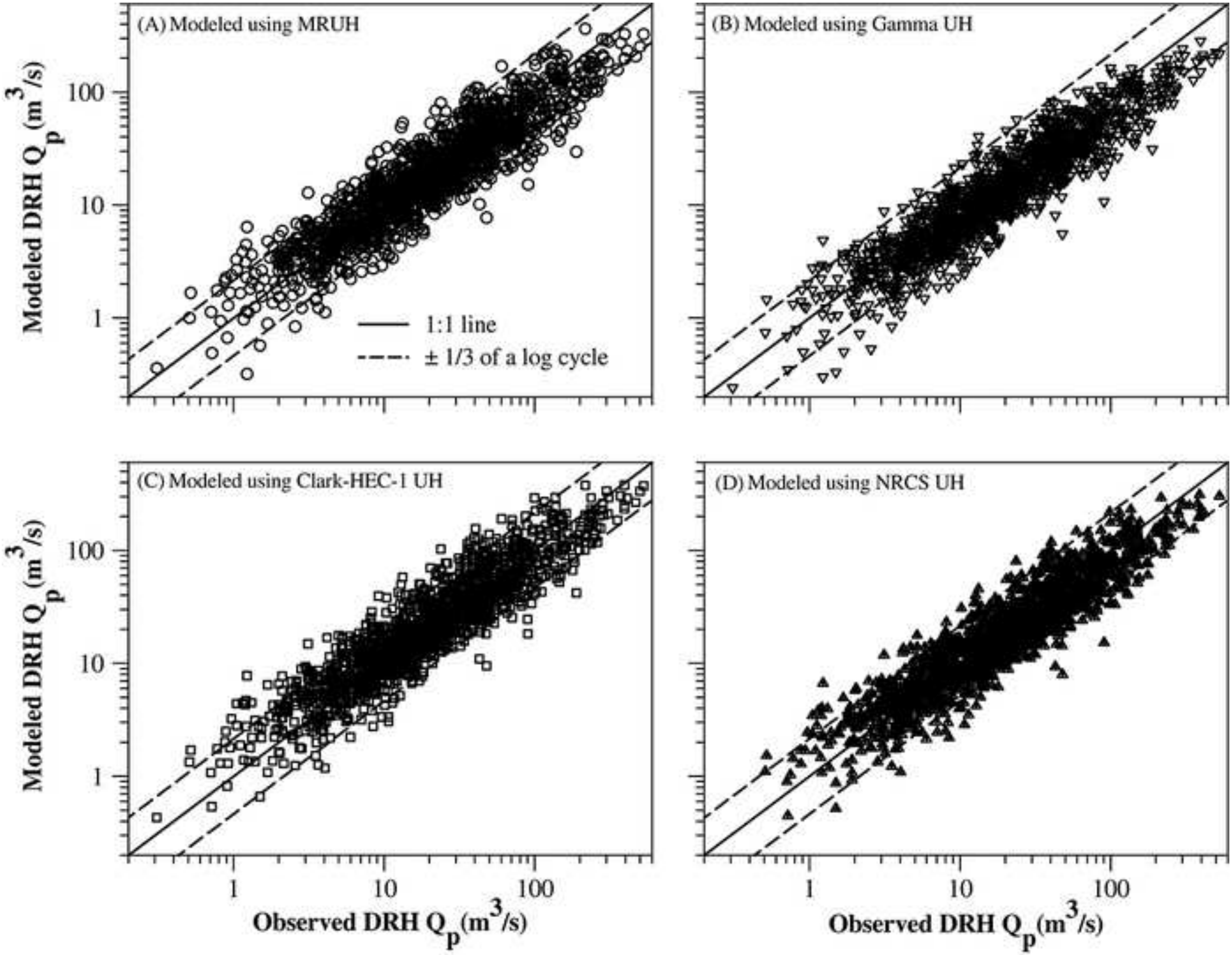


Figure 9

Click here to download high resolution image



\# **Thank you for accepting our paper for publication in Water Management. We did consider most of the minor changes suggested.**

EDITORIAL PANEL COMMENTS:

1) The authors use the abbreviation 'cms' for cubic metres per second on the figures. This is not standard, and I am more familiar with either 'cumecs' or m^3/s (preferably this latter).

 # We have now used m^3/s in all the figures and text wherever required.

2) The authors spell gauge 'gage', which I assume is American - would you want to see this changed?

 # We have now changed ‘gage’ to ‘gauge’ at all places.

REVIEWER COMMENTS:

Authors have revised the manuscript incorporating the suggestions made. Text quality has improved significantly, and the adopted procedure illustrated clearly. I hope it will be helpful to its intended audience. Thus, the manuscript can be recommended for publication in the journal including following minor corrections.

EDITORIALS
Replace with?.
\>> <alpha> = shape parameter of gamma unit hydrograph (GUH)
 # Updated as suggested.

\>> Qp = peak discharge of DRH in m^3/s or ft^3/s
 # Updated as suggested.

\>>Tp = time to peak of DRH in min. or hr.
 # Updated as suggested.

\>>TpU = time to peak of UH in min. or hr.
 # Updated as suggested.

\>>Page-14 Line-24: However, the UH is a mathematical model
 # Updated as suggested.

\>>As peak discharge/discharges repeated several times in text, use respective notations.
 # Thank you. The notations have been used at most of the places now.

Page 14 :
\>>Check sentence
"are listed in Table 1 for applications of the MRUH method with five combinations of Tc and C."

Table 1 indicates results for four combinations only is it???

 # Thank you. “five” has been changed to “four”.

>>Avoid repetition as in
"for applications of the MRUH method with five combinations of Tc and C. Applications of the MRUH method"

#Thank you. We have removed repetitions throughout the manuscript.

>>Rewrite as "slightly inferior to others with respect to median value of TB"
 # The sentence has been rewritten as suggested

Page 15 :
>>Improve sentence structure "Simulated peak discharges derived
1. using the MRUH method with the forward-computed"

 # The sentence has been rewritten as :

“Simulated Q_p results obtained from the MRUH method using the forward-computed (literature-based) runoff coefficient C_{lit} are compared against the Q_p results obtained using the back-computed runoff coefficient C_{vbc} (Figure 7). For both the cases, T_c values were estimated using the Kirpich equation.”

Page 18:
>>Rewrite as "well with the observed value (Figure 8B)."
 # The sentence has been rewritten as suggested.

JOURNAL COORDINATOR COMMENTS:

>>1. Please supply each of your figures separately and in high resolution. If you have created any of your figures using either Microsoft Word, Excel or Powerpoint you can upload these original files. The images must be editable. Figures created using any other programme must be uploaded either in .tiff or vectored .eps file format and have a minimum resolution of 600dpi when viewed at 10cm. Larger/wider figures must have a higher dpi of 800-1200 dpi as they will be printed larger on the page.

#Separated graphs in 600 dpi are provided.

2. Please upload a completed author copyright form.

 # It was provided.

Thank you.

Table
Click here to download Table: Tables-Sep-2013.doc

List of Tables

Table 1. Quantitative measures of the success of the DRH Q_p and T_p modeled using C_{vbc} and MRUHs with T_c estimated using four equations

Table 2. Quantitative measures of the success of the DRH Q_p and T_p modeled using MRUH with T_c estimated using the Kirpich equation and C estimated using two different methods (C_{vbc} and C_{lit})

Table 3. Sensitivity (change in EF) of DRH derived from MRUH on T_c and C for the rainfall event on 05/07/1972 for the USGS streamflow-gauging station 08178600 Salado Creek, San Antonio, Texas

Table 4. Statistic summary of watershed-averaged root mean squared errors between modeled and observed DRHs normalized by observed peak discharges

Table 5. Quantitative measures of the success of DRH Q_p and T_p modeled using four UH models for 1,400 rainfall-runoff events in 80 Texas watersheds

Table 1. Quantitative measures of the success of the DRH Q_p and T_p modeled using C_{vbc} and MRUHs with T_c estimated using four equations

Statistical Parameters	Using the Haktanir-Sezen equation [1]	Using the Johnstone-Cross equation [2]	Using the Williams equation [3]	Using the Kirpich equation [4]
R^2 for Q_p	0.75	0.80	0.75	0.80
EF for Q_p	0.66	0.79	0.48	0.73
Median value of QB	-0.19	0.00	-0.41	-0.10
Fraction of storms with $-0.5 \leq QB \leq 0.5$	0.70	0.72	0.60	0.75
% of storms within ± 1/3 of a log cycle (Q_p)	82.4	88.7	67.5	88.6
R^2 for T_p	0.75	0.72	0.74	0.73
EF for T_p	0.74	0.71	0.74	0.72
Median value of TB	0.00	-0.05	0.10	-0.01
Fraction of storms with $-0.5 \leq TB \leq 0.5$	0.72	0.73	0.65	0.72
% of storms within ± 1/3 of a log cycle (T_p)	82.1	80.5	78.2	82.3

[1] T_c computed using the Haktanir-Sezen equation ranged from 0.8 to 6.5 hours in the study watersheds, with median and mean values of 2.6 hours and 2.9 hours, respectively.

[2] T_c computed using the Johnstone-Cross equation ranged from 0.7 to 5.0 hours in the study watersheds, with median and mean values of 1.7 hours and 1.9 hours, respectively

[3] T_c computed using the Williams equation ranged from 1.2 to 11.7 hours in the study watersheds, with median and mean values of 4.0 hours and 4.5 hours, respectively

[4] T_c computed using the Kirpich equation ranged from 0.6 to 7.1 hours in the study watersheds, with median and mean values of 2.2 hours and 2.4 hours, respectively

Table 2. Quantitative measures of the success of the DRH Q_p and T_p modeled using MRUH with T_c estimated using the Kirpich equation and C estimated using two different methods (C_{vbc} and C_{lit})

Statistical Parameters	Using C_{vbc}	Using C_{lit}
R^2 for Q_p	0.80	0.44
EF for Q_p	0.73	0.42
Median value of QB	-0.10	0.45
Fraction of storms with $-0.5 \leq QB \leq 0.5$	0.75	0.45
% of storms within ± 1/3 of a log cycle (Q_p)	88.6	63.0
R^2 for T_p	0.73	0.73
EF for T_p	0.72	0.72
Median value of TB	-0.01	-0.01
Fraction of storms with $-0.5 \leq TB \leq 0.5$	0.72	0.72
% of storms within ± 1/3 of a log cycle (T_p)	82.3	82.3

Table 3. Sensitivity (change in *EF*) of DRH derived from MRUH on T_c and *C* for the rainfall event on 05/07/1972 for the USGS streamflow-gauging station 08178600 Salado Creek, San Antonio, Texas

Change in T_c %	Change in *EF*	Change in *C* %	Change in *EF*
-50	-0.18	-50	-0.27
-25	-0.02	-25	-0.02
-10	0.01	-10	0.02
10	-0.03	10	-0.06
25	-0.09	25	-0.21
50	-0.22	50	-0.66

Table 4. Statistic summary of watershed-averaged root mean squared errors between modeled and observed DRHs normalized by observed peak discharges

Statistical Parameters	Using MRUH	Using Gamma UH	Using Clark-HEC-1 UH	Using NRCS UH
Maximum	1.78	1.61	1.95	1.74
Minimum	0.25	0.19	0.23	0.22
Mean	0.61	0.61	0.62	0.57
Median	0.52	0.53	0.53	0.51

Table 5. Quantitative measures of the success of DRH Q_p and T_p modeled using four UH models for 1,400 rainfall-runoff events in 80 Texas watersheds

Statistical Parameters	Using MRUH	Using Gamma UH	Using Clark-HEC-1 UH	Using NRCS UH
R^2 for Q_p	0.80	0.82	0.81	0.83
EF for Q_p	0.73	0.63	0.79	0.76
Median value of QB	-0.10	-0.32	0.02	-0.12
Fraction of storms with $-0.5 \leq QB \leq 0.5$	0.75	0.71	0.71	0.77
% of storms within ± 1/3 of a log cycle (Q_p)	88.6	80.6	88.5	90.9
R^2 for T_p	0.73	0.73	0.71	0.71
EF for T_p	0.72	0.72	0.70	0.70
Median value of TB	-0.01	0.03	-0.02	0.00
Fraction of storms with $-0.5 \leq TB \leq 0.5$	0.72	0.73	0.75	0.75
% of storms within ± 1/3 of a log cycle (T_p)	82.3	84.1	81.8	82.4