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MODELING GROUND WATER FLOW USING FLUX BOUNDARY CONDITIONS[1]

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ABSTRACT: Determination of the boundary conditions for modeling ground water flow is a critical point especially in regional models. Normally the regional models require model areas that are greater than the given area of interest. This work focuses on the prediction of hydraulic heads in regional models using flux boundary conditions. The model uses flux boundary conditions that were estimated using a radial flow analog and Darcy's law. The regional model that is presented uses no parameter identification (inverse estimation) procedures. In the present work, the Houston area was used. The simulation of the hydrological conditions of the Chicot and Evangeline Aquifers that underlie the Houston area were made using the available information about the geological profile in the Houston region and the current information about the existing production wells. The regional model works as a forward problem. The system parameters such as hydraulic conductivity, specific storage, and hydrological stresses were specified, and the model predicts the hydraulic head. Actual data from piezometers operated by the U.S. Geological Survey (USGS) in many places throughout Houston were used as initial conditions. Some piezometric head data were generated using the regional variable theory called kriging to supply head estimates in areas where data were unavailable. The Modular Three Dimensional Finite Difference Groundwater Flow Model developed by the USGS was used to predict the hydraulic heads. The predicted ground water heads are compared to the actual data. The results show that the model performs well for locations where data were available.

(KEY TERMS: ground water; prescribed head; flux boundary; inverse estimation; regional model.)

INTRODUCTION

The major water bearing units in the Houston-Galveston area are the Chicot and Evangeline aquifers. The Chicot aquifer overlies the Evangeline aquifer that overlies the Burkeville confining layer. The relationship of the Chicot aquifer, the Evangeline aquifer, and the Burkeville layer is shown in Figure 1. The Chicot and Evangeline aquifers consist of unconsolidated and discontinuous layers of sand and clay that dip toward the Gulf of Mexico. A detailed description of the subsurface geology is given by Ryder (1988). Tables 1 and 2 show the hydrogeological characteristics of these two aquifers.

Ground water in the Houston area is used for public supply, industry, and irrigation. Generally speaking, water levels in the Houston area (regional basis) declined from 1943, the beginning of development, until 1977. Since late 1976, changes in the pumping distribution resulting from efforts to control subsidence and the introduction of surface water from Lake Livingston have altered the pattern of water level changes. The average daily withdrawals of ground water in Harris County and parts of Fort Bend and Waller Counties between 1975-1984 was 20 m^3/s (464 Mgal/day). The percentage of ground water to total average daily use during 1975-1984 was between 48 to 58 percent (Williams and Ranzau, 1987). During 1985-1989, the City of Houston's water supply averaged about 55 percent ground water. During the decade 1980-1989, the average daily withdrawal of ground water in Harris County and parts of Fort Bend and Waller Counties was 19.26 m^3/s (439.77 Mgal/day).

The purpose of the present work is to present a flow model to determine the head distributions in the underlying aquifers. This work describes a methodology to estimate flux boundary conditions and uses regional variable theory to estimate initial conditions for locations where there are no data.

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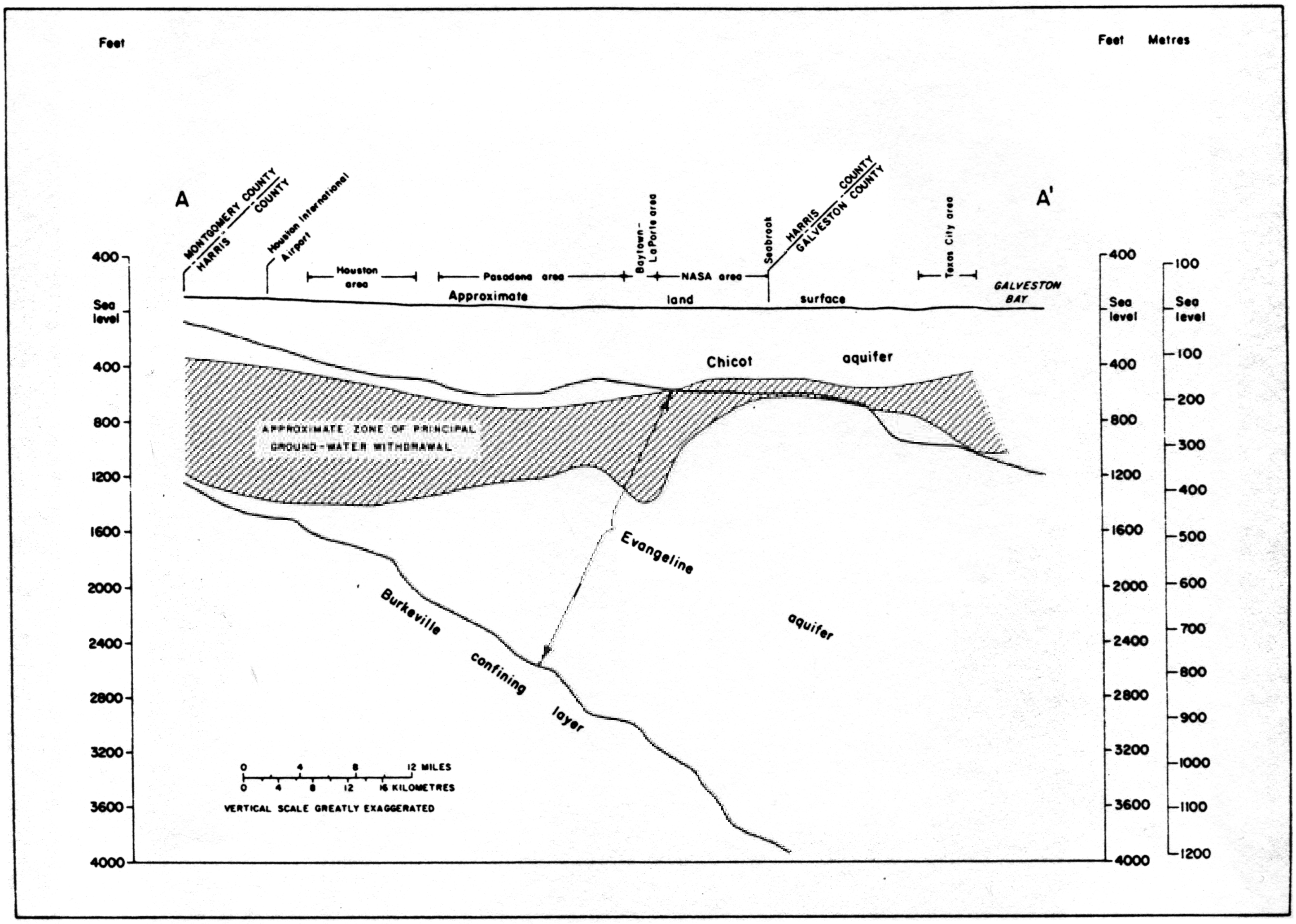

Figure 1. Hydrological Profile from the Houston Area (from Gabrysch and Bonnet, 1975).

TABLE 1. Thickness of the Chicot and Evangeline Aquifers (at the locations shown in Figure 2).

	Thickness			
	Chicot		Evangeline	
Location	ft	m	ft	m
Texas City-Moses Lake	1000	305	5000	1520
Johnson Space Center	770	235	3400	1036
Clear Lake	700	213	3072	1036
Seabrook	600	183	4000	120
Pasadena	600	183	2800	854
Baytown	500	152	3000	915
East End	600	183	2500	762
Southwest	700	213	2500	762
Northeast	500	152	2000	610
Addicks	600	183	1800	549
Lake Houston	500	152	2000	610

TABLE 2. Some Properties of the Texas Gulf Coast Geohydrological Formations.

	Horizontal Hydraulic Conductivity (ft/day)	Vertical Hydraulic Conductivity (ft/day)	Mean Water Density (lbs/ft^3)
Holocene Upper Pleistocene Permeable Zone	170	1×10^{-2}	62.64
Lower Pleistocene Upper Pliocene Permeable Zone	20	1×10^{-3}	63.52
Lower Pliocene Upper Miocene Permeable Area	60	1×10^{-4}	64.45

CONCEPTUAL HYDROLOGICAL MODEL

The subsurface lithology of the Houston area is composed of sand and clay layers of varying thickness. Bravo (1990) studied sonic, spontaneous-potential, and conductivity logs for five of the 11 borings shown in Figure 2 (Baytown, Clear Lake, Johnson Space Center, Southwest, and Addicks). The logs were manually interpreted to generate geologic profiles of the subsurface at the five sites (Keys and MacCary, 1971). Interpreted geologic profiles for the Baytown, Clear Lake, and Johnson Space Center boring are shown in Bravo *et al.* (1991).

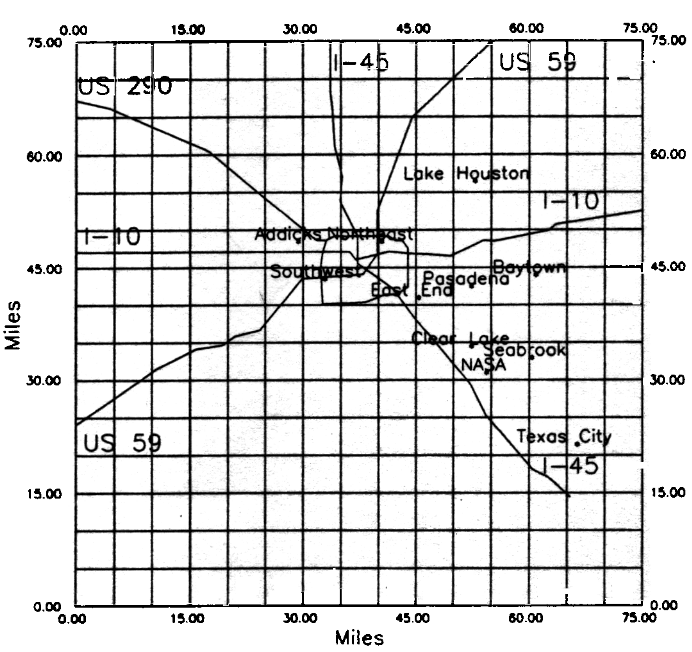

Figure 2. Location of the USGS Borings and Boundary and Grid of the Model.

The representation of the subsurface geology was further simplified by concentrating (condensing) the sand and clay layers in a manner consistent with the stratigraphy in the East-West direction and developing the eight layers conceptual model shown in Figure 3. The previous regional models of ground water flow in the Houston area used five layers and assumed the Chicot aquifer as confined (Meyer and Carr, 1979). The North-South subsurface geology was modeled using the conceptual model and adjusting the thickness of each layer so that the overall aquifer thickness follows the transept shown in Figure 1. The East-West subsurface model was extrapolated horizontally beyond the limits shown in Figure 3 because there was no further stratigraphic information.

CONCEPTUAL AQUIFER FLOW MODEL

The conceptual hydrogeologic model and flow model are described mathematically by (Bear and Verruijt, 1987)

$$\bar{V} \cdot (T_i \nabla \phi_i) + \frac{\phi_i - \phi_{i-1}}{\sigma_i} + R_i - P_i = S\frac{\partial \phi_i}{\partial t} + S_k \frac{\partial \phi_i}{\partial t} \tag{1}$$

subject to the boundary conditions

$$\phi_i(x,y,t) = \textit{known on } d\Gamma 1 \ \textit{(Dirichlet condition)} \tag{2}$$

$$T_i \frac{\partial \phi_i}{\partial x}\frac{\partial x}{\partial n} = \textit{known on } d\Gamma_2 \ \textit{(Flux condition)} \tag{3}$$

and the initial conditions

$$\phi_i(x,y,0) = \textit{known on } \Gamma \tag{4}$$

where $\phi_i(x,y,t)$ is the piezometric head in sand layer i, $T_i(x,y)$ is the transmissivity in sand layer i, $\sigma_i(x,y)$ is the conductance of the clay layer between sand layer i-1 and i, $R_i(x,y)$ is the recharge in sand layer i, $P_i(x,y)$ is the pumping from layer i, n is the outward unit normal vector along the boundary of the flow domain, $d\Gamma$ is the boundary of the flow domain ($d\Gamma_1 \cup \Gamma_2 = d\Gamma$), Γ is the flow domain, S is the storage coefficient of the aquifer, and S_k is the storage of the semi-pervious clay layers.

The Chicot Aquifer was modeled as an isotropic aquifer with the potential for either confined or unconfined horizontal flow. The Evangeline Aquifer sand layers were modeled as confined leaky isotropic aquifers. The intervening clay layers were modeled as semi-pervious formations. The effects of delayed storage in the clay layers were modeled as a source term in the flow model. Unconfined flow in the Chicot Aquifer, when it occurs, was modeled using the Dupuit assumptions and a backwards time linearization. The transmissivity was calculated as the product of saturated thickness and hydraulic conductivity. A prescribed piezometric head boundary condition (Dirichlet) was applied along the edge of the model that intersects Galveston Bay, while a prescribed flux boundary condition (Neuman) was applied along the rest of the boundary.

The previous regional models of ground water flow in the Houston area used prescribed head everywhere along the boundary (Meyer and Carr,1979). The boundaries of the Meyer and Carr model were extended outwards the Houston area to areas of minimal pumping to reduce the boundary effects and to eliminate the necessity of imposing flux boundaries. The area covered in their model was 27,000 square miles.

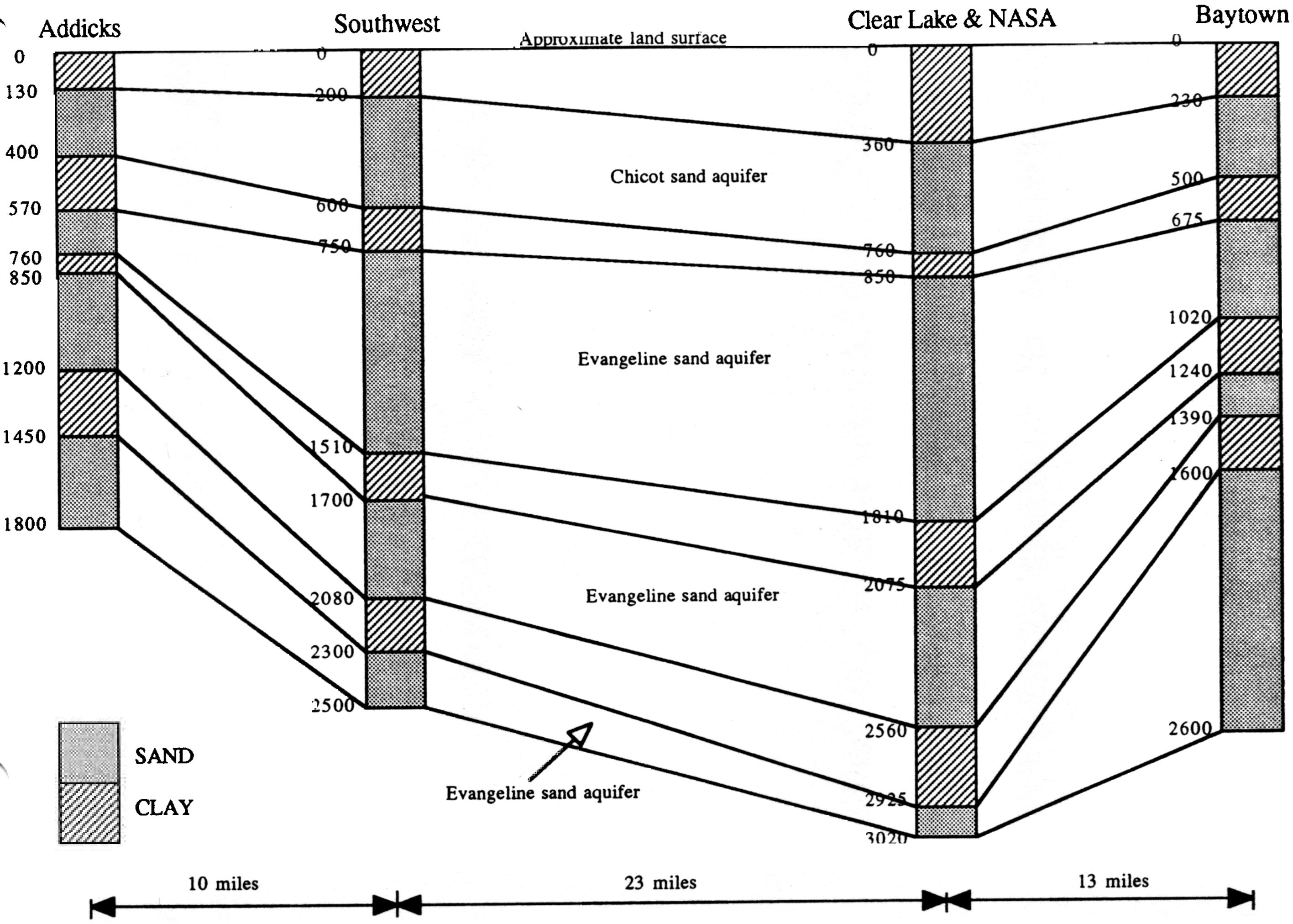

Figure 3. Conceptual Model of the Ground Water Hydrology of the Houston Area. The numbers indicated meters below land surface.

The present work used a flux boundary condition because there were not sufficient data to determine a prescribed head boundary condition for the area studied. The flux boundary condition allows the reduction of the extent of the modeling area to 5625 square miles. The extent of the region studied was chosen to cover the withdrawal areas (pumping areas). Using the flux boundary, there is no need to model areas that are greater that the given area of interest.

Piezometric contour maps from 1980 to 1989 were observed to have the same appearance as contour maps that would be expected for radial steady flow to a well. The steady radial flow to a well in a confined aquifer using Dupuit's assumption is given by the expression

$$Q = 2\pi K b \frac{h - h_w}{\ln \frac{r}{r_w}} \qquad (5)$$

where Q is the flow rate, K is the hydraulic conductivity, b is the thickness of the confined aquifer, h is the piezometric head at a distance r from the well, h_w is the piezometric head at a distance equal to the well radius (r_w), and r is the distance to the piezometric head h from the well. This fact suggested a test of the relationship between radial distances from a hypothetical origin and the piezometric head. The tests were performed for the decade 1980-1989. Eight directions were taken (north, north-east, east, south-east, south, south-west, west, and north-west). In most directions the relationship between the piezometric head and the logarithm of the radial distance was found to be linear, and the slopes of the regression lines were almost the same for the ten years studied (Bravo, 1990). These slopes were used to estimate the hydraulic gradient at the boundary. Figure 4 shows the regression lines for the altitude of water levels at

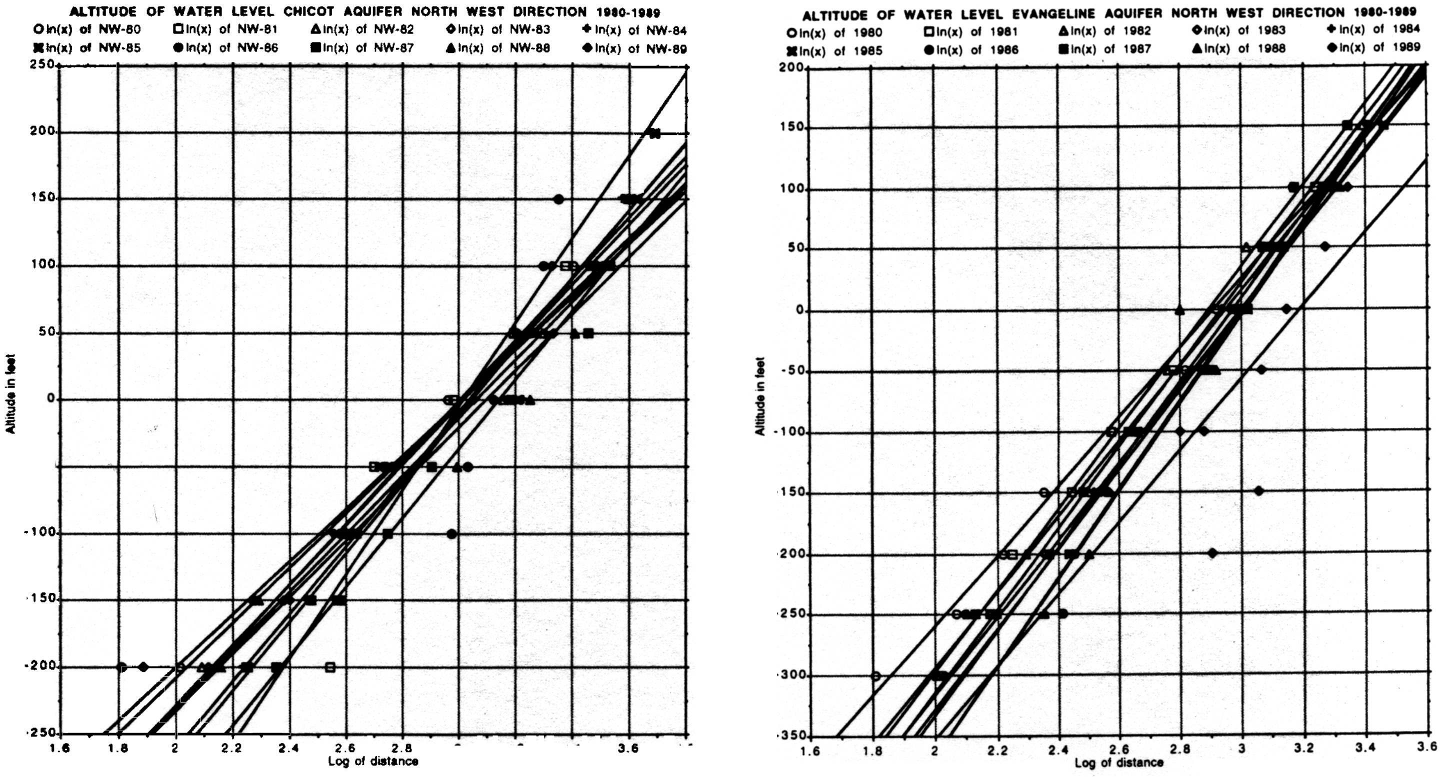

Figure 4. Regression Lines for the Altitude of Water Levels at the Chicot and Evangeline Aquifers North-West Direction from 1980 to 1989.

the Chicot and Evangeline aquifer in the North-West and South-East direction from 1980 to 1989.

The extent of the region studied was chosen to cover the withdrawal areas (pumping areas) for the same decade that was used to generate the regression lines. The boundary is shown in Figure 2. The radial flow analog and Darcy's law were used to estimate the flux into the domain of interest. The values of the fluxes at each cell across the boundary was calculated using Darcy's law. The pumping rates for the ten years studied varied from 440 million m^3/year to 741 million m^3/year; yet the values for the fluxes were relatively constant. Because of this behavior, it was assumed that the fluxes remain constant for prediction horizons of several years. A ground water budget that assumes the clay layers contribute an amount of water equal to 25 percent of pumping (the proportion concluded by Meyer and Carr (1979) was satisfied using this flux boundary condition, further strengthening confidence in the flux boundary methodology.

INITIAL CONDITIONS

Even though maps of the piezometric heads are available, initial piezometric heads for all cells in the model are unavailable. A regional variable theory (kriging) was used to estimate the initial piezometric heads in the cells for which there was no data (Marsily, 1986). A circular search pattern for kriging the data assumed that the variation of the heads in the North-South direction were statistically independent for the variation of the heads in the East-West direction (Davis, 1986). This assumption was consistent with the methodology used to determine the flux boundary conditions.

AQUIFER HYDRAULIC CHARACTERISTICS

The transmissivities and storage coefficients for the sand layers were taken from previous studies (Jorgensen, 1975; Meyer and Carr, 1979). The vertical hydraulic conductivities and storage coefficients of the semi-pervious layers were determined independently using the methods developed by Bravo (1990) and

Bravo *et al.* (1991) and explained in detail by Cleveland *et al.* (1992).

FLOW MODEL SOLUTION

The flow model described by Equations (1) through (4) was solved using the Modular three-dimensional Finite Difference Ground Water Flow Model (MODFLOW) developed by the U.S. Geological Survey (McDonald and Harbaugh, 1988). MODFLOW is a block-centered finite difference code that can simulate all types of aquifers. This computer program has the ability to model conceptually complicated flow systems and temporarily varying excitations. Transient flow for the geometry defined by the boundary shown in Figure 2 was modeled using injection wells to simulate the fluxes along the boundary.

RESULTS

The flow model was operated for simulation periods of one year, using initial data from 1980. As a sample, Figure 5 shows the observed 1984 head distribution in the Chicot Aquifer. Contours are head in feet. The vertical and horizontal scales are in miles. Figure 6 is the simulated head distribution (missing data are estimated by kriging) using initial data from 1980. Figures 7 and 8 are the observed and simulated head distributions in 1984 for the Evangeline Aquifer system. If the observed and the simulated hydraulic heads are overlaid, one can observe that the contour lines almost coincide in many areas of the grid. To measure the performance of the model locally, the relative prediction error for the Chicot and Evangeline Aquifers were calculated. The formula used was

$$RPE(x,y) = \frac{\phi^{predict}_{(x,y)} - \phi^{actual}_{(x,y)}}{\phi^{actual}_{(x,y)}} \qquad (6)$$

where RPE(x,y) is the relative prediction error of the flow model.

Figure 9 shows a map of the 1984 RPE for the Chicot Aquifer. The model performed well in predicting piezometric heads in the Chicot Aquifer at locations where actual data were available. Figure 10 shows a map of the 1984 RPE for the Evangeline Aquifer. Again the model performs well for those locations where there were data. These maps are to be interpreted as a representation of the zones where the model performs well. The zero piezometric head contour line and its neighborhood presents high RPE values because of the small denominator of the formula used for the calculations. What is remarkable is that no parameter identification (inverse estimation) procedures beyond determining the boundary conditions were used, yet the model performed adequately.

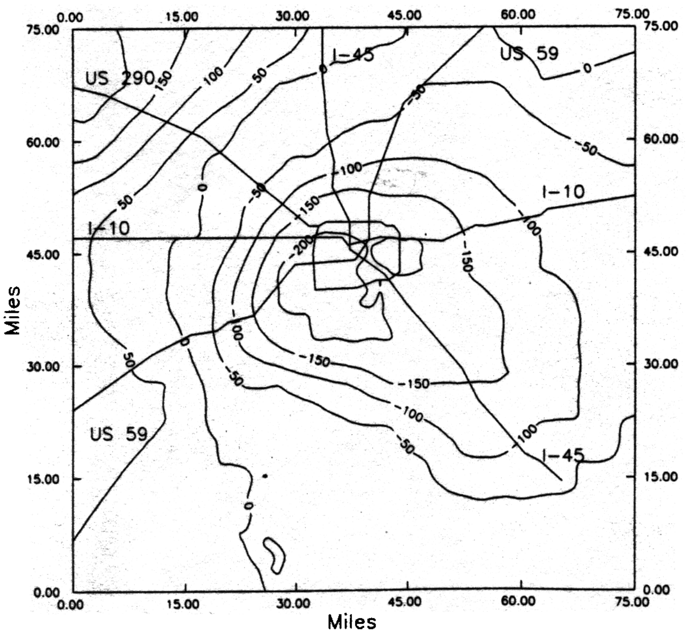

Figure 5. Chicot Aquifer 1984 Heads (observed).

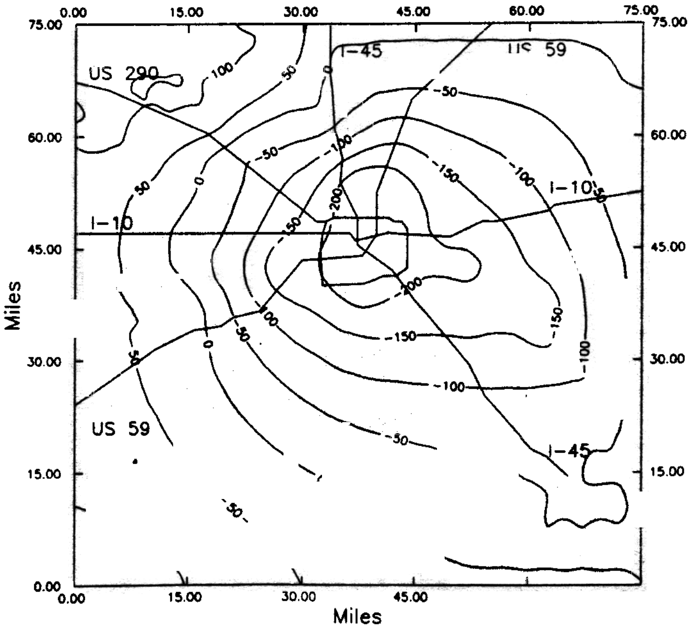

Figure 6. Chicot Aquifer 1984 Heads (simulated)

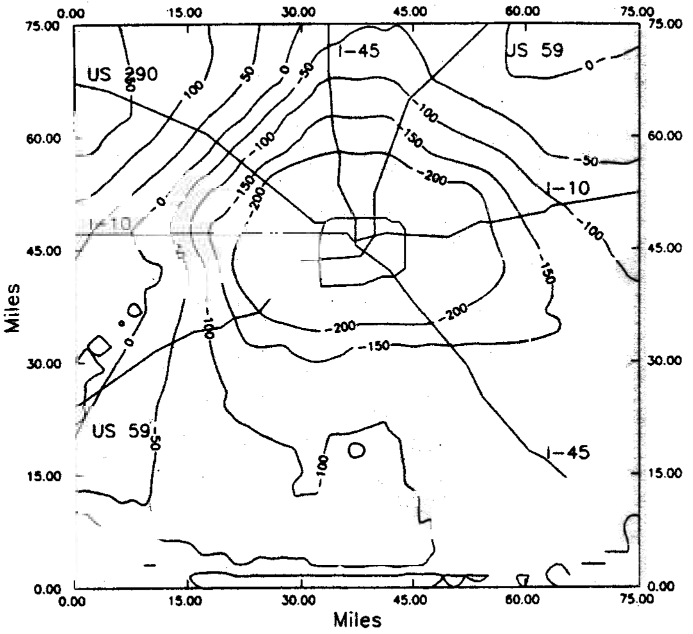

Figure 7. Evangeline Aquifer 1984 Heads (observed).

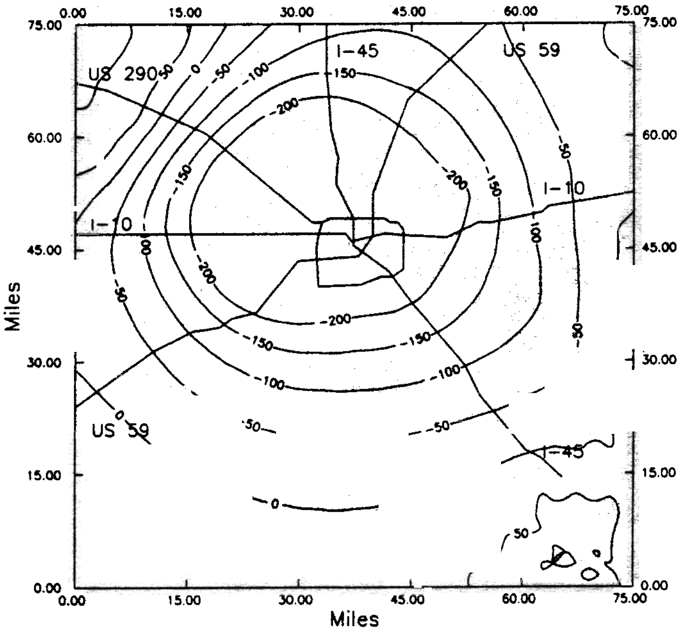

Figure 8. Evangeline Aquifer 1984 Heads (simulated)

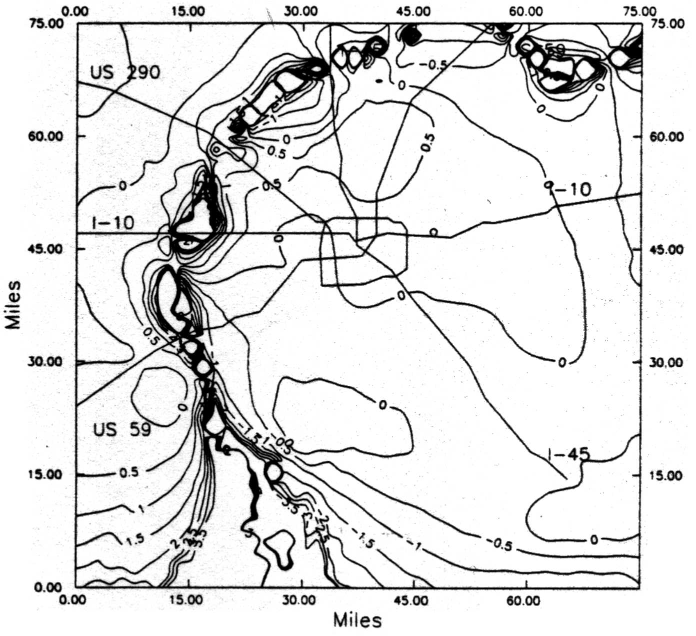

Figure 9. Relative Prediction Error for the 1984 Chicot Aquifer Heads.

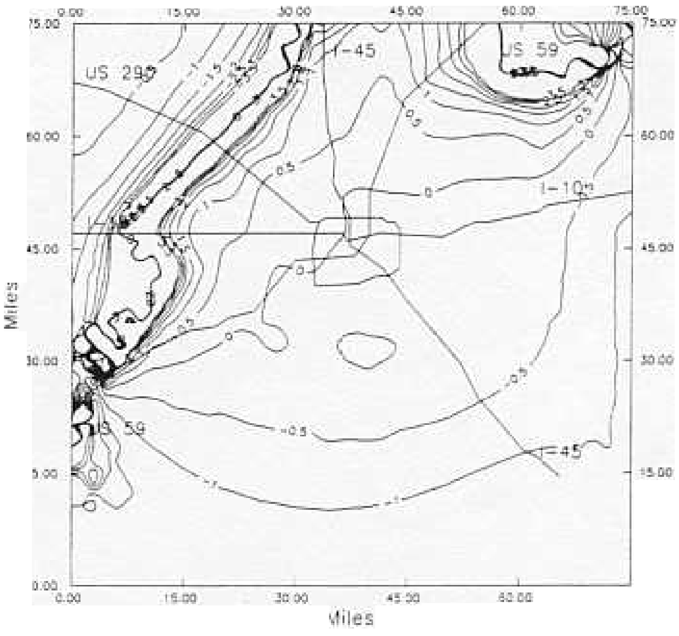

Figure 10. Relative Prediction Error for the 1984 Evangeline Aquifer Heads.

HYDRAULIC HEADS GEOLOGY AND TOPOGRAPHY

The geological formations (Ryder, 1988) from which most of the ground water is pumped in the Houston-Galveston region are the following: the Catahoula Sandstone and Fleming Formation of Miocene age; the Goliad Sand of Pliocene age; the Willis sand, Bentley and Montgomery Formations, and Beaumont Clay of Pleistocene age; and Alluvium of quaternary age. The unit composed of the Willis sand, Bentley Formation, Montgomery Formation, Beaumont Clay, and Quaternary Alluvium is known as the Chicot aquifer. The Evangeline aquifer is composed of the Goliad Sand unit. The reason to separate the Chicot aquifer from the Evangeline is a difference in the hydraulic conductivities noted in the two aquifers.

The downtown of Houston has an elevation of about 49 ft (15 m) above mean sea level whereas the Johnson Space Center south of Houston and 20 miles away from the downtown area has an elevation of 17 ft (5.2 m). Therefore, the topography of the region is almost flat. Observing the maps of piezometric heads for both aquifers, one could conclude that the ground water flow is primarily due to the hydraulic head difference in two very well defined aquifers.

CONCLUSIONS

Based on the results, the flow simulation appeared to perform well in areas where data were available. The model used regional variable theory for estimating initial conditions at locations where there were no data, and a radial flow analog to estimate flux boundary conditions in the Houston area. The techniques used may be applicable to similar regions; the flux boundary condition eliminates the need to model areas that are greater than the given area of interest.

Further research includes a study to determine the sensitivity of the model to changes in aquifer parameters, a study of the influence of storage in the clay layers when the vertical flow assumption is relaxed, and a study of the influence of the search pattern in the kriging algorithm when the assumption of statistical independence of the variation of head with direction is relaxed.

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