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Instantaneous Unit Hydrograph Evaluation for Rainfall-Runoff Modeling of Small Watersheds in North and South Central Texas

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Abstract: Data from over 1,600 storms at 91 stations in Texas are analyzed to evaluate an instantaneous unit hydrograph (IUH) model for rainfall-runoff models. The model is fit to observed data using two different merit functions: a sum of squared errors function, and an absolute error at the peak discharge time (Q_pMAX) function. The model is compared to two other models using several criteria. Analysis suggests that the Natural Resources Conservation Service Dimensionless Unit Hydrograph, Commons' Texas hydrograph, and the Rayleigh IUH perform similarly. As the NRCS and Commons' models are tabulations, the Rayleigh model is an adequate substitute when a continuous model is necessary. The adjustable shape parameter in the Rayleigh model does not make any dramatic improvement in overall performance for these data, thus fixed shape hydrographs are adequate for these watersheds.

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Introduction

The instantaneous unit hydrograph (IUH) is the direct runoff hydrograph (DRH) resulting from a unit depth of an excess rainfall hyetograph (ERH) applied uniformly over a watershed for a short ($dt \rightarrow 0$) duration. A major advantage of an IUH is that the IUH does not require uniform excess precipitation for a specific duration.

The direct runoff hydrograph is computed as the convolution of the ERH and the IUH function as described by

$$Q(t) = A\int_0^t i(\tau)u(t-\tau)d\tau \tag{1}$$

where $i(t)$=EPH (precipitation rate as a function of time); $u(t)$=IUH (unit response rate as a function of time); $Q(t)$=DRH (direct runoff rate as a function of time); A=watershed area; and τ=time lag (time between a particular precipitation event and its associated runoff).

The IUH function is required to exhibit linearity with respect to effective precipitation and integrates to unity; these are each properties shared by probability distributions. The similarity is not coincidental, and one interpretation is that the IUH is a residence time distribution of precipitation on the watershed.

Nash (1958), Leinhard (1964), Dooge (1973), and others, through conceptual approaches ranging from cascade of linear reservoirs to statistical–mechanical methods, have derived various IUH functions from observed DRH and ERH. Many of these IUH functions are gamma-family probability distributions. Singh (2000) developed methods to represent the Natural Resources Conservation Service (NRCS) dimensionless unit hydrograph as a gamma distribution (Singh 2000).

In this paper a gamma-family distribution for the IUH functions is tested using over 1,600 storms on 91 watersheds in North and South central Texas. The analysis determined distribution parameters for each storm, and compared the IUH model to conventional hydrograph models for applicability to the Texas data.

Data Assembly and Preparation

The United States Geologic Survey (USGS) conducted small watershed studies in Texas during the period spanning the early 1960s to the middle 1970s, with some studies extending into the 1980s. The storms documented in the USGS studies were used to evaluate unit hydrographs and these data are critical for unit hydrograph investigations in Texas. A substantial database was assembled from stations with paired rainfall and runoff data. Asquith et al. (2004) provide details of the studies and database used in this paper.

The stations used in this paper are listed in Table 1. The table lists the USGS station number, common watershed or river basin name, the nominal drainage area, and the number of paired events. The rainfall and runoff data for each station were as-

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Table 1. List of Stations, Drainage Area, and Storm Count

Watershed	Area (km^2)	Station ID	Number of events
(a) Austin			
Barton Creek	232.31	08155200	5
Barton Creek	300.43	08155300	8
Bear Creek	31.60	08158810	8
Bear Creek	62.16	08158820	2
Bear Creek	54.39	08158825	2
Boggy Creek	33.93	08158050	10
Boggy South Creek	9.27	08158880	14
Bull Creek	57.75	08154700	13
Little Walnut Creek	13.52	08158380	2
Onion Creek	321.15	08158700	6
Onion Creek	429.92	08158800	2
Shoal Creek	7.23	08156650	13
Shoal Creek	18.21	08156700	16
Shoal Creek	19.58	08156750	13
Shoal Creek	31.86	08156800	24
Slaughter Creek	21.34	08158840	9
Slaughter Creek	59.83	08158860	2
Waller Creek	5.98	08157000	40
Waller Creek	10.70	08157500	38
Walnut Creek	14.43	08158400	10
Walnut Creek	31.34	08158500	14
Walnut Creek	32.63	08158100	15
Walnut Creek	67.86	08158200	17
Walnut Creek	132.86	08158600	22
West Bouldin Creek	8.08	08155550	10
Wilbarger Creek	11.94	08159150	29
Williamson Creek	16.32	08158920	14
Williamson Creek	49.21	08158930	18
Williamson Creek	71.48	08158970	16
(b) San Antonio			
Alazar Creek	8.44	08178300	30
Leon Creek	14.43	08181000	9
Leon Creek	38.85	08181400	15
Leon Creek	3.08	08181450	29
Olmos Creek	0.85	08177600	12
Olmos Creek	53.61	08177700	23
Olmos Creek	6.29	08178555	10
Salado Creek	24.71	08178600	13
Salado Creek	10.49	08178620	3
Salado Creek	6.35	08178640	7
Salado Creek	6.03	08178645	5
Salado Creek	0.67	08178690	39
Salado Creek	1.17	08178736	12
(c) Dallas			
Ash Creek	17.92	08057320	5
Bachman Branch	25.90	08055700	39
Cedar Creek	24.40	08057050	3
Coombs Creek	12.30	08057020	7
Cotton Woode Creek	22.01	08057140	6
Duck Creek	20.85	08061620	8
Elam Creek	3.24	08057415	8
Five Mile Creek	19.81	08057418	7
Five Mile Creek	34.19	08057420	10
Floyd Branch	10.80	08057160	8
Joes Creek	5.02	08055580	7
Joes Creek	19.45	08055600	10
Newton Creek	15.31	08057435	4
Prairie Creek	23.39	08057445	8
Rush Branch	3.16	08057130	7
South Mesquite	34.70	08061920	9
South Mesquite	59.57	08061950	31
Spanky Creek	17.53	08057120	5
Turtle Creek	20.67	08056500	42
Whites Branch	6.55	08057440	4
Woody Branch	29.78	08057425	10
(d) Fort Worth			
Dry Branch	2.80	08048550	25
Dry Branch	5.57	08048600	27
Little Fossil	14.61	08048820	19
Little Fossil	31.86	08048850	24
Sycamore	45.84	08048520	24
Sycamore	2.51	08048530	28
Sycamore	3.50	08048540	24
Sycamore	0.98	SSSC	21
(e) Small rural sheds			
Brazos Basin/Cow Bayou	13.60	08096800	49
Brazos Basin/Green	8.24	08094000	28
Brazos Basin/Pond Elm	57.50	08098300	19
Brazos Basin/Pond Elm	125.87	08108200	20
Colorado Basin/Deep	8.86	08139000	25
Colorado Basin/Deep	14.01	08140000	28
Colorado Basin/Mukewater	56.46	08136900	22
Colorado Basin/Mukewater	10.41	08137000	38
Colorado Basin/Mukewater	182.33	08137500	4
San Antonio Basin/Calaveras	18.16	08182400	23
San Antonio Basin/Escondido	8.52	08187000	31
San Antonio Basin/Escondido	21.83	08187900	21
Trinity Basin/Elm Fork	1.99	08050200	34
Trinity Basin/Honey	3.26	08058000	28
Trinity Basin/Honey	5.54	08057500	31
Trinity Basin/Little Elm	5.44	08052630	28
Trinity Basin/Little Elm	195.54	08052700	57
Trinity Basin/North	17.66	08042650	14
Trinity Basin/North	55.94	08042700	69
Trinity Basin/Pin Oak	45.58	08063200	33

sembled into file-pairs (rainfall file and runoff file for each storm).

The over 1,600 file pairs were parsed to extract the date/time, accumulated runoff, and accumulated weighted precipitation. In all cases, an artificial record was added so that all the data start at 00:00:00 hrs of the event day. These files were further processed using linear interpolation to produce time-series of cumulative precipitation and runoff on 1-min increments. The 1-min interval was selected because the analysis follows the method described by Weaver (2003) and attributed to O'Donnell (1960), where each rainfall increment is treated as an individual storm and the runoff from these individual storms are convolved using a unit hydrograph to produce the model of the observed storm. The underlying data for rainfall and runoff are from larger time intervals, typically in the range of 5–15 min, and the 1-min resolution in these derived time series is artificial.

A constant discharge baseflow separation method was used because it is simple to automate and apply to hydrographs with multiple peaks. Prior researchers (e.g., Laurenson and O'Donell 1969; Bates and Davies 1988) have reported that unit hydrograph derivation is insensitive to baseflow separation method when the baseflow is a small fraction of the flood hydrograph—a situation satisfied in this work. The implementation determined when a rainfall event began on a particular day, all discharge before that time was accumulated and converted into an average rate. This average rate was then removed from the observed discharge data, and the remaining flow was considered to be direct runoff.

Excess precipitation was modeled using a constant fraction loss model (McCuen 1998), where some constant ratio of precipitation becomes runoff. This approach again was selected for simplicity with regard to automated analysis. This loss model is represented as

$$[i(t)] = C_r p(t) \qquad C_r = \frac{\int Q(t)dt}{A\int p(t)dt} \tag{2}$$

where $[i(t)]$=excess rainfall rate; $p(t)$=raw (observed) rainfall rate; C_r=fraction of precipitation that appears as runoff; $A\int p(t)dt$=cumulative rainfall for the storm (a volume); A=watershed area; and $\int Q(t)dt$=cumulative direct runoff for the storm (a volume).

Additional details of the data preparation, separation techniques, and rainfall loss models are reported in He (2004).

Hydrograph Models

Three different hydrograph models are examined in this paper; the NRCS dimensionless hydrograph, the Commons hydrograph, and an IUH based on the Rayleigh distribution.

The NRCS dimensionless unit hydrograph (DUH) used by the NRCS (formerly the SCS) was developed in the late 1940s (NRCS 1972). The NRCS analyzed a large number of unit hydrographs for watersheds of different sizes and in different locations and developed a generalized dimensionless unit hydrograph in terms of t/t_p and q/q_p where t_p=time to peak. The point of inflection on the unit graph is approximately 1.7 the time to peak and the time to peak was observed to be 0.2 the base time (hydrograph duration) (T_b).

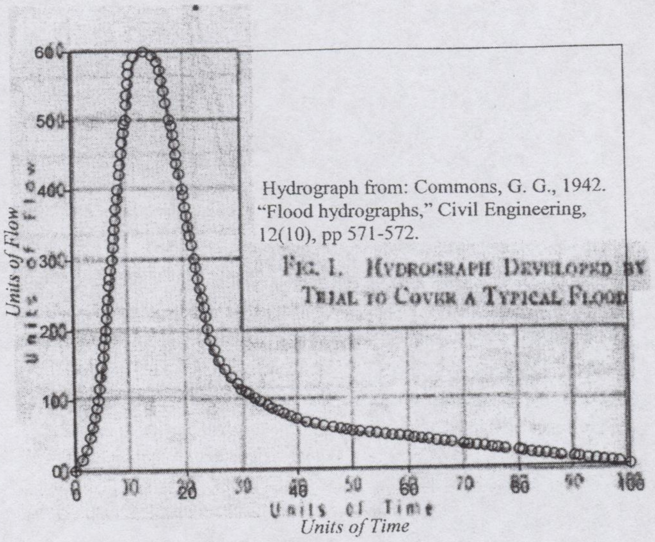

Fig. 1. Texas unit hydrograph. Circles are from manual digitization of original figure [shown in background; Commons (1942), ASCE].

Commons (1942) developed a dimensionless unit hydrograph for use in Texas, but details of how the hydrograph was developed are not reported. The labeling of axes in the original document indicates that the hydrograph is dimensionless (i.e., units of flow and units of time). Fig. 1 is a plot of the Commons hydrograph in its original units. The markers are the manually digitized values, obtained from the original figure (shown as the background) in Fig. 1. A dimensionless hydrograph is created by dividing the ordinates of the Commons hydrograph by its peak units of flow value, and by dividing the units of time by the time when the peak occurs. In contrast to the NRCS hydrograph, the inflection point occurs at about $2.0t_p$, and the time to peak occurs at about $0.13T_b$.

He (2004) developed several instantaneous unit hydrographs based on gamma-family distributions following the conceptual approach of Nash (1958) and others. The behavior of a single reservoir element was postulated, then the elements were combined in a cascade structure and the behavior of this structure was analyzed to develop hydrograph distributions. Eq. (3) is the Rayleigh instantaneous unit hydrograph

$$u(t) = \frac{2}{\bar{t}}\left(\frac{t}{\bar{t}}\right)^{2N-1}\left[\frac{1}{\Gamma(N)}\right]\exp\left[-\left(\frac{t}{\bar{t}}\right)^2\right] \tag{3}$$

where $\bar{t}$=distribution residence time; and N=reservoir number (a shape parameter).

Eq. (3) is a special case of the hydrograph distribution developed by Leinhard (1964) and Leinhard and Meyer (1967) who postulated the distribution entirely from statistical–mechanical principles, then demonstrated its applicability on two watersheds in Iowa. N can take noninteger values (Nauman and Buffham 1983), although the concept of a fractional reservoir has little physical analogy.

The Rayleigh hydrograph can be expressed as a dimensionless hydrograph using conventional terms by the following transformations:

$$T_p^2 = \frac{(2N-1)}{2}\bar{t}^2 \tag{4}$$

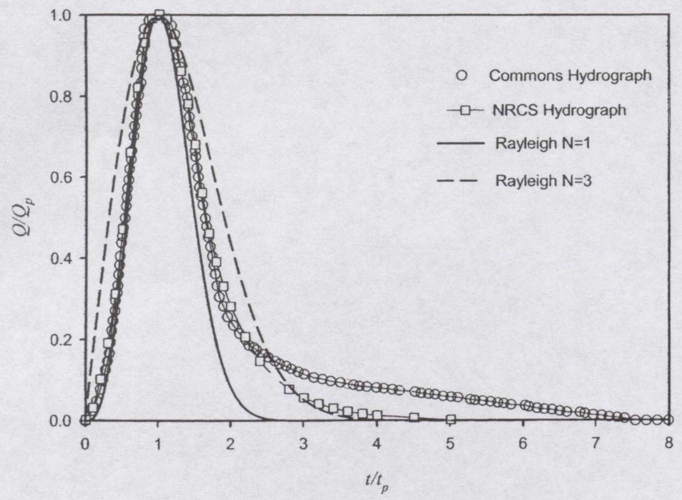

Fig. 2. Commons, NRCS, and Rayleigh dimensionless hydrographs. Rayleigh hydrograph is shown for two different values of N (shape parameter).

$$Q_p = F^* u(T_p)^* A \tag{5}$$

where T_p=time to peak (conventional meaning); Q_p=peak discharge rate (conventional meaning); A=watershed area; and F=unit conversion [e.g., *645.33* if $u(T_p)$ is in inches/hour and A is square miles].

The peak rate from Eq. (3) is obtained from

$$u(T_p) = \frac{(2N-1)^N}{2^{N-1}\Gamma(N)} \frac{1}{T_p} \exp\left[-\frac{(2N-1)}{2}\right] \tag{6}$$

and the dimensionless unit hydrograph is expressed as

$$\frac{u(t)}{u(t_p)} = \left(\frac{t}{t_p}\right)^{2N-1} \exp\left\{-\left[\frac{t_p^2 - t^2}{2t_p^2}(2N-1)\right]\right\}$$
$$= \left\{\frac{t}{t_p}\exp\left[\frac{1}{2}\left(1 - \frac{t^2}{t_p^2}\right)\right]\right\}^{2N-1} \tag{7}$$

Eq. (4) relates the shape and residence time to the time to peak, T_p, with its conventional meaning, and Eq. (5) is the peak discharge, Q_p, also with its conventional meaning. Eq. (6) is the peak rate (of the Rayleigh distribution), and Eq. (7) is a dimensionless representation of the instantaneous unit hydrograph.

Fig. 2 is a plot of the three hydrographs in dimensionless space. The Commons hydrograph is quite different in shape after the peak than the other dimensionless hydrographs—it has a very long time base on the recession portion of the hydrograph. The Rayleigh hydrograph has adjustable shape in our work, but otherwise is more similar to the NRCS hydrograph than to the Commons hydrograph.

The Rayleigh rainfall-runoff model for this paper is the convolution of Eqs. (2) and (3).

$$Q(t)_m = \int_0^t [i(\tau)] A \frac{2}{\bar{t}} \frac{1}{\Gamma(N)} \left[\frac{(t-\tau)^{2N-1}}{\bar{t}^{2N-1}}\right] \exp\left\{-\left[\frac{(t-\tau)}{\bar{t}}\right]^2\right\} d\tau \tag{8}$$

In the case of either the NRCS or Commons hydrograph, the unit hydrograph component is computed by the appropriate tabulation and dimensionalization (Q_p, T_p).

Determining Parameters

The resulting rainfall-runoff models require one or two parameters: T_p for NRCS and Commons models, and N and $\bar{t}$ for the Rayleigh model. The excess rainfall is determined by the storm properties. The parameters for each storm are determined by calculating the direct runoff hydrograph from Eq. (8) using the observed rainfall and comparing the modeled response to the observed response for the same storm on the same watershed. Two different merit functions considered were the sum of squared errors (SSE) and a maximum absolute deviation at peak discharge (Q_pMAD). Mathematically these merit functions are represented as

$$\mathrm{SSE} = \sum_{i=1}^{\mathrm{NOBS}} (Q_m - Q_o)_i^2 \tag{9}$$

and

$$Q_p\mathrm{MAD} = |Q_m(t_{\mathrm{peak}}) - Q_o(t_{\mathrm{peak}})| \tag{10}$$

where Q=discharge (L^3/T); the subscripts O and M represent observed and model discharge; NOBS=total number of values in a particular storm event; and t_{peak}=actual time in the observations when the peak observed discharge occurs. The first merit function produces results that sacrifice peak matching in favor of matching the runoff volume, while the second merit function favors matching the peak discharge magnitude with little regard for the rest of the hydrograph. A search technique was used instead of more elegant methods, principally to ensure a result. The search systematically computes the value of a merit function using every permutation of model parameters described by the set-builder notation in Eq. (11).

Rayleigh model

$$\bar{t} \in \{1,2,3,\ldots,720\}$$

$$N \in \{1.00, 1.01, 1.02, \ldots, 9.00\}$$

NRCS and Commons model

$$T_p \in \{1,2,3,\ldots.720\} \tag{11}$$

Fractions of a minute were ignored (hence the 1-minute increments in the timing parameters) and higher resolution in the shape parameter was deemed unnecessary. The set of parameters that produces the smallest value of the merit function is saved as the storm optimal set for that storm. This approach, while inelegant, is robust and produces values that are qualitatively reasonable.

Typical Results

Fig. 3 is a plot of a typical result. The figure compares the model hydrograph and the observed hydrograph for a particular storm. Qualitatively this plot suggests that all the models produce different, but comparable results. For the particular example in Fig. 3, the Rayleigh model appears to perform better with regard to peak fit than the other models, but this better performance does not generalize to the entire set of storms.

Fig. 4 is a plot of the peak discharge produced by Eq. (8) for each storm versus the observed peak discharge for that storm for the three models. Fig. 4 is for parameters based on the SSE merit function. The solid line is the 1:1 line along which all the markers should plot. The figure illustrates that the SSE merit function marker cloud falls below the 1:1 line and thus generally under-

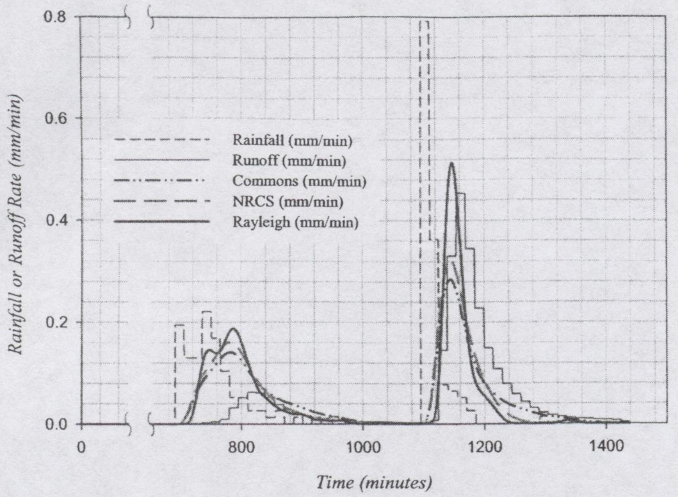

Fig. 3. Typical results for single storm using QpMAX merit function and three different hydrograph models. Station is located on Ash Creek in Dallas, Tex. Storm date is June 3, 1973.

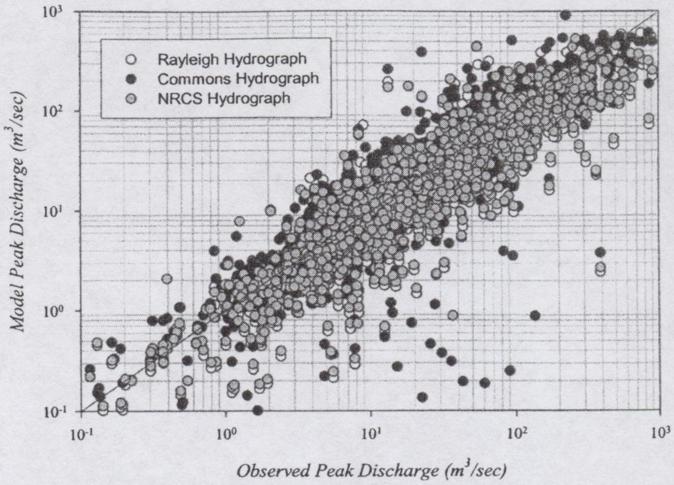

Fig. 5. Rayleigh, Commons, and NRCS model peak discharge versus observed peak discharge for parameters determined by QpMAX criterion

predicts peak magnitudes. The Rayleigh model has greater variability than either the NRCS or Commons model, and the NRCS and Commons model produce very similar results despite their different appearance in dimensionless space.

Fig. 5 is the same information as Fig. 4 except the parameters are based on the QpMAX merit function. Fig. 5 illustrates that the 1:1 line passes through the center of the marker cloud, with slightly more variability for all the models. Qualitatively this figure demonstrates that if peak discharge prediction is important, this merit function is preferred. This result is not surprising as the merit function compares differences in peak flows.

Figs. 6 and 7 are plots of the time when the model peak discharge occurs (in dimensional time) versus the time when the observed peak discharge occurs (also in dimensional time). In both these figures the 1:1 line passes through the centroid of the marker cloud, suggesting that either merit function is adequate for finding the timing parameter of any of the hydrograph models.

Figs. 8 and 9 are plots of the dimensionless parameters Q_p and T_p for each storm plotted against watershed area. The Rayleigh model parameters are expressed in Q_p and T_p form

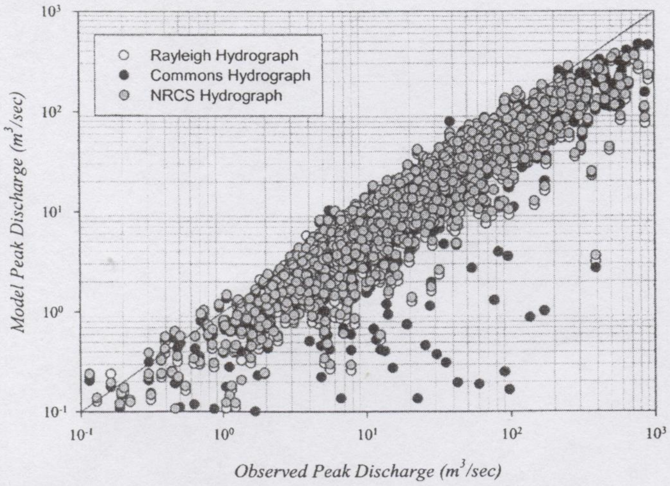

Fig. 4. Rayleigh, Commons, and NRCS model peak discharge versus observed peak discharge for parameters determined by SSE criterion

using Eqs. (4) and (5). The plots indicate that the stations have multiple storms, and illustrate the range of drainage areas used in this study (e.g., small watersheds). Trendlines in each figure are used to illustrate that both the general trend of the marker clouds and the relative location of the cloud centroid are about the same for each hydrograph model.

Acceptance Analysis

We examined three measures of acceptability as a quantitative approach to determine if one of the three models performed better for these data. These measures are calculated after determining the parameters. The measures are a normalized mean square error, a peak discharge relative error, and a peak time absolute error.

Normalized mean square error emphasizes scatter in a data set. Smaller values of NMSE suggest better performance. NMSE is calculated (Patel and Kumar 1998) using

$$\mathrm{NMSE} = \frac{\overline{(Q_o - Q_m)^2}}{\bar{Q}_o \bar{Q}_m} \tag{12}$$

where $\bar{Q}_o$=arithmetic mean of the observed runoff values; and $\bar{Q}_m$=arithmetic mean of the model runoff values.

The peak relative error is the difference in magnitude between the observed and model peak rate and defined as

Table 2. Acceptance Measures for Two Merit Functions and Three Unit Hydrograph Models

	Hydrograph model		
Measure	Commons	NRCS	Rayleigh
Median_NMSE_SSE	1.29E−07	4.09E−11	2.56E−10
Median_NMSE_PEAK	3.69E−09	2.75E−11	1.48E−14
Median_QB_SSE	0.38	0.34	0.29
Median_QB_PEAK	0.20	0.12	−0.08
Median_TB_SSE	−3.00	−6.00	−10.00
Median_TB_PEAK	20.00	19.00	2.00

Table 3. Fraction of Storms within Prescribed Peak and Temporal Error Criteria

Measure	Hydrograph model		
	Commons	NRCS	Rayleigh
QB_SSE	0.29	0.35	0.42
QB_PEAK	0.40	0.45	0.48
TB_SSE	0.59	0.57	0.56
TB_PEAK	0.54	0.52	0.62

merit function is expected to produce hydrograph parameters that will underpredict the peak discharge as compared to the QpMAX criterion. This conclusion is supported by Figs. 4 and 5 and by the difference in the median QB values for all analyzed storms.

The Rayleigh model is demonstrated to be an adequate substitute for the Commons and NRCS tabulations. It is expected that other gamma-family models are also suitable substitutes. The adjustable shape parameter in the Rayleigh model did not produce any dramatic improvement on model performance, thus for these data the use of unit hydrographs with fixed shapes is adequate.

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