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THEORY OF FILTRATION OF HIGHLY COMPACTABLE BIOSOLIDS

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ABSTRACT

Compactability of particulate structures is a key factor in the behavior of thickeners, filters, centrifuges, and presses. Aggregates in slurries are deposited at a cake or sediment surface under null stress. As more deposits cover the surface, developing stresses compact the particulate bed. Principal sources of stress originate from (1) sediment weight in thickening, (2) frictional drag in filters, (3) centrifugal forces in centrifuges, and (4) surface forces in belts or diaphragms. Only frictional forces in filters are considered in the present work.

Stress applied to cakes results in a decrease in porosity and an increase in resistance to flow (decrease in permeability). The rate at which the permeability decreases with pressure has a profound effect on cake behavior. For highly compactable beds of biosolids or fragile flocs, doubling of the pressure may result in more than a doubling of the local resistance. Consequently, at applied pressures frequently below one atmosphere, increasing pressure neither increases the flow rate nor decreases the average cake porosity.

For highly compactable biosolids, theoretical equations show that (1) filtrate volume vs. t is independent of pressure drop across the cake, Δp_c, (2) the average specific resistance is proportional to Δp_c, and (3) the average volume fraction of solids is independent of Δp_c.
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KEY WORDS

Filtration, solid/liquid separation, compactability, average cake porosity, biosolids

INTRODUCTION

Solid/liquid separations begin with mixtures of liquids and particles. Depending on original chemical formation, particles may range from hard, dense crystals to soft cellular materials. After suspensions are formed, repulsive and attractive forces play important roles in determining the structures developed by interacting particles. Attractive interparticle forces produce aggregates which may contain anywhere from a few to thousands of particles. Properties of the aggregates (or flocs) and their response to stress account for compactibility and flow resistance.

Particulate structures encountered in sediments vary enormously in resistance to compressive forces. The nature of the chosen pretreatment agents and the mechanical mixing process determine the properties of cakes formed during deposition in thickeners, filters, and centrifuges. Generally, increasing sedimentation velocity, producting permeable cakes, and decreasing the liquid content of the wet cake are objectives of pretreatment operations.

Aggregates are deposited at a growing cake surface under a null stress. The cake structure is a function of the internal porosity and aggregate shape. As each successive surface layer is covered by new deposits, it is compacted by developing stresses from sediment weight in gravity thickeners, centrifugal forces in centrifuges, drag forces in filters, and direct mechanical forces in belts, pistons, and diaphragms.

When compressive stress is applied to a matrix of particles, the solidosity, ε_s (volume fraction of solids) decreases and the permeability, K, increases. Several different types of response to compressive stress (effective pressure), p_s, are shown in Fig. 1 below.
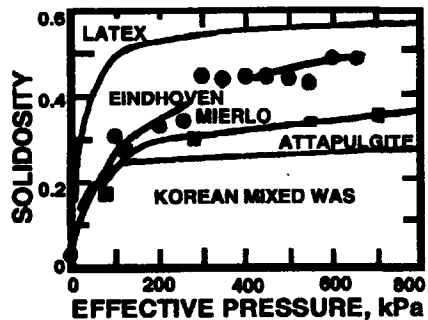

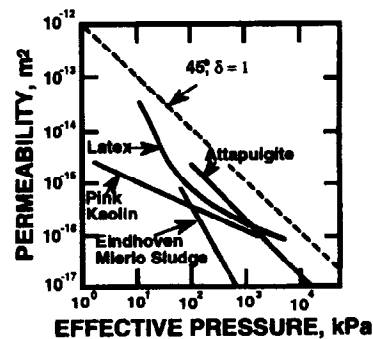Figure 1. Behavior of porous cakes under pressure

Figure 2. Permeability vs. effective pressure

Sources of data for Figures 1 and 2: Latex, Grace (1953); Eindhoven-Mierlo Sludge, 15% FeCL₃, 20% lime, dry basis, LaHeij (1994); attapulgite, Yeh (1985); Hong Kong Pink Kaolin, Shirato (1960).

Permeabilities of a number of materials are shown in Fig. 2 on a logarithmic plot as a function of the effective pressure p_s. There is a direct correlation between the degree of cake compaction shown in Fig. 1 and the rate of decrease in permeability shown in Fig. 2. The solidosity of the fragile latex flocs increases rapidly up to a pressure of approximately 75-85 kPa and then undergoes a more gradual increase at higher pressures. This behavior is reflected in the 100 fold decrease of the permeability in the pressure range up to 75-85 kPa. This supercompactability is followed by a region of moderate compactibility where the compressed latex flocs mimic the behavior of Hong Kong Pink Kaolin. With a slope of -0.45 for the log K vs. log p_s of Pink Kaolin in Fig. 2, doubling the pressure at a given location in a filter or centrifuge cake would reduce the permeability to 68% of its former value. This reduction in K is considerably less than would be experienced by the latex, attapulgite, and Endhoven-Mierlo sludge (60% domestic, 40% industrial; 80% organic, 20% inorganic; average dry solids density = 1200-1300 kg/m³ treated with 15% FeCl₃ and 10% lime on a dry basis). The slopes of the plots of log K vs. log p_s for both latex and Eindhoven-Mierlo (EM) sludge indicate similar supercompactability, but the permeability of the EM sludge is much lower than the latex at pressures above 100 kPa.

EMPIRICAL CONSTITUTIVE EQUATIONS

The development of empirical models for representing porosity, solidosity (volume fraction of solids), specific flow resistance, and permeability as functions of effective pressure has depended on power law approximations. Although the Kozeny-Carman equation is frequently mentioned in the literature, it is

inadequate for compactable cakes. The authors are unaware of any theoretical equations that can be used with confidence, so the following empirical relationships are used in the present work:

$$(\varepsilon_s / \varepsilon_{so})^{1/\beta} = (\alpha / \alpha_o)^{1/n} = (K / K_o)^{-1/\delta} = 1 + p_s / p_a \tag{1}$$

where the subscript o refers to the null stress values of ε_s, K, and α (specific resistance); β, n, and δ are compactibility parameters; p_a is an arbitrary parameter. The permeability, specific resistance, and solidosity and corresponding exponents in the empirical models are related by:

$$\alpha K \varepsilon_s = 1, \qquad \delta = n + \beta \tag{2}$$

THEORY OF FLOW THROUGH COMPACTABLE BEDS

Developing an approximate theory of flow through compactable, porous media in filters requires a static force balance (in the absence of momentum changes) to give effective stress in terms of the liquid pressure and Darcy's law to relate flow rate to the liquid pressure gradient. For planar filtration, the effective pressure, p_s, and the liquid pressure, p_L, are related by

$$p_L + p_s = p \qquad dp_L + dp_s = 0 \tag{3}$$

The quantities p_L and p_s vary with position and time. The applied pressure is either constant or a function of time alone. Darcy's law relates the liquid pressure gradient to the flow rate in the form

$$\frac{dv}{dt} = q_L = -\frac{K}{\mu}\frac{dp_s}{dx} = -\frac{1}{\mu\alpha}\frac{dp_s}{d\omega} \tag{4}$$

where v is the filtrate volume per unit filter area; t is the time; q_L is the liquid flow rate per unit filter area; x is the distance from the supporting medium to an arbitrary point in the cake; ω is the volume of solids per unit filter area in the distance x. Eq. 4 assumes negligible solids velocity in the cake. The liquid pressure differential, dp_L, in Darcy's law has been replaced by - dp_s in Eq. 4. As K and α are functions of p_s, the expressions in Eq. 4 can be integrated to yield q_L as a function of the pressure drop across the cake. The differential volume of solids $d\omega$ in dx and the total volume of solids in a cake of thickness L are given by

$$d\omega = \varepsilon_s dx, \qquad \omega_c = \int_0^L \varepsilon_s dx = \varepsilon_{sav} L \tag{5}$$

Most developments in the theory of filtration are based on the mass of solids per unit filter area w. The mass basis is related to the volumetric basis by $w = \rho_s \omega$ and $dw = \rho_s d\omega$ where ρ_s is the solids density. The volumetric basis is superior, but in wastewater processing the solids densities are not generally known and the mass basis must be used.

Integrating the material coordinate formula in Equation 4 and defining α_{av} leads to

$$\mu q_L \omega_c = \int_0^{\Delta p_c} \frac{dp_s}{\alpha} = \frac{\Delta p_c}{\alpha_{av}} = \mu \frac{dv}{dt} \omega_c \tag{6}$$

Eq. 6 involves: q_L the filtrate rate, ω_c, total volume of dry solids per unit filter area, and Δp_c, liquid pressure drop across the cake. Eq. 6 states that if the amount of deposited solids and rate of filtration are known, then the pressure drop can be calculated. A similar integration in spatial coordinates leads to

$$\mu q_L = \int_0^{\Delta p_c} K dp_s = K_{av} \Delta p_c \tag{7}$$

Dividing Eq. 6 by Eq. 7 provides a useful relationship for the average volume fraction of solids

$$\frac{\omega_c}{L} = \varepsilon_{sav} = \frac{1}{\alpha_{av} K_{av}} \tag{8}$$

This relationship does not apply where radial flow is involved (centrifuges, candle filters) or to gravity thickeners.

MATERIAL BALANCES

A volumetric balance on a per unit filter area basis in the form: slurry volume = cake volume + filtrate volume, gives

$$\frac{\omega_c}{\phi_s} = \frac{\omega_c}{\varepsilon_{sav}} + v = L + v, \qquad v = \left(\frac{1}{\phi_s} - \frac{1}{\varepsilon_{sav}} \right) \omega_c \tag{9}$$

where ϕ_s is the volume fraction of solids in the slurry and ω_c is the total volume of solids in the cake and slurry. Multiplication by the solids density converts ω_c to w_c (a mass basis). Solving for ω_c in terms of v yields

$$\omega_c = \phi_s v / (1 - \frac{\phi_s}{\varepsilon_{sav}}) \tag{10}$$

If there is no sedimentation (a dubious assumption; Tiller et al., 1995), ϕ_s will remain constant. On the other hand, the average volume fraction of solids ε_{sav} in the cake will generally increase with time. An increase in ε_{sav} can have a significant effect on calculated values of ω_c when concentrated slurries are involved. Slurries are considered "dilute" or "concentrated" depending on the ratio ϕ_s/ε_{sav}. For dilute slurries with the ratio less than 0.1, the value of the expression in Eq. 10 does not change much with variations in ε_{sav}. However for concentrated slurries, small variations in ε_{sav} have relatively large effects on Eq. 10 and calculated values of ω_c based on measured values of v. Although the ratio is assumed constant in most theoretical studies, it may undergo large changes in filtering biosolids. In the example in the next section, it varies from 0.33-0.50.

VOLUME VS. TIME FOR HIGHLY COMPACTABLE CAKES

At the start of a filter run, all of the pressure drop is across the medium; the first "layer" of cake is deposited under null stress. As the cake grows and its resistance increases, the pressure drop across the cake increases and relative pressure drop across the medium diminishes. The pressure drop across the cake Δp_c increases and approaches the total applied pressure p.

The pressure drop across the cake in Eq. 6 can be replaced by $\Delta p_c = p - \mu q_L R_m$ where R_m is the medium resistance. Substituting ω_c from Eq. 10 into Eq. 6 yields

$$\mu q_L \omega_c = \left(\frac{1}{\phi_s} - \frac{1}{\varepsilon_{sav}} \right) = \mu q_L v = \left(\frac{1}{\phi_s} - \frac{1}{\varepsilon_{sav}} \right) \frac{\Delta p_c}{\alpha_{av}} \tag{11}$$

Replacing ε_{sav} by $1/\alpha_{av} K_{av}$ and solving for v leads to

$$v = \frac{1}{\mu q_L}\left(\frac{1}{\phi_s \alpha_{av}} - K_{av}\right)\Delta p_c = \frac{1}{\mu q_L}\int_0^{p-\mu q_L R_m} (\frac{1}{\phi_s \alpha} - K)dp_s \qquad (12)$$

where the average values have been replaced by the integrals in Eqs. 6 and 7. Use of Eq. 12 avoids the assumption of constant ε_{sav} and α_{av}. Eq. 12 provides a relationship that yields v as a function of q_L, which generally must be solved numerically. As $q_L = dv/dt$, the time required to produce volume v can be obtained from

$$t = \int_0^v \frac{1}{q_L}dv \qquad (13)$$

Substituting the formulas in Eq. 1 for α and K into Eq. 12 gives

$$v = \frac{1}{\mu q_L}\int_0^{p-\mu q_L R_m} [\frac{(1+p_s/p_a)^{-n}}{\phi_s \alpha_o} - K_o(1+p_s/p_a)^{-\delta}]dp_s \qquad (14)$$

Integrating and substituting limits yields

$$\frac{\mu q_L v}{p_a} = \frac{(1+\Delta p_c/p_a)^{1-n}-1}{\phi_s \alpha_o(1-n)} - K_o\frac{(1+\Delta p_c/p_a)^{1-\delta}-1}{1-\delta} \qquad (15)$$

As long as $n < 1$ and $\delta < 1$, $(1-n)$ and $(1-\delta)$ are positive; and Eq. 15 can be used as is. However, when $n > 1$ and $\delta > 1$, both $(1-n)$ and $(1-\delta)$ are negative; and a profound change take place in the nature of calculations based on Eq. 15. Rearranging Eq. 15 and substituting $K_o = 1/\alpha_o \varepsilon_{so}$ leads to

$$\frac{\mu q_L \alpha_o v}{p_a} = \frac{1}{\phi_s(n-1)}\left[1 - \frac{1}{(1+\Delta p_c/p_a)^{n-1}}\right] - \frac{1}{\varepsilon_{so}(\delta-1)}\left[1 - \frac{1}{(1+\Delta p_c/p_a)^{\delta-1}}\right] \qquad (16)$$

EXAMPLE USING DATA FOR MIERLO BIOSOLIDS

Solution of Eq. 16 yields v as a function of q_L. The area under the plot of $1/q_L$ vs. v yields filtration time. To start the process, the initial flow rate is calculated assuming $\Delta p_c = 0$ and $q_L = p/\mu R_m$. Subsequently smaller values of q_L are chosen, Δp_c is calculated; and v is obtained from Eq. 16. Values of parameters for a Mierlo sludge flocculated with 1.5 wt. % of polyelectrolyte (Rohm KF975) on a dry solid basis follow: $\varepsilon_{so} = 0.03$, $K_o = 8.3 \cdot 10^{-12}$ m^2, α_o=4.016 $\cdot 10^{12}$ m^{-1}, $p_a = 1000$ Pa, $n = 1.83$, $\beta = 0.47$, $\delta = 2.3$ (LaHeij, p.66, 1994). Values of other parameters required for the solution include $p = 1.0 \cdot 10^5$ Pa, $\phi_s = 0.015$, $\mu = 0.001$ Ps•s, and $R_m = 5, 25, 50 \cdot 10^{10}$ m^{-1}. Using these values, the initial value $q_L = 1.0 \cdot 10^8/R_m$ m/s. With $R_m = 5 \cdot 10^{10}$, Eq. 16 takes the following form

$$q_L v = 2 \bullet 10^{-5}\left[1 - \frac{1}{(1+\Delta\Pi_c)^{0.83}}\right] - 6.385 \bullet 10^{-6}\left[1 - \frac{1}{(1+\Delta\Pi_c)^{1.3}}\right] \qquad (17)$$

where $\Delta\Pi_c = \Delta p_c/1000$. The two terms involving $(1+\Delta\Pi_c)$ decrease rapidly with increasing $\Delta\Pi_c$. In Fig. 3, plots of $1/q_L$ vs. v are shown for the three values of R_m. All three plots approach the same asymptote which has a zero intercept.

Plots of dt/dv and t/v are commonly used in analyzing constant pressure filtration data. For the conventional parabolic relation between v and t to be valid, dt/dv must be linear in v. The theoretical plots in Fig. 3 provide evidence that a parabolic relation does not exist in the initial stages. Practically, the time required to reach the linear region can be very short and may be missed in experiments. The three points A,B,C at which linearity is approached occur respectively at 10, 120, and 500-600 seconds. When the time to linearity is very short, problems related to introducing slurry into the filter, increasing the pressure, and starting measurements frequently preclude the possibility of accurate v vs. t data prior to the linear portions of the plots. If an experimenter only identifies the straight line portion of the curves, he may draw erroneous conclusions about cake compactibility and the true initial medium resistance. It is essential to start with a relatively large medium resistance if the initial stages are to be studied.
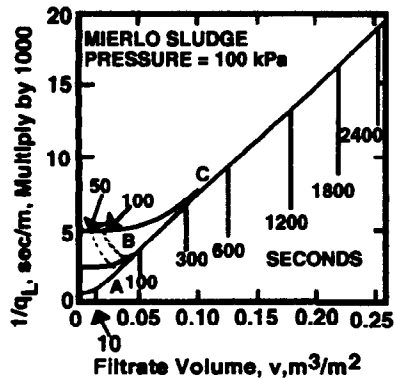

Figure 3. Graph showing determination of time based on Eq. 13 and using parameters for Mierlo sludge (LaHeij, 1994) flocculated with 1.5% dry basis of Rohm KF 975 polyelectrolyte and varying values of R_m
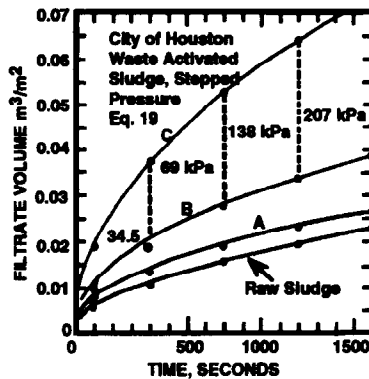
Figure 4. Volume vs. time with pressure increase at 400 second intervals. Curves A,B,C refer to City-Chem Chemfloc 5878 polyelectrolyte dosages of 0.7, 1.4, and 2.8% dry basis.

ASYMPTOTIC RELATIONSHIP

As Δp_c becomes larger and approaches the applied pressure p, the terms in brackets in Eq. 16 approach unity; and the equation assumes the form

$$\frac{\mu \alpha_o}{p_a} = v \frac{dv}{dt} = \left[\frac{1}{\phi_s(n-1)} - \frac{1}{\varepsilon_{so}(\delta-1)} \right] \tag{18}$$

When the medium resistance is small and the linear region is approached in seconds, integration of Eq. 18 yields

$$v^2 = \frac{2p_a}{\mu \alpha_o} \left[\frac{1}{\phi_s(n-1)} - \frac{1}{\varepsilon_{so}(\delta-1)} \right] t \tag{19}$$

This v vs. t relationship is independent of applied pressure. For highly compactable cakes with sufficiently large values of n and δ, increasing pressure beyond some minimum value has little effect on v vs. t. When the medium resistance is large, the integration must be carried out over the linear region.

The average solidosity can be obtained by combining Eqs. 8 and 16 to give

$$\varepsilon_{sav} = \frac{\int_{0}^{\Delta p_c} \frac{dp_s}{\alpha}}{\int_{0}^{\Delta p_c} K dp_s} = \varepsilon_{so} \frac{\delta-1}{n-1} \frac{[1-1/(1+\Delta\Pi_c)^{n-1}]}{[1-1/(1+\Delta\Pi_c)^{\delta-1}]} \qquad (20)$$

As the pressure drop increases the average solidosity approaches

$$\varepsilon_{sav} = \varepsilon_{so} \frac{\delta-1}{n-1} \qquad (21)$$

This expression represents the maximum possible average volume concentration of solids and is independent of pressure. Combining the definition of average specific resistance in Eq. 6 with Eqs. 11 and 21 leads to

$$\alpha_{av} = \alpha_o (n-1) \frac{p}{p_a} \qquad (22)$$

$$K_{av} = K_o \frac{1}{\delta-1} \frac{p_a}{p} \qquad (23)$$

Thus the average specific resistance is proportional to the pressure and the average permeability is inversely proportional to the pressure for highly compactable cakes.

In Fig. 4, volume vs. time data are shown for a City of Houston waste activated sludge (3.34% by weight of solids) treated with differing amounts of cationic acrylamide copolymer in mineral oil. After mixing diluted polymer with the sludge, the slurry was poured into a filter and pressure was applied with compressed nitrogen. Starting with an initial pressure of 34.5 kPa, the pressure was increased incrementally to 69, 138, and 207 kPa at intervals of 400 sec. Each time the pressure was increased, a slight jump in dv/dt occurred. The scale in Fig. 4 is too small to show the small discontinuities in the slope.

The large slopes at $t = 0$ for all the curves shown in Fig. 4 are an indication the medium resistance was small. The lack of substantial change in dv/dt at each time increment of pressure is an indication of supercompactibility.

A rather severe test of Eq. 19 involved using only the last point on each curve to calculate $t/v2$ and then using the result to predict the entire v vs. t history. The points in Fig. 4 represent values calculated as the basis of Eq. 19. Although the fit is excellent, it is essential to be conservative in interpreting parabolic fits, because over limited ranges, many functions can be approximated by second degree polynomials. Sorensen and Hansen (1993), Bruus (1992), LaHeij (1994), and Tiller and Yeh (1985) have provided data that support Eq. 18. The methodology for obtaining data similar to the results shown in Fig. 4 offers promise for the evaluation of flocculants.

NOTATION

K = permeability, m^2
K_{av} = average permeability, m^2

K_o = unstressed cake permeability, m^2

L = cake thickness, m

n = compressibility coefficient, Eq. 1

p = filtration applied pressure, Pa

p_a = empirical constant, Eq. 1, Pa

p_s = effective of compressive pressure, Pa

Δp_c = hydraulic pressure across cake, Pa

q_L = superficial (specific discharge) velocity of liquid, m/s

R_m = medium resistance, m^{-1}

t = time, s

v = filtrate volume per unit filter area, m^3/m^2

w_c = mass of inert solids per unit area of filter in cake, kg/m^2

x = distance from medium, m

α = local specific flow resistance, m^{-2}

α_{av} = average specific flow resistance, m^{-2}

α_o = unstressed cake specific flow resistance, m^{-2}

β = compressibility coefficient, Eq. 1

δ = compressibility coefficient, Eq. 1

ε_s = volume fraction of solids (solidosity)

ε_{sav} = average volume fraction of solids in cake

ε_{so} = unstressed value of ε_s

μ = liquid viscosity, Pa·s

$\Delta\Pi_c$ = $\Delta p_c/p_a$

ρ_s = solids density, kg/m^3

ϕ_s = volume fraction of solids in slurry

ω = volume of inter solids per unit filter area between 0 and x, m^3/m^2

ω_t = total volume per unit filter area of inert solids in cake, m^3/m^2

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