Instantaneous Unit Hydrographs for Central Texas

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Abstract

An instantaneous unit hydrograph is a hydrograph resulting from one unit of excess rainfall uniformly distributed over a watershed over an infinitesimal time period. This research examines if a simple instantaneous unit-hydrograph model can be used to mimic runoff data for 1600+ rainfall-runoff events from 80 locations in central and north central Texas, and if the parameters are invariant within a watershed or a data base module (a collection of geographically nearby watersheds).

The model was characterized for its ability to reproduce peak flows, total volumes, and peak arrival times. Qualitatively, the model produced reasonable estimates of peak arrival time, but has a systematic bias in predicting peak flows. The timing estimates are different from each other for most modules (Dallas, Fort Worth, San Antonio, Austin, and a Rural Watershed Network), and are weakly correlated to watershed area for the database as a whole. A feasibility argument for synthesis for un-gaged areas is presented.

Introduction

The unit hydrograph (UH) method is a technique to represent the causal relationship between rainfall and surface runoff. Numerous studies have been conducted on the unit hydrograph ranging from the empirical approaches (e.g. Snyder, 1938) to conceptual linear models (Clark, 1943; Nash 1958, -1959; Dooge, 1959; Leinhard 1972). Likewise, numerous studies have been conducted relating simple watershed measures to generate synthetic unit hydrographs (e.g. Gray, 1962; Wu, 1963; Gupta, et. al. 1980; Wilson and Brown, 1992).

The Texas Department of Transportation (TxDOT) uses unit hydrograph methods to estimate the magnitude and duration of discharges for design of drainage structures when watershed drainage area exceeds 200 acres, but is less than about 20 square miles. Typically, Natural Resources Conservation Service (NRCS) methods are used. In this paper, an instantaneous unit hydrograph (IUH) is used to examine runoff-producing storms for watersheds in central Texas, in order to evaluate the applicability of IUHs for use in un-gaged regions of Texas, and to evaluate the NRCS approach.

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Methodology

A principal objective of this paper is to determine an IUH from observed rainfall-runoff data. The assumption is made that an IUH exists, and that it is the response function to a simple linear system, and a research task is to determine suitable parameters of the transfer function. A database is assembled that contains rainfall and runoff values for the analysis. Once the data are assembled, the runoff is analyzed for the presence of any baseflow, and this component of runoff is removed. Once the baseflow is removed, the remaining streamflow is called the direct runoff hydrograph (DRH). The total volume of discharge is determined and the rainfall input is analyzed for rainfall losses. The losses are removed so that the total rainfall input volume is equal to the total discharge volume. The rainfall input after this process is called the effective precipitation. By definition, the cumulative effective precipitation is equal to the cumulative direct runoff.

When the rainfall-runoff transfer function and its coefficients are known a-priori, then the DRH should be obtainable by convolution of the rainfall input signal with the IUH response function. Convolution is the summation (integration) at a particular point in time of all the responses to prior inputs. The difference between the observed DRH and the model DRH should be negligible if the data have no noise, the system is truly linear, and we have selected both the correct function and the correct coefficients. In practice we do not expect negligible differences.

If the analyst postulates a functional form then searches for correct values of coefficients, the process is called de-convolution. In this research we accomplish de-convolution by guessing at coefficient values, convolving the effective precipitation, and compare the model output with the actual output. A merit function is used to quantify the error between the modeled and observed output, and a simple searching scheme is used to record the guesses that reduce the value of the merit function. When this search is completed the parameter set is called a non-inferior set of coefficients of the transfer function.

Conceptual Model

A unit hydrograph is the runoff hydrograph that results from one unit of excess rainfall depth uniformly distributed over the entire watershed over one unit of time. An instantaneous unit hydrograph (IUH) is the unit hydrograph produced when the excess rainfall is applied over a very short time period (an instantaneous pulse). The development of an IUH requires assumptions about how the watershed converts rainfall into runoff (a transfer function) – in this paper a simple conceptual model based on a series of connected reservoirs was used to simulate how a watershed converts rainfall into runoff. The use of a cascade of reservoirs is a well-studied conceptual model that has been used for many unit hydrograph analyses (e.g. Nash, 1958; Dooge, 1959; Dooge, 1973; Croley, 1980).

A schematic is shown on Figure 1 of a watershed conceptualized as a series of identical reservoirs without feedback. The outflow of each reservoir is assumed to be proportional to the accumulated storage in the reservoir (Equation 1).

$$Aq_{i,t} = az_{i,t} \tag{1}$$

A mass balance for a reservoir is expressed in Equation 2, where z_i represents the accumulated storage depth, a is the reservoir discharge coefficient, and q_i is the outflow for a particular reservoir (Equation 1), and the watershed area (through which the excess precipitation enters the system) is A.

$$A\dot{z}_{i,t} = Aq_{i-1,t} - az_{i,t} \tag{2}$$

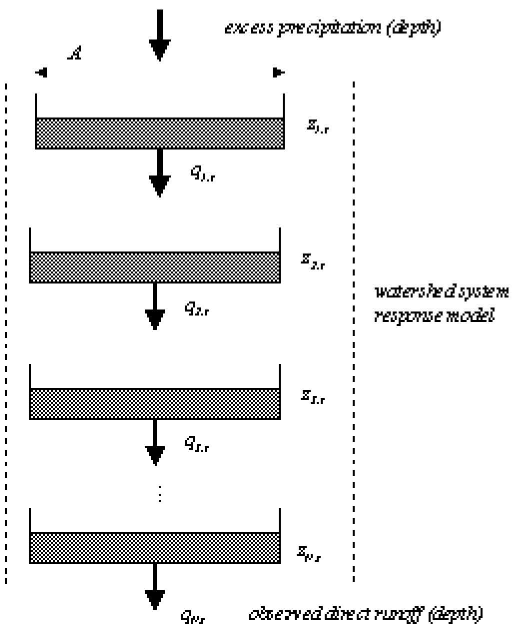

Figure 1. Watershed Conceptual Model

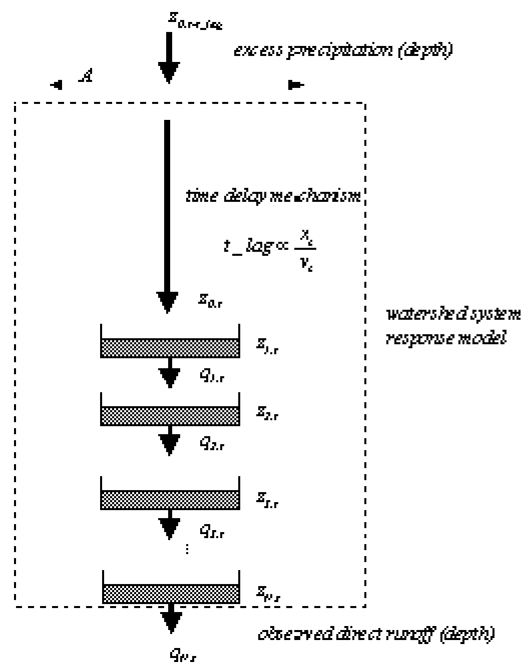

Figure 2. Watershed Conceptual Model (with Time-Delay)

In Equation 2, the first reservoir receives the initial charge of water, z_o over an infinitesimally small time interval. The ratio of discharge coefficient and watershed area will have dimensions of time, and is called the residence time (Equation 3).

$$\bar{t} = \frac{a}{A} \tag{3}$$

The entire watershed response is expressed as the system of linear ordinary differential equations.

$$
\begin{array}{llllll}
\dot{z}_1 = & z_o & -\bar{t}z_1 & & & \\
\dot{z}_2 = & & \bar{t}z_1 & -\bar{t}z_2 & & \\
\dot{z}_3 = & & & \bar{t}z_2 & -\bar{t}z_3 & \\
\vdots & & & \vdots & \vdots & \\
\dot{z}_N = & & & & \bar{t}z_{N-1} & -\bar{t}z_N
\end{array}
\tag{4}
$$

The analytical solution to this system for the last reservoir is expressed in Equation 5.

$$
q_{N,t} = z_0 \cdot \left(\frac{t}{\bar{t}}\right)^N \left(\frac{1}{(N-1)!}\right)\left(\frac{1}{t}\right)\exp(-\frac{t}{\bar{t}})
\tag{5}
$$

This result is identical to the Nash model (Nash 1958) and is incorporated into standard hydrology programs such as the COSSARR model (Rockwood et. al. 1972). The factorial can be replaced by the Gamma function and the result can be extended to a conceptual model with a non-integer number of reservoirs. To model a time-series of precipitation inputs, the individual responses are convolved and the result of the convolution is the output from the watershed. If each input is represented by the product of a rate and time interval ($z_o(t) = q_o(t)\ dt$) then the individual response is (note the Gamma function is substituted for the factorial)

$$
dq_{N,t} = q_0(t)\cdot\left(\frac{t-t}{\bar{t}}\right)^N \left(\frac{1}{\Gamma(N)}\right)\left(\frac{1}{t-t}\right)\exp(-\frac{t-t}{\bar{t}})dt
\tag{6}
$$

The accumulated responses are given by

$$
q_N(t) = \int_0^t q_0(t)\cdot\left(\frac{t-t}{\bar{t}}\right)^N \left(\frac{1}{\Gamma(N)}\right)\left(\frac{1}{t-t}\right)\exp(-\frac{t-t}{\bar{t}})dt
\tag{7}
$$

In addition to the reservoir number and residence time, it is observed in real data that there is a lag in time between the input sequence and the output sequence. The physical explanation of this lag time is to observe that the cascade model does not account for travel time between the reservoirs representing the watershed. A simple approach to account for the observed time lag is to include a time delay related to some mean travel time in the watershed. Figure 2 is a schematic including this delay in response. The linear system is unchanged except time from the input is shifted by the amount *t_lag* which is assumed to be proportional to the ratio of a characteristic length x_c and some characteristic velocity v_c.

The analytical solution for this conceptual model is

$$
q_N(t) = \int_0^t q_0(t - t\_lag)\cdot\left(\frac{t-t}{\bar{t}}\right)^N \left(\frac{1}{\Gamma(N)}\right)\left(\frac{1}{t-t}\right)\exp(-\frac{t-t}{\bar{t}})dt
\tag{9}
$$

where the input sequence in the integrand (q_0) has zero value at times smaller than or equal to zero. These two models are identical except that in Equation 8 the inputs are lagged *t_lag* units – that is if elapsed time is smaller than the lag time, the input depths are zero, otherwise the input depths are those at time (t - *t_lag)*. Thus a precipitation event at time zero will not produce an output until time *t_lag*, and so on.

Equation 9 is used to predict the watershed response to a time-series of rainfall inputs. The unknown watershed characteristics are the residence time ($\bar{t}$; *t_bar*), the reservoir number (N), and the lag time (*t_lag*). These three parameters are found by a search method for each storm, and then aggregated for each station and module for statistical evaluation.

<u>Database Development</u>

USGS small watershed studies in Texas were conducted from the 1960's to the middle 1970's. There are a significant number of individual storm events contained within these studies, but the data were not digitally available and the printed reports represented the sole data source. Figure 3 is a map illustrating the locations of the watersheds used in the study.   The obvious urban areas are displayed, and the small rural watersheds (many of which are in the urban clusters) comprise the remainder of the stations. The distances between stations are apparent from the map scale.

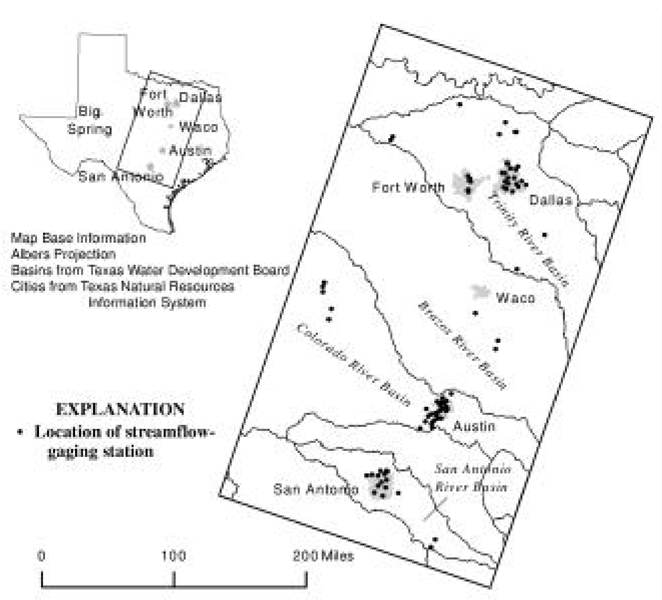

Figure 3. Map Showing Station Locations

(From Asquith, 2003)

The resulting database has about 1600 storms over the entire set of gauging stations with a minimum of two storms at each station and some stations having over 30 storms. Table 1 is a summary of the 88 stations used in this research and the storm event count per station. All the files are ASCII files so the data should be forward compatible for many years. Asquith (2003) provides further details about the database and structure.

Table 1. Stations and Number of Storms in Study

Austin			
Watershed	Sub-Shed	Station ID	#Events
BartonCreek		08155200	5
BartonCreek		08155300	8
BearCreek		08158810	8
BearCreek		08158820	2
BearCreek		08158825	2
BoggyCreek		08158050	10
BoggySouthCreek		08158880	14
BullCreek		08154700	13
LittleWalnutCreek		08158380	2
OnionCreek		08158700	6
OnionCreek		08158800	2
ShoalCreek		08156650	13
ShoalCreek		08156700	16
ShoalCreek		08156750	13
ShoalCreek		08156800	24
SlaughterCreek		08158840	9
SlaughterCreek		08158860	2
WallerCreek		08157000	40
WallerCreek		08157500	38
WalnutCreek		08158100	15
WalnutCreek		08158200	17
WalnutCreek		08158400	10
WalnutCreek		08158500	14
WalnutCreek		08158600	22
WestBouldinCreek		08155550	10
WilbargerCreek		08159150	29
WilliamsonCreek		08158920	14
WilliamsonCreek		08158930	18
WilliamsonCreek		08158970	16

Dallas			
Watershed	Sub-Shed	Station ID	#Events
AshCreek		08057320	5
BachmanBranch		08055700	41
CedarCreek		08057050	3
CoombsCreek		08057020	7
CottonWoodCreek		08057140	6
DuckCreek		08061620	8
ElamCreek		08057415	8
FiveMileCreek		08057418	7
FiveMileCreek		08057420	10
FloydBranch		08057160	8
JoesCreek		08055600	14
NewtonCreek		08057435	3
PrairieCreek		08057445	8
RushBranch		08057130	5
SouthMesquite		08061920	9
SouthMesquite		08061950	31
SpankyCreek		08057120	4
TurtleCreek		08056500	42
WoodyBranch		08057425	13
Fort Worth			
Watershed	Sub-Shed	Station ID	#Events
DryBranch		08048550	25
DryBranch		08048600	27
LittleFossil		08048820	20
LittleFossil		08048850	24
Sycamore		08048520	24
Sycamore		08048530	28
Sycamore		08048540	24
Sycamore		SSSC	21

San Antonio			
Watershed	Sub-Shed	Station ID	#Events
AlazanCreek		08178300	30
LeonCreek		08181000	10
LeonCreek		08181400	15
LeonCreek		08181450	29
OlmosCreek		08177600	12
OlmosCreek		08177700	23
OlmosCreek		08178555	10
SaladoCreek		08178600	13
SaladoCreek		08178640	10
SaladoCreek		08178645	5
SaladoCreek		08178690	39
SaladoCreek		08178736	12

SmallRuralSheds			
Watershed	Sub-Shed	Station ID	#Events
BrasosBasin	CowBayou	08096800	48
BrasosBasin	Green	08094000	28
BrasosBasin	Pond-Elm	08098300	19
BrasosBasin	Pond-Elm	08108200	21
ColoradoBasin	Deep	08139000	27
ColoradoBasin	Deep	08140000	28
ColoradoBasin	Mukewater	08136900	22
ColoradoBasin	Mukewater	08137000	38
ColoradoBasin	Mukewater	08137500	4
SanAntonioBasin	Calaveras	08182400	24
SanAntonioBasin	Escondido	08187000	31
SanAntonioBasin	Escondido	08187900	21
TrinityBasin	ElmFork	08050200	34
TrinityBasin	Honey	08057500	31
TrinityBasin	Honey	08058000	29
TrinityBasin	LittleElm	08052630	29
TrinityBasin	LittleElm	08052700	58
TrinityBasin	North	08042650	14
TrinityBasin	North	08042700	56
TrinityBasin	PinOak	08063200	33

Baseflow Separation

Many of the events in the database contained non-zero runoff values that occurred before the precipitation events, so some baseflow separation technique was indicated. The principal criterion for method selection was based on the need for a method that was simple to automate because hundreds of events needed processing. Appleby (1970) reports on a baseflow separation technique based on a Ricatti-type equation for baseflow. The general solution of the baseflow equation is a rational functional that is remarkably similar in structure to either a LaPlace transform or Fourier transform. Unfortunately Appleby's paper omits the detail required to actually infer an algorithm from the solution, but it is useful in that principles of signal processing are clearly indicated in the model.

Nathan and McMahon (1990) examined automated baseflow separation techniques. The objective of their work was to identify appropriate techniques for determination of baseflow and recession constants for use in regional prediction equations. Two techniques they studied in detail were a smoothed minima technique and a recursive digital filter (a signal processing technique similar to Appleby (1970)). Both techniques were compared to a graphical technique that extends pre-event runoff (just before the rising portion of the hydrograph) with the point of greatest curvature on the recession limb (a constant-slope method, but not aimed at the inflection point). They concluded that the digital filter was a fast objective method of separation but their paper suggests that the smoothed minima technique is for all practical purposes indistinguishable from either the digital filter or the graphical method. Furthermore the authors were vague on the constraint techniques employed to make the recursive filter produce non-negative flow values and to produce peak values that did not exceed the original streamflow. Press et.al. (1989) provide convincing arguments against time-domain signal filtering and especially recursive filters. Nevertheless the result for the smoothed minima is still meaningful, and this technique appears fairly straightforward to automate, but it is intended for relatively continuous discharge time series and not the episodic data in the present application.

The constant slope and concave methods (McCuen, 1998) are not used in this work because the observed runoff hydrographs have multiple peaks and it is impractical to locate the recession limb inflection point with any confidence. The master depletion curve method (McCuen, 1998) is not used because even though there is a large amount of data, there is insufficient data at each station to construct reliable depletion curves, and the time scale is inadequate. Recursive filtering and smoothed minima was dismissed because of the type of events in the present work (episodic and not continuous). Therefore in the present work the constant discharge method was chosen because it is simple to automate and apply to multiple peaked hydrographs.

Prior researchers (e.g. Laurenson and O'Donell, 1969; Bates and Davies, 1988) have reported that unit hydrograph derivation is insensitive to baseflow separation method when the baseflow is not a large fraction of the flood hydrograph – a situation satisfied in this work. The particular implementation in this research determined when the rainfall event began on a particular day, all discharge before that time was accumulated and converted into an average rate. This average rate was then removed from the observed discharge data, and the result was considered to be the direct runoff hydrograph.

Effective Precipitation

The effective precipitation is the fraction of actual precipitation that appears as direct runoff (after baseflow separation). Typically the precipitation signal (the hyetograph) is separated into three parts, the initial abstraction, the losses, and the effective precipitation. Initial abstraction is the fraction of rainfall that occurs before direct runoff. Operationally several methods are used to estimate the initial abstraction. One method is to simply censor precipitation that occurs before direct runoff is observed. A second method is to assume that the initial abstraction is some constant volume. The NRCS method assumes that the initial abstraction is some fraction of the maximum retention that varies with soil and land use. Losses after initial abstraction are the fraction of precipitation that is stored in the watershed (depression, interception, soil storage) that does not appear in the direct runoff hydrograph. Typically depression and interception storage are considered part of the initial abstraction, so the loss term essentially represents infiltration into the soil in the watershed. Several methods to estimate the losses include: Phi-index method, Constant fraction method, and infiltration capacity approaches (Horton's curve, Green-Ampt model).

For the present effort a combined censoring and constant fraction method was selected, again because of ease of automation. Pre-runoff precipitation is removed, then the remaining cumulative precipitation depth and cumulative runoff depth are computed. The loss function is derived from the precipitation rate by multiplication of the rate and the ratio of cumulative runoff depth and cumulative precipitation depth.

De-convolution/Parameter Estimation

The IUH parameters are estimated by simulating the DRH from the effective rainfall signal and adjusting values until some merit function is minimized. The functions considered are the classic sum of squared errors (SSE), the root mean squared error (RMSE), and the maximum absolute deviation (MAD). Mathematically these merit functions are

$$SSE = \sum_{i=1}^{N} (Q_S - Q_O)_i^2 \tag{10}$$

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (Q_S - Q_O)_i^2} \tag{11}$$

$$MAD = \max \left| Q_S - Q_O \right|_i \tag{12}$$

where Q is the discharge (L^3/T); the subscripts $_O$ and $_S$ represent observed and simulated discharge, respectively, and *N* is the total number of values in a particular storm event. The search procedure used initial estimates determined by graphing a single storm at each station and guessing a reasonable value for *t_bar*, *t_lag*, and *N=1*.

A search routine that systematically adjusted these guesses by increments of 1.0 was employed. The algorithm was programmed to continue adjustment(s) as long as an adjustment improved the merit function. When no further improvement could be detected, the algorithm then randomly selected 100 adjustments from a uniform distribution centered on the last best guess as a check that a local minimum was not stopping the processing. If the program could still not improve the merit function, then processing for that storm stopped, and the algorithm moved onto the next storm. In the present paper, only the SSE results are presented, the analysis using the other functions is not complete.

Results

Typical results of the analysis are presented on Figure 4, which is a plot of the instantaneous (rate) precipitation and runoff as well as the cumulative effective precipitation and runoff. The dashed lines represent the instantaneous values, while the solid lines are cumulative values. The solid black line is the result of convolving the instantaneous effective precipitation using the IUH model (Equation 9) with the parameter values shown on the chart (*t_bar = 49*, *t_lag = 34*, *N = 1*).

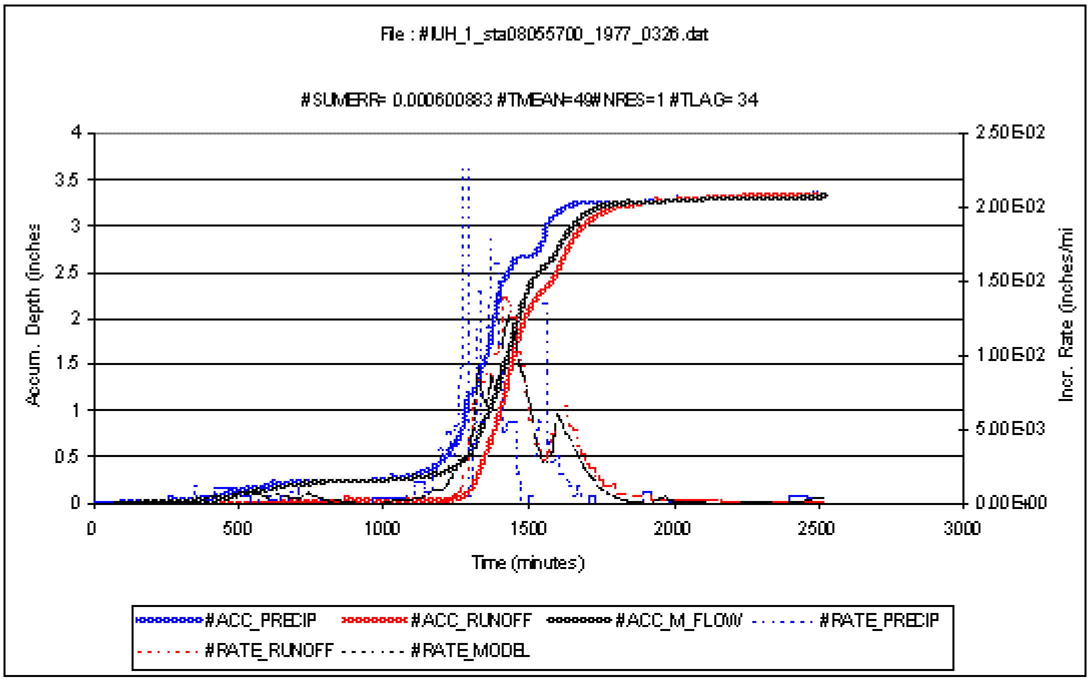

Figure 4. Typical IUH Results

For the particular storm in Figure 4, the total precipitation was 5.62 inches, of which 3.34 inches was effective precipitation; the remainder was treated as rainfall loss. Thus in this particular example the runoff coefficient (r_c) (ratio of cumulative runoff to cumulative precipitation) was nearly 0.6 (relatively high for the Dallas module). The analysis of each storm produces a different result and the promise of the IUH is that some aggregate measure (mean, median, etc.) of these parameters can be used for a station or module, and that this aggregate value can be estimated from some geometric watershed properties. This step towards synthesis is beyond the scope of the present report, but is the ultimate goal of the research.

Table 2 lists the aggregate IUH values for each module in the study. The arithmetic mean values were subjected to two-sample t-tests of their differences (e.g. are the differences in the values in the table explainable by chance?). The details are omitted for brevity, but the results are that the reservoir number is essentially the same for all modules, *t_bar* for Dallas, Fort Worth, and San Antonio are the same, while those for Austin and Small Rural are different from the rest of the modules (and each other), *t_lag* for Austin, Dallas, and San Antonio are the same, and the runoff coefficients for Austin, San Antonio and Small Rural are the same.

Table 2. IUH Statistics from Central Texas Analysis

Column (1) is the Module Name (All modules is the aggregate of the entire data base); Column (2), T_bar is $\bar{t}$ in Equation 9; Column (3), N_res is N in Equation 9; Column (4), T_lag is *t_lag* in Equation 9; Column (5), RawAccRain is the mean or median cumulative rainfall before losses; Column (6), RawAccFlow is the mean or median cumulative runoff before baseflow separation; Column (7), RainFrac is the mean/median of runoff coefficient (approx. Col.(6)/Col.(5)); Column (8) ,Num_Obs is the number of storms analyzed in given module.

Arithmetic Mean Values

(1)	(2) T_bar (min)	(3) N_Res	(4) T_lag (min)	(5) RawAccRain (inches)	(6) RawAccFlow (inches)	(7) RainFrac	(8) Num_Obs
All Modules	185.01	1.14	78.81	2.62	0.92	0.32	1624
Austin	157.45	1.09	64.90	3.03	0.96	0.26	398
Dallas	96.94	1.05	65.62	2.86	1.50	0.52	239
FortWorth	104.54	1.29	34.16	2.18	0.90	0.39	192
SanAntonio	84.74	1.38	61.95	2.65	0.74	0.27	207
SmallRuralSheds	301.03	1.08	114.10	2.39	0.73	0.28	588

Median Values

(1)	(2) T_bar (min)	(3) N_Res	(4) T_lag (min)	(5) RawAccRain (inches)	(6) RawAccFlow (inches)	(7) RainFrac	(8) Num_Obs
All Modules	84.00	1.00	42.00	2.26	0.58	0.28	1624
Austin	79.00	1.00	38.00	2.50	0.54	0.23	398
Dallas	52.00	1.00	43.00	2.50	1.18	0.50	239
FortWorth	66.00	1.00	17.00	1.81	0.64	0.36	192
SanAntonio	45.00	1.00	33.00	2.26	0.48	0.25	207
SmallRuralSheds	160.00	1.00	61.00	2.05	0.38	0.20	588

Equation 9 (Repeated)

$$q_N(t) = \int_0^t q_0(t - t\_lag) \cdot \left(\frac{t-t}{\bar{t}}\right)^N \left(\frac{1}{\Gamma(N)}\right)\left(\frac{1}{t-t}\right) \exp\left(-\frac{t-t}{\bar{t}}\right) dt$$

These findings imply that the IUH model presented here is unlikely to be able to be applied to all of central Texas without some geographical adjustment – much like the NRCS methods require. Also of some importance is that the runoff coefficient for Dallas and Fort Worth are larger than the other modules, and different from the other modules (and each other), yet the cumulative average and median rainfall amounts are about the same. This result suggests that the rainfall-runoff behavior is fundamentally different for these two locations as compared to the rest of the locations in the database.

A qualitative assessment of IUH performance is made by using an actual storm, and using the aggregate IUH for the module, for all modules, and the NRCS DUH (with some modification) to produce a predicted DRH. The NRCS DUH example in this paper is simply a Gamma-based IUH that was forced to have the shape as the NRCS DUH, the timing parameter is obtained by trial-and-error for this paper. These DRHs are then visually compared to the observed hydrograph. The results for the particular storm in Figure 4 are displayed in the following figures.

Figure 5 is the same storm convolved using Equation 9, but with different values of the IUH constants. The values are those listed in Table 2 for arithmetic mean IUH for the Dallas module. Equation 13 illustrates the insertion of parameters into Equation 9.

$$q_N(t) = \int_0^t q_0(t-66)\cdot\left(\frac{t-t}{96.94}\right)^{1.05}\left(\frac{1}{\Gamma(1.05)}\right)\left(\frac{1}{t-t}\right)\exp(-\frac{t-t}{96.94})dt \tag{13}$$

In Equation 13, $q_o(t)$ is the effective rainfall time series obtained by multiplication of the original rainfall time series and the runoff coefficient for the module. The greater lag time moves the model peak to the right (as compared to Figure 4) and the larger residence time lowers the value of the peak.

Figure 6 is also the same storm again convolved using Equation 9, but with the values in Table 2 for median IUH for the Dallas module. Equation 14 shows the "parameterization" used in constructing Figure 6.

$$q_N(t) = \int_0^t q_0(t-43)\cdot\left(\frac{t-t}{52}\right)^{1.05}\left(\frac{1}{\Gamma(1.00)}\right)\left(\frac{1}{t-t}\right)\exp(-\frac{t-t}{52})dt \tag{14}$$

In this case the result is not much different than the "non-inferior" result.

Figure 7 is again the same storm, but the timing parameters are taken from the scatter plots below. In essence, Figure 7 represents an attempt at a synthetic IUH based on the watershed area as predicted by a power-law fit to the cloud of points in the scatter plots. Surprisingly, the result is not horrible – in fact it is close to Figure 5 results (because a regression strongly depends on arithmetic means this result is not entirely unexpected). The parameterization used was (r_c*=0.5,t_bar = 119,t_lag = 42, N = 1*).

Figure 8 is yet the same storm again, only in this Figure the reservoir number is much higher and was determined by fitting Equation 9 so that it produced a UH with the same shape as the NRCS DUH. The only parameter remaining in this case is the timing parameter that was estimated by trial and error. The parameterization used was (r_c*=0.5,t_bar = 15,t_lag = 0, N = 5*). It is remarkable that this particular example agrees nicely with the median IUH model, but it also illustrates the non-unique nature of the IUH approach outlined in this paper. Further analysis with the NRCS DUH is in-progress.

Table 3 is a list of the relative error in predicting the peak discharge in all these examples. The values reported are the difference between the model and the observed peak discharge rate (in/min), normalized by the observed peak discharge rate. The storm-specific value is expected to be closest, but remarkably the median IUH and the NRCS-DUH approximation do nearly as well.

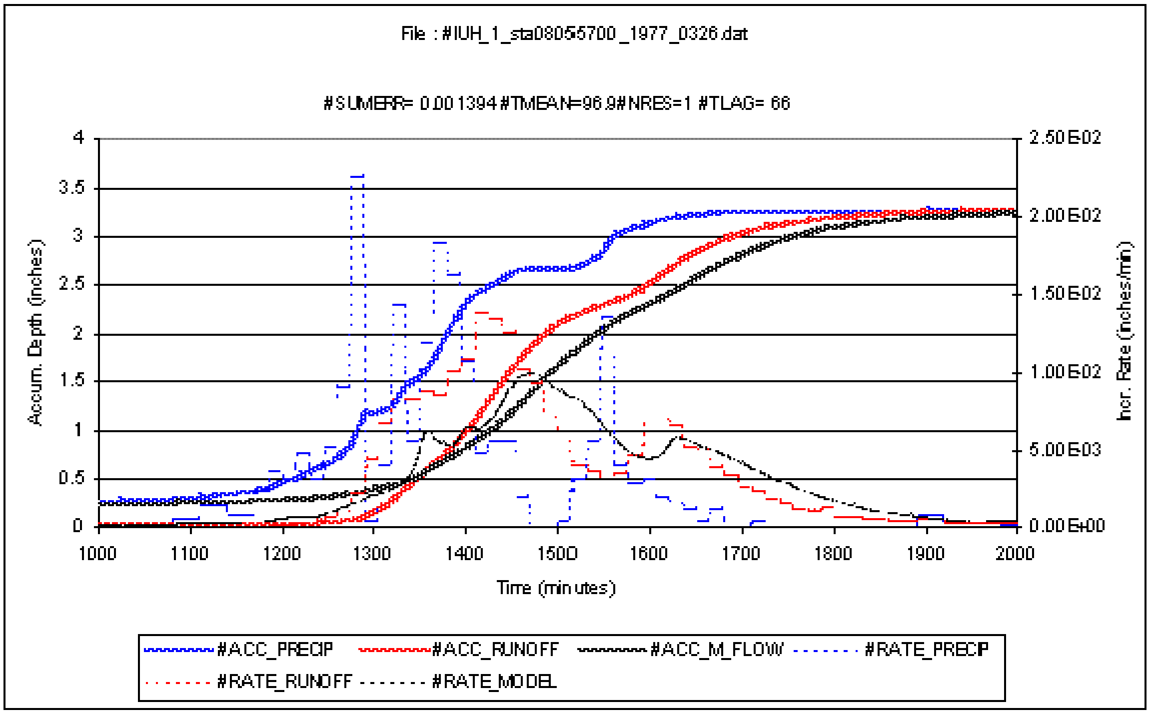

Figure 5. Dallas Module Arithmetic Mean IUH (Table 2).

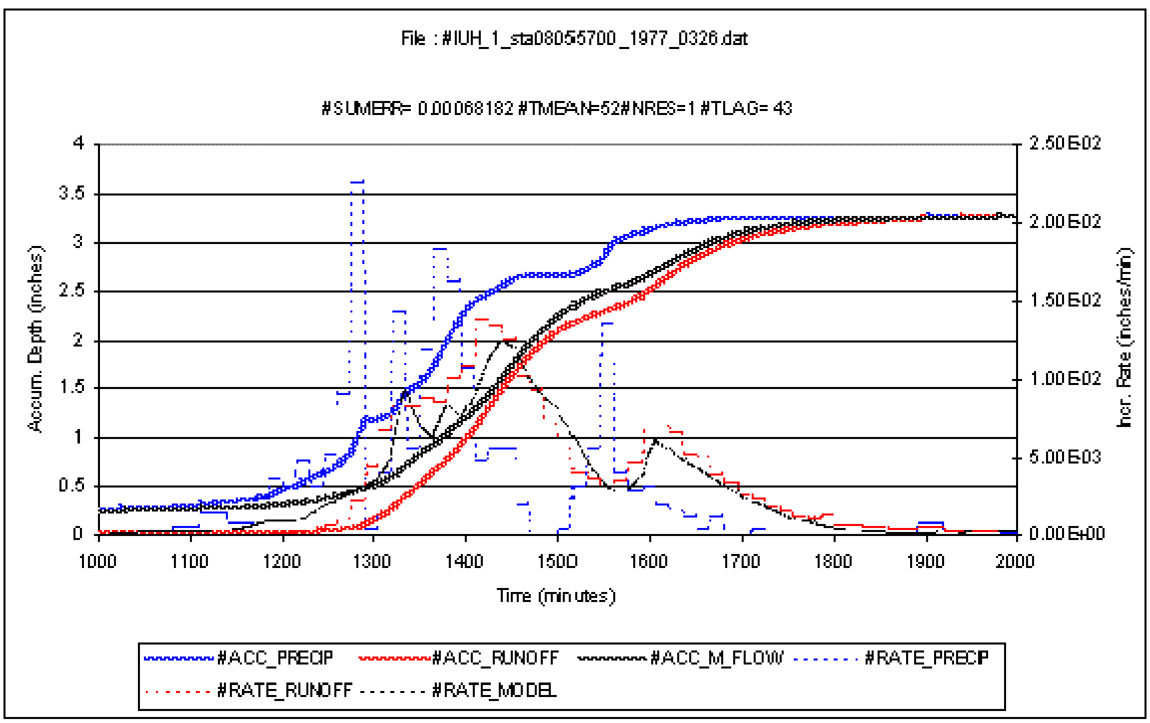

Figure 6. Dallas Module Median IUH (Table 2).

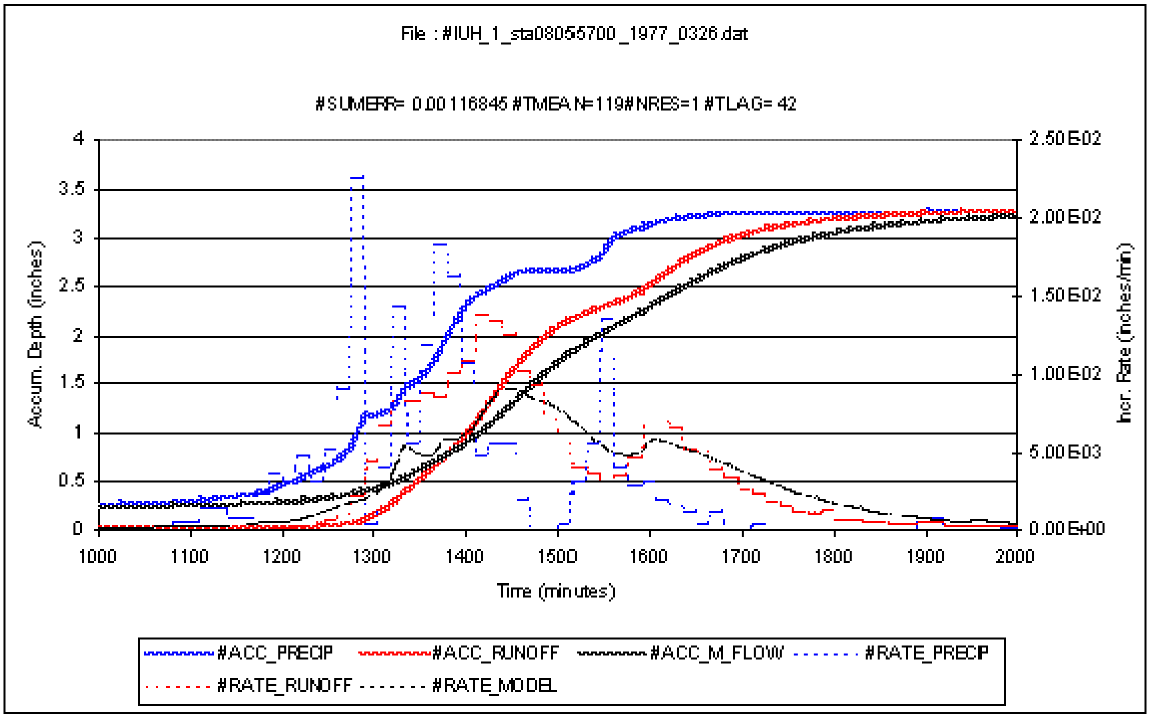

Figure 7. Area Correlation IUH

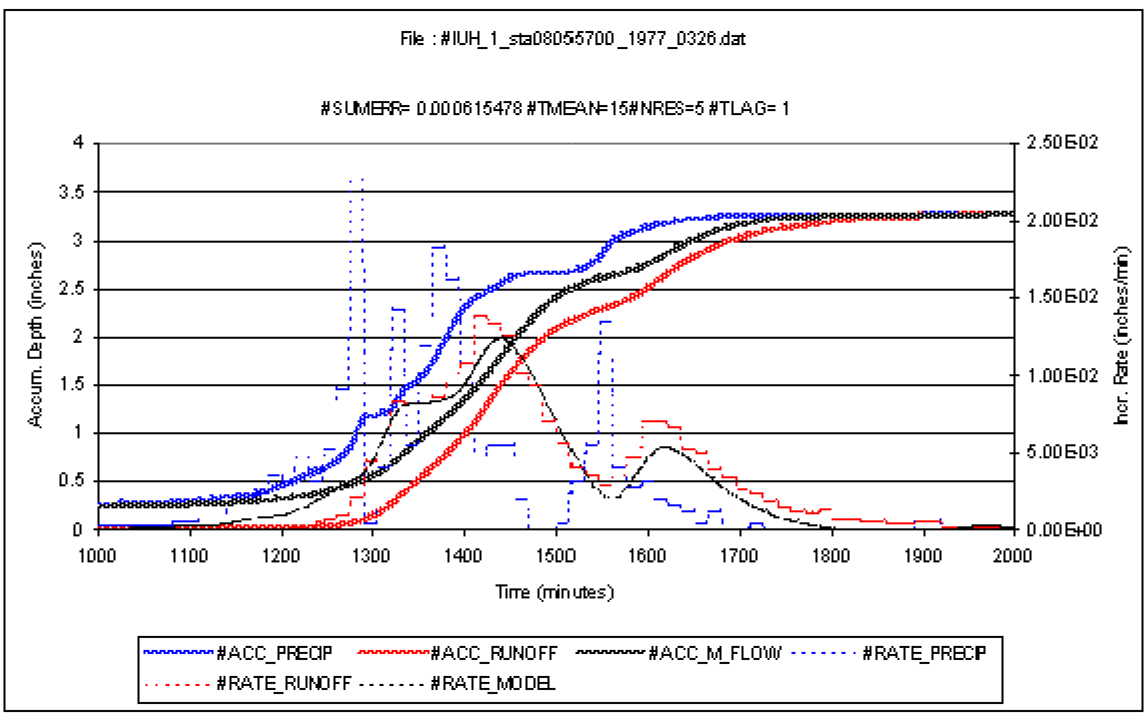

Figure 8. NRCS Same Shape IUH

Table 3. Peak Discharge Prediction Error

Storm Specific IUH (Figure 4)	Module Mean IUH (Figure 5)	Module Median IUH (Figure 6)	Area-Correlation IUH (Figure 7)	NRCS-DUH (Shape Fit) (Figure 8)
-8.8%	-27.9%	-10.4%	-33.9%	-10.21%

The qualitative performance values illustrated in Table 3 was typical for the entire database. All the IUH models under predict the peak discharge. Additional hypothesis tests are being conducted to address the following research questions: Are the mean/median values in each station/module for each IUH parameter different? Is the median estimated volume of discharge biased (lower or higher) than the observed volume of discharge? Is the estimated peak discharge biased (lower or higher) that the observed peak discharges? Is the estimated peak arrival time biased relative to the observed peak arrival time?

Correlation to Geometric Properties

Scatter plots of IUH parameter values as a function of watershed area, without regard to location are presented as a qualitative assessment of the potential to develop synthetic IUH models as functions of watershed properties. As an example, the DRH in Figure 7 was produced by reading *t-lag* and *t_bar* from the middle of the "scatter" in the two figures below at the 10.0 mi.^{2} area (the area of the station in the example). The IUH from this approach performed poorly for the example, but not hopelessly (it is about the same as using the arithmetic mean values for all the Dallas data).

Figure 9 is a scatter plot of *t_bar* versus watershed area for the 88 stations. Each station has a single drainage area, but there are as many timing values as there are storms for the station, thus the vertical dimension of the cloud of data represents quite a large number of duplicate points. The diagonal line is a 1:1 proportionality line. If data were along this line, then it would mean that the parameter value scales proportional to area (e.g. 1 min per 1 mi^2.), however the data cloud approaches the guide line as area increases which indicates that the scaling is not 1:1.

Figure 10 is a scatter plot of t-lag versus watershed area for the 88 stations. The two scatter plots suggest that there is a correlation between watershed area and the significant timing parameters, but the correlation is extremely weak at best. Generally, these scatter plots essentially show that as area increases, the timing value increases, but not at the same rate. This result makes some sense when one considers that characteristic times should be proportional to a length characteristic, while the area is proportional to the square of that characteristic (there might be geometries where the relationship is 1:1, but these are likely to be the exception).

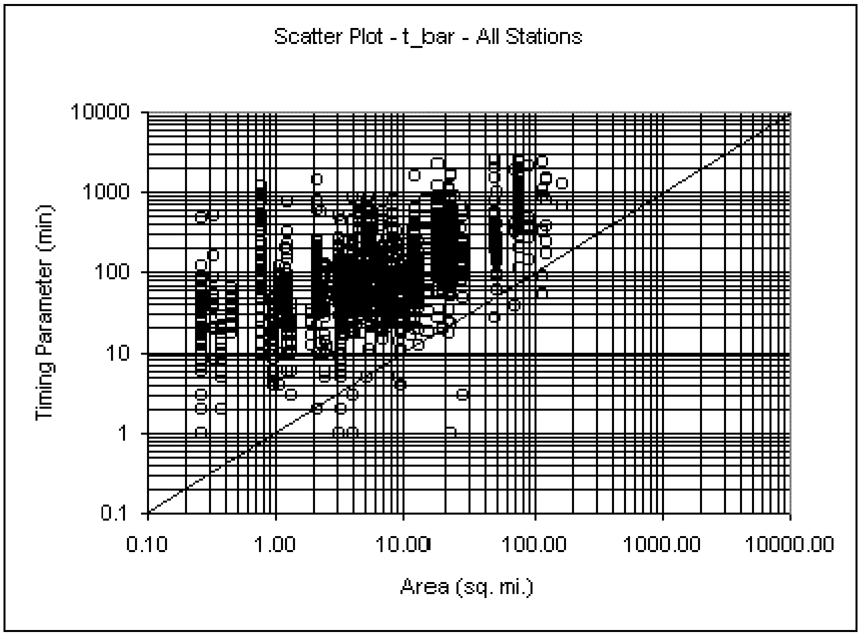

Figure 9. Residence Time - Area Scatter Plot (Log-Log Scale)

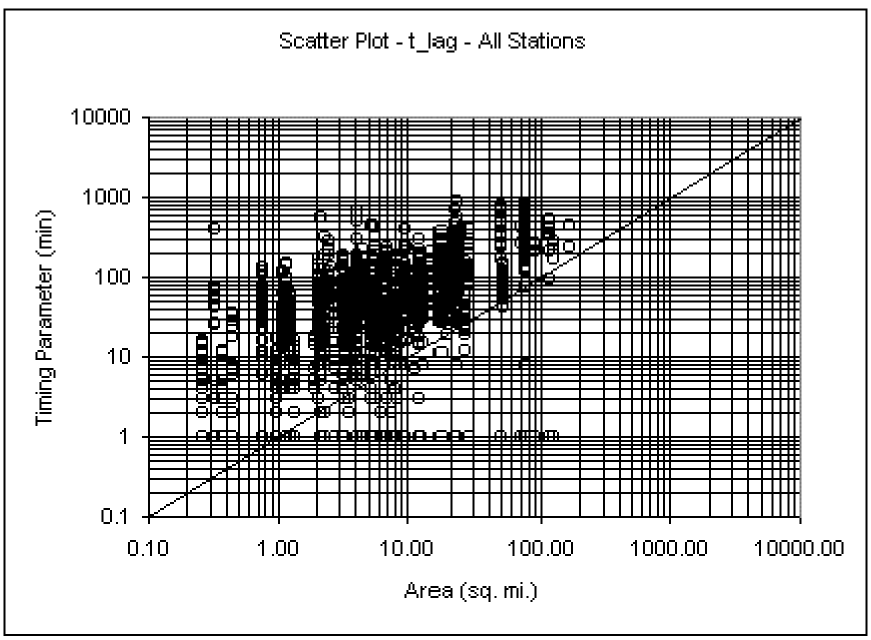

Figure 10. Time Lag - Area Scatter Plot (Log-Log Scale).

Other geometric measures are being investigated as better predictors of the IUH constants for a watershed. It is postulated that a factor-type predictor equation can be developed that can relate simple watershed measures to the IUH constants.

Conclusions

This research illustrates that IUH-type models are feasible for rainfall-runoff modeling of watersheds in Texas in the size range of 0.1 to ~100 mi^2. A simplistic IUH model based on a cascade of reservoirs can match peak discharge rates to within 15% of observed values, and match the arrival time of the peak within an hour or so. The comparison of the IUH method to NRCS methods is incomplete, but an IUH selected to have the same shape as the NRCS DUH also has the ability to produce a model output reasonably close to observed values (at least for the one case shown here). There is some systematic bias in the approach that needs resolution or at least explanation. A crude relationship between area and timing parameters was demonstrated to have some value in predicting runoff with a peak discharge error on the same order of magnitude as using an arithmetic mean IUH from the Dallas module.

On-going work will resolve the systematic bias issue, and explore the use of geometric measurements of watersheds as a predictive tool for the IUH parameters that in-turn are used to produce the DRH from a rainfall time series. Additional work on baseflow separation and other rainfall-loss models (also predictable a-priori) are in progress. Even though significant efforts are required to refine the approach presented, much of the up-front work is now in-place and rapid progress is expected. Combined with the hyetograph efforts of other members of the research team, this tool could provide an alternative for NRCS methods in some circumstances.

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