Synthesis of Unit Hydrographs from a Digital Elevation Model

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Abstract

Characterization of hydrologic processes of a watershed requires estimation of the specific time-response characteristics of the watershed. In the absence of observations these characteristics are estimated from watershed physical characteristics. An exploratory assessment of a particle-tracking approach for parameterizing unit hydrographs from topographic information for applicable Texas watersheds is presented.

The study examined 126 watersheds in Texas, for which rainfall and runoff data were available with drainage areas ranging approximately from 0.65 to 388 square kilometers, main channel lengths ranging approximately from 1.1 to 80 kilometerss, and dimensionless main channel slopes ranging approximately from 0.0002 to 0.02. Unit hydrographs based on entirely on topographic information were generated and used to simulate direct runoff hydrographs from observed rainfall events. These simulated results are compared to observed results to assess method performance. Unit hydrographs were also generated by a conventional analysis (of the observed data) approach to provide additional performance comparison.

The results demonstrate that the procedure is a reasonable approach to estimate unit hydrograph parameters from a relatively minimal description of watershed properties, in this case elevation and a binary development classification. The method produced unit hydrographs comparable to those determined by conventional analysis as thus is a useful synthetic hydrograph approach.

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Introduction

The unit hydrograph (UH) is a model to predict the streamflow hydrograph from a rainfall hyetograph at the outlet of a basin. It can be expressed as

$$q(t) = \int_0^T r(\tau) f(t-\tau) d\tau \tag{1}$$

where where $q(t)$ is unit discharge from a basin at time t, $r(t)$ is an input function that represents either rainfall or excess rainfall, $f(t-\tau)$ is a response function (the unit hydrograph), and T is the duration of the input. Equation 1 assumes that basins respond as linear systems and this assumption is the main criticism of unit hydrograph theory. Despite this criticism, unit hydrographs are used to estimate streamflow from relatively small basins, typically for engineering purposes and often produce reasonable results. With the linearity assumption, the response, $f(t-\tau)$, has the same properties as a probability density function specifically, it integrates to unity on the range $(-\infty, \infty)$, and $f(t-\tau) \geq 0$ for any values of $(t-\tau)$.

Traditionally as suggested by Sherman (1932) and explained in many references, the UH of a watershed is derived from observed runoff and rainfall records. For ungaged watersheds, such data are unavailable, and synthetic methods are used to infer the unit hydrograph. These methods vary in how the geomorphic information from the watershed is incorporated to produce estimates of the unit hydrograph.

Clark (1945) developed a method for generating unit hydrographs for a watershed based on routing a time-area relationship through a linear reservoir. Excess rainfall covering a watershed to some unit depth is released instantly and allowed to traverse the watershed and the time-area relation represents the translation hydrograph. The time-area relationships are usually inferred from a topographic map. The linear reservoir is added to reflect storage effects of the watershed. Clark's method clearly attempts to relate geomorphic properties to watershed response.

Leinhard (1964) derived a unit hydrograph model using a statistical-mechanical analogy and two important assumptions. The first is that the travel time taken by an excess raindrop landing on the watershed to the outlet is proportional to the path-line distance the raindrop must travel. The second assumption is that the area swept by any characteristic distance is proportional to some power of that characteristic distance. Dimensionally, the ratio of the travel time to path length would be a characteristic velocity. Lienhard's derivation did not attempt to relate watershed properties that might appear on a map to the hydrologic response, but the connection was implied.

Rodriguez-Iturbe and Valdez (1979) and Gupta and others (1980) examined the structure of unit hydrographs conceptualized as residence time distributions from a geomorphic perspective and provided guidance to parameterize the hydrographs in terms of Horton's bifurcation ratio, stream length ratio, and stream area ratio and an independently specified basin lag time. In these works the result was called a geomorphic unit hydrograph (GUH). Like Leinhard's derivation the relationships of path, path length, and travel time are fundamental in the development of the unit hydrographs. Furthermore, all these derivations rely on the concept of representing the excess rainfall as an ensemble of particles distributed on the watershed.

Jin (1992) developed a GUH based on a gamma-distribution and suggested a way to parameterize the distribution based on path types and a streamflow velocity. Like the prior work, the concept of distance, velocity and time was crucial. In Jin's GUH the initial estimate of velocity was based on a peak observed discharge for a basin, thus some kind of streamflow record was required, or some estimate of bankflow discharge would be required.

Maidment (1993) developed a GIS-based approach using the classical time-area method and GIS scripts. Muzik (1996) approached the time-area modeling in a similar fashion. These works used flow routing based on a constant velocity or subjectively predetermined velocity map independently incorporating concepts of a GUH.

Kull and Feldman (1998) assumed that travel time for each cell in the watershed was simply proportional to the time of concentration scaled by the ratio of travel length of the cell over the maximum travel length. Thus the velocity from any point to the outlet is uniform and constant. Each cells excess rainfall is lagged to the outlet based on the travel distance from the cell. Travel time in overland and channel flow are determined beforehand. This approach is essentially a version of Clarks (1945) methodology and is implemented in HEC-GEOHMS (HEC 2000).

Saghafian and Julien (1995) derived a GIS-based time-to-equilibrium approach for any location on a watershed based on a uniform overland flow model, that incorporated elevation information. Saghafian and others (2002) used this concept to develop a time-variable isochrone GIS technique to generate runoff hydrographs for non-uniform hyetographs (non-uniform in space and time).

Olivera and Maidment (1999) developed a raster-based, spatially distributed routing technique based on a first-passage-time response function (a gamma-type unit hydrograph at the cell scale).

Lee and Yen (1997) recognized that a kinematic-wave model could be used to estimate travel times over a path to the watershed outlet and developed a procedure to parameterize a GUH by relating slope along a set of planes defined by stream order that are linked to each other an the watershed outlet.

Additionally numerous methods in the literature relate properties such as selected lengths, slopes, fraction of sewer served areas, etc. to unit hydrograph timing parameters. A selection of the more common methods appears in Rousseau and others (2005). The methods in the Rousseau report generally use a handful of measures to estimate the timing parameters and were developed prior to common availability of digital elevation data.

The significance of all these studies is that the concepts of distance, velocity and time need to be linked to physical characteristics of the watershed to parameterize a unit hydrograph in the absence of observed runoff and rainfall data. Additionally, the GIS studies appeared to have evolved in parallel to the GUH theory incorporating similar ideas while implicitly parameterizing the underlying GUH by various methods. Shamseldin and Nash (1998) argue that GUH theory is equivalent to the assumption of a generalized UH equation described by a distribution whose parameters must subsequently be related by regression (or otherwise) to appropriate catchment characteristics.

This paper presents the results of a hybrid approach to estimate the parameters of a GUH by analysis of an arrival time distribution of rainfall particles, whose travel speeds and paths are determined by local watershed slope. A particle-tracking program was used to generate the arrival time distribution, and 30-meter digital elevation model (DEM) data were used to compute local slopes and travel paths. A UH equation was then fit to the arrival time distribution to extract a timing parameter and a shape parameter, unique to each watershed; an approach similar to Shamsheldin and Nash's (1998) argument.

The study examined 126 watersheds in Texas, for which rainfall and runoff data were available. For the selected watersheds, the drainage areas range approximately from 0.25 to 150 square miles, main channel lengths range approximately from 0.7 to 49 miles, and dimensionless main channel slopes range approximately from 0.0002 to 0.02. Because a rainfall-runoff database exists for the study watersheds, the resulting unit hydrographs can be used to generate runoff hydrographs for the historical rainfalls and these modeled hydrographs compared to the observed hydographs to evaluate the performance of the particle tracking approach. UH were also generated by a conventional analysis (of the observed data) approach to provide a performance comparison.

Rainfall-Runoff Database

A digital database of rainfall and runoff values for over 1,600 storms from 126 developed and undeveloped watersheds in Texas was used for the research. A portion of the database is described and tabulated in Asquith and others (2004), and additional 33 watersheds in the Houston area supplements the Asquith database. A watershed properties database was developed from 30-meter digital elevation models. The watershed properties database is described in Roussel and others (2005), it too is supplemented with properties from the 33 Houston area watersheds. Figure 1 is a map of the study watershed locations that illustrates the spatial distribution of the study.

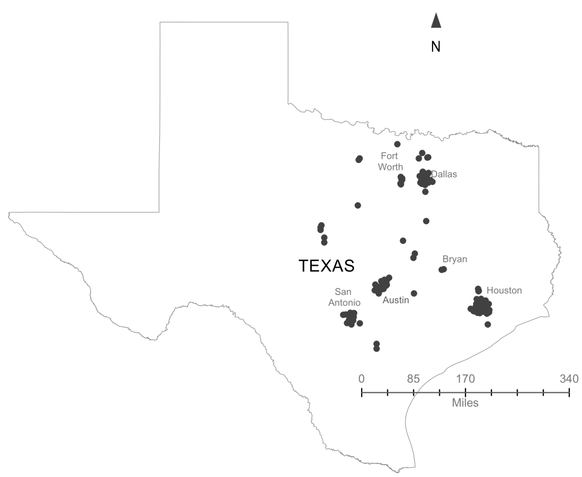

Figure 1: Map of study watersheds

Methodology

Generating an excess rainfall arrival time distribution at the watershed outlet was addressed by placing a computational particle on each cell of a DEM grid, computing the direction this particle would move from an 8-cell pour point model (OCalligan and Mark, 1984), and computing the speed of the particle according to a uniform flow equation whose velocity term is determined by the slope along the particle path at the particle's current position. A short interval of time is allowed to pass, and the particle's new position is calculated and the entire computational process is repeated.

Over the short time interval, the particle will move a distance along its pathline determined by the product of the appropriate characteristic velocity and the time interval. Figure 2 illustrates the relationship between cartesian and pathline coordinates. This work assumed the square of velocity is proportional to watershed slope at any location, and therefore the velocity field depends on the particle positions.

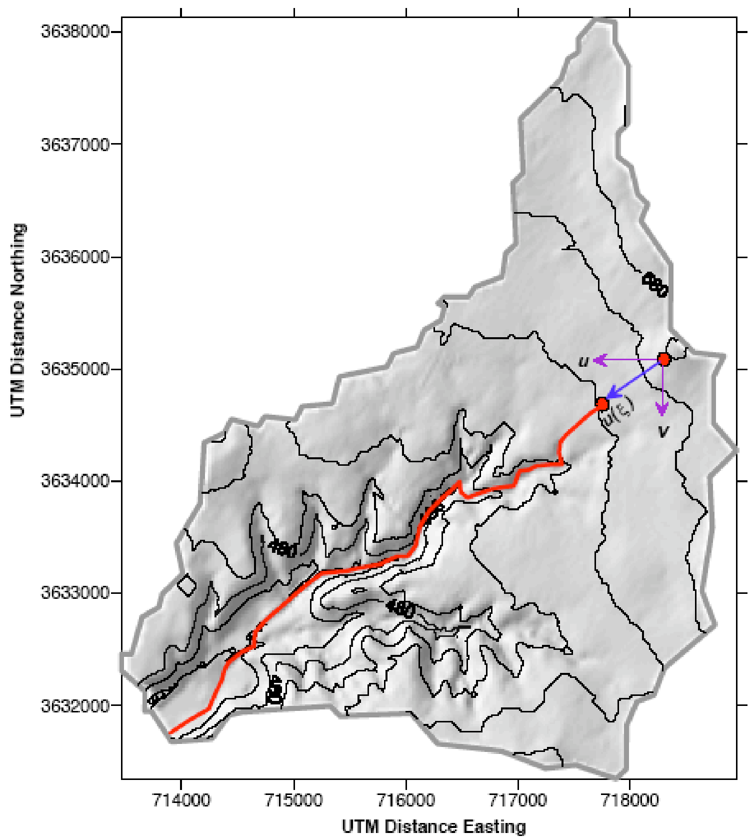

Figure 2: Shaded relief map of watershed associated with USGS gaging station 08057320. A particle pathline, pathline and Cartesian velocities are depicted for a single runoff particle.

Equation 2 represents the formula in a path line coordinate system used to determine the velocity at any location in the watershed.

$$u(\xi)|u(\xi)| = k^2 \frac{\partial z}{\partial \xi} \tag{2}$$

The value u is the velocity of the particle along the path, ξ is distance or location on the particle's flow path, z is the watershed elevation at the current particle position, and k^2 represents the square of velocity of the particle on a unit slope. The absolute value formulation is used so that the numerical method preserves correct directional information (flow is always downslope). This approach is similar to existing methods, but makes no distinction between channel and overland flow. All results presented in this paper are based on this velocity model.

In the present work we have adopted the following structure for k

$$k = \frac{1.5}{n_f} d^{\frac{2}{3}} \tag{3}$$

where n_f is a frictional term (an adjustable parameter) that is conceptually analogous but not numerically equal to Manning's n, and d is a mean flow depth (an adjustable parameter). This particular structure is selected to make the procedure look like Manning's type physics is incorporated. The resulting particle kinematics are analogs to Woodings (1965) kinematic wave analysis for overland flow and similar to the isochrone derivation technique of Sagafian and Julien (1995) who adapted the kinematic wave theory for distributed rainfall-runoff modeling and presented an example (Saghafian and others, 2002) for a watershed in West Africa. Other than a desire to illustrate that the k term could be made look like a Manning's equation, the writer's prefer the concept of unit-slope characteristic velocity

The applicability of the velocity model is subject to an important consideration regarding the backwater effect from downstream. In this work we have implicitly assumed that there is no backwater effect, but the Houston watersheds are known to have backwater effects at the gaging stations as well as tidal influence. Additionally the Houston data have slopes one order of magnitude smaller than the remaining watersheds and the applicability of a kinematic-wave type flow is questionable.

Figure 2 displays a single path for clarity. On the illustrated watershed, using a 30-meter resolution DEM, 20,639 paths were identified (one for each grid cell on the approximately 7 square mile watershed) that drain the the outlet located in the lower left corner of the figure. On some of the larger watersheds, over 500,000 paths were identified. Each path is defined by an individual particle's starting point, and each particle follows its own unique path. Equation 3 is evaluated at least once for each path, and multiple times for paths that traverse long distances across the watershed as particles move down-slope towards the outlet. The entire ensemble of particles is moved contemporaneously and the arrival times of individual particles at the watershed outlet are recorded. The cumulative arrival time distribution of the particle ensemble is the residence time distribution of excess rainfall on the watershed and contains information equivalent to an S-curve hydrograph. Alternatively, one could compute the total travel time along each path and rank order these arrival times to construct the arrival time distribution. By fitting a unit hydrograph model to this empirical S-curve, unit hydrograph parameters are recovered. Figure 3 is one such cumulative arrival time distribution for the Ash Creek Watershed in Dallas, TX.

The computational burden is extreme, even though the approach as presented is highly parallel (the particles do not interact). A purpose-built cluster computer (Cleveland, 2004) was used to speed the computational throughput, by distributing the particle position computations among multiple processors. Despite taking advantage of the parallel structure of the problem, it still takes considerable time to complete the description of even a single watershed.

The unit hydrograph model selected for this research is a generalized gamma distribution (Leinhard, 1964; 1967) and is expressed as

$$f(t) = \frac{\beta}{\Gamma(n/\beta)} \left(\frac{n}{\beta}\right)^{n/\beta} \frac{1}{t_{rm\beta}} \left(\frac{t}{t_{rm\beta}}\right)^{n-1} \exp\left[-\frac{n}{\beta}\left(\frac{t}{t_{rm\beta}}\right)^{\beta}\right] \tag{4}$$

The distribution parameters n and $t_{rm\beta}$ have physical significance in that $t_{rm\beta}$, is a mean residence time of an excess raindrop on the watershed, and n, is an accessibility number, roughly proportional to the exponent on the distance-area relationship (a shape parameter). β, is the degree of the moment of the residence time; $\beta = 1$ would be an arithmetic mean, while for $\beta = 2$ the residence time is a root-mean-square time. $\beta = 2$ is used throughout this work, in part to be faithful to Leinhard's original derivation. Equation 4 can also be expressed as a dimensionless hydrograph using the following transformations (Leinhard, 1972) to express the distribution in conventional dimensionless form.

$$t_{rm\beta} = \left(\frac{n}{n-1}\right)^{1/\beta} T_p \tag{5}$$

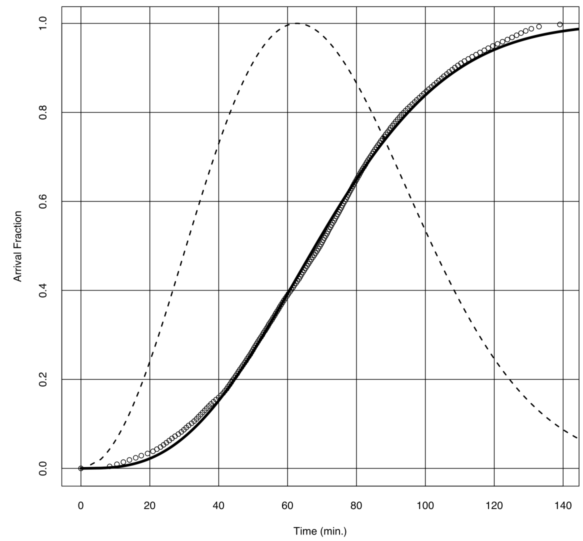

Figure 3: Empirical cumulative arrival time distribution (open circles) and fitted cumulative unit hydrograph distribution (solid line). The cumulative unit hydrograph is the integral of Equation 4. The dashed line is the dimensionless unit hydrograph for this watershed (Equation 7).

$$Q_p = f(T_p) \tag{6}$$

Expressed as a dimensionless hydrograph distribution equation 1 becomes

$$\frac{Q}{Q_p} = \left(\frac{t}{T_p}\right)^{n-1} \exp\left[-\frac{n-1}{\beta}\left(\left(\frac{t}{T_p}\right)^{\beta} - 1\right)\right] \tag{7}$$

The cumulative distribution function is determined by integrating Equation 4 and this cumulative distribution is fit to the empirical S-curve hydrograph using a least square error minimization criterion. Once the distribution parameters, n and $t_{rm\beta}$ are recovered, they are then converted into conventional hydrograph parameters using Equations 5 and 6. Figure 3 that shows the cumulative arrival time distribution for Ash Creek Watershed also displays the "fitted" Leinhard unit hydrograph, which is the source of the timing parameters for subsequent rainfall-runoff modeling.

In principle, the time of concentration, T_c, should be the time at which the cumulative hydrograph is unity, but the cumulative hydrograph approaches unity asymptotically, so the authors selected T_c as the time when the cumulative hydrograph obtained a value 0.98, a fraction of the total distribution. The choice of the value 0.98 is strictly ad hoc, and no rigorous selection method was applied.

The result is that the values of n and $t_{rm\beta}$ are determined from a terrain model, which is conceptually equivalent to determining unit hydrograph parameters from physical watershed characteristics (for example: main channel length, slope, etc.), except this work considers the ensemble of characteristics (all the potential flow paths, all the slopes along these paths, etc.).

In addition to the generation of UH from the arrival time distribution a conventional analysis of the observed data to generate UH parameters was performed using the method described in Cleveland and others (2006).

Application

The computer program that generated the arrival time distribution is referred to in this work as the Digital Terrain Runoff Model (DTRM). The DTRM was applied to the entire set of watersheds using 30-meter digital elevation data. The watersheds were classified into ”developed” and ”undeveloped” watersheds. Representatives of each classification existed in all the database modules, thus the classification does not reflect a particular geographic location. The values used in equation 3 for generating the cumulative hydrographs for developed watersheds are $n_f = 0.04$ and $d = 0.2$. These values were determined by trial-and-error using the Ash Creek watershed (a developed watershed) and the June 3, 1973 storm to calibrate the particle-tracking model. These two values were applied to all developed watersheds regardless of size and location. The values used in equation 3 for generating the cumulative hydrographs for undeveloped watersheds are $n_f = 0.08$ and $d = 0.2$, and were determined by a single-storm trial and error ”calibration” of the Little Elm watershed. These two values were applied to all undeveloped watersheds regardless of size and location.

For each watershed, DRTM was run once using the appropriate n_f and d values and a single Leinhard hydrograph, with two parameters, n and $t_{rm\beta}$, is generated for each watershed. These two values are determined entirely from topographic data and the assumed n_f and d; no actual rainfall-runoff data is used by the DTRM.

To evaluate the performance of the estimation procedure, historical rainfall data are applied to the watershed and the runoff is simulated. These simulated runoff hydrographs are compared to observed runoff hydrographs. Figure 4 is a representative example of output from this testing using observed data from the authors database. The observed hydrograph is the dashed line with the step-wise changes in value, while the smooth curve is the model result using the same hyetograph (input rainfall) and convolving this rainfall with the Lienhard unit hydrograph using the watershed values for n and $t_{rm\beta}$. The plot in Figure 4 is typical, but not all storms were reproduced equally well.

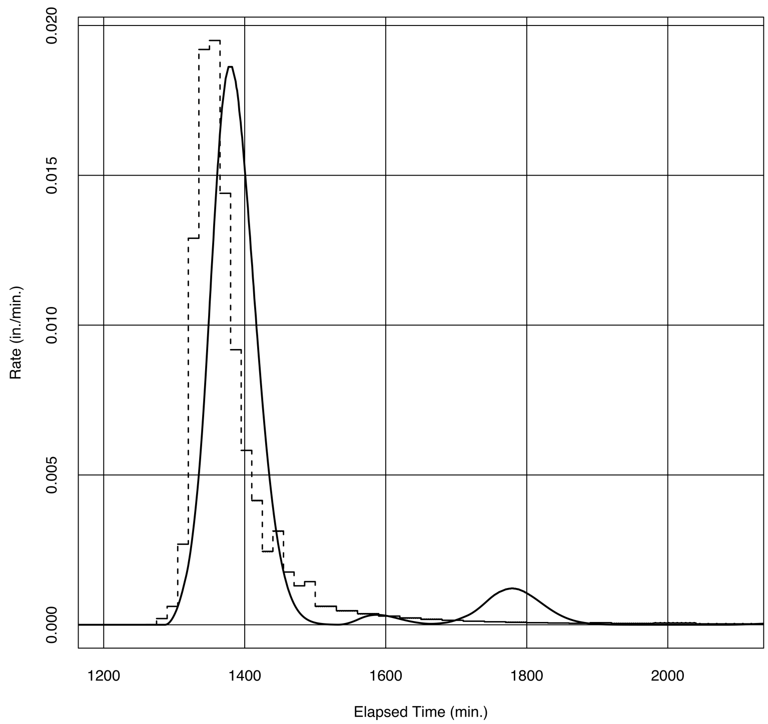

Figure 4: Observed (dashed) and Simulated (solid) runoff hydrograph for Ash Creek, May 27, 1975 storm

Results and Discussion

Figure 5 is a set of plots that illustrate the unit hydrograph parameters estimated using the DTRM procedure and by conventional analysis. The conventional analysis produces a different pair of Q_p and T_p for each storm, and median of these values is compiled and reported for each station, while the DRTM model only produces a single pair of Q_p and T_p for each station. The conventional-derived values are shown on

Figure 5 as open markers. The DTRM-derived values are plotted as closed markers. The two left panels present the results for the Central Texas watersheds (excluded Houston) and the right panels present the results for all the study watersheds. The horizontal axis is the ratio of main channel length to slope. This particular explanatory variable was chosen as a way to represent different watershed sizes and slopes on a single plot.

In right hand panels (includes Houston), there is an increase in variability at the larger values of MCL/S. These larger values are from the Houston data that have relatively small slopes for a given watershed size and thus plot far to the right on the MCL/S axis. As mentioned earlier, the Houston watersheds not only have low slope, but backwater effects are known to be significant and contribute the the variability in both the T_p and Q_p plots. If the Houston data are removed from the plots, the variability is reduced, as in the left hand panels.

Hypothesis tests that the median T_p and Q_p values estimated by either procedure, when classified by watershed development, showed that there was no evidence to reject the null hypothesis that the median values are the same for either method of estimation at a level of significance of $\alpha = 0.05$. Figure 5 and the statistical tests support a conclusion that the DTRM model generates unit hydrographs that are comparable to hydrographs generated by conventional analysis of rainfall-runoff data.

Figure 6 is a set of plots that qualitatively illustrate the performance of the approach on over 2600 storms. The left panels are the results when the unit hydrographs are generated using the DTRM procedure, the right panels are the same storms, except the hydrograph parameters were determined by conventional analysis (i.e. rainfall and runoff data are used, no knowledge of watershed physical characteristics is used). The upper plots are the observed peak discharge and simulated peak discharge for individual storms. An equal-value line is plotted that represents an ideal result. The variability of the DTRM procedure is larger, and the DTRM result is more symmetric around the equal value line. The increased variability is anticipated as the method has no access to rainfall data to estimate hydrologic response.

The lower plots are the time when the peak discharge occurred in either the observations or the simulations. As in the upper plots, the variability for the DTRM procedure is larger. The median values of the peak discharge or time of peak discharge (for roughly 2600 storms) are similar regardless of classification (observed, simulated-DTRM, simulated-Conventional). A Kruskal-Wallis test supports this conclusion – there is no evidence to reject the null hypothesis that the median values do not differ

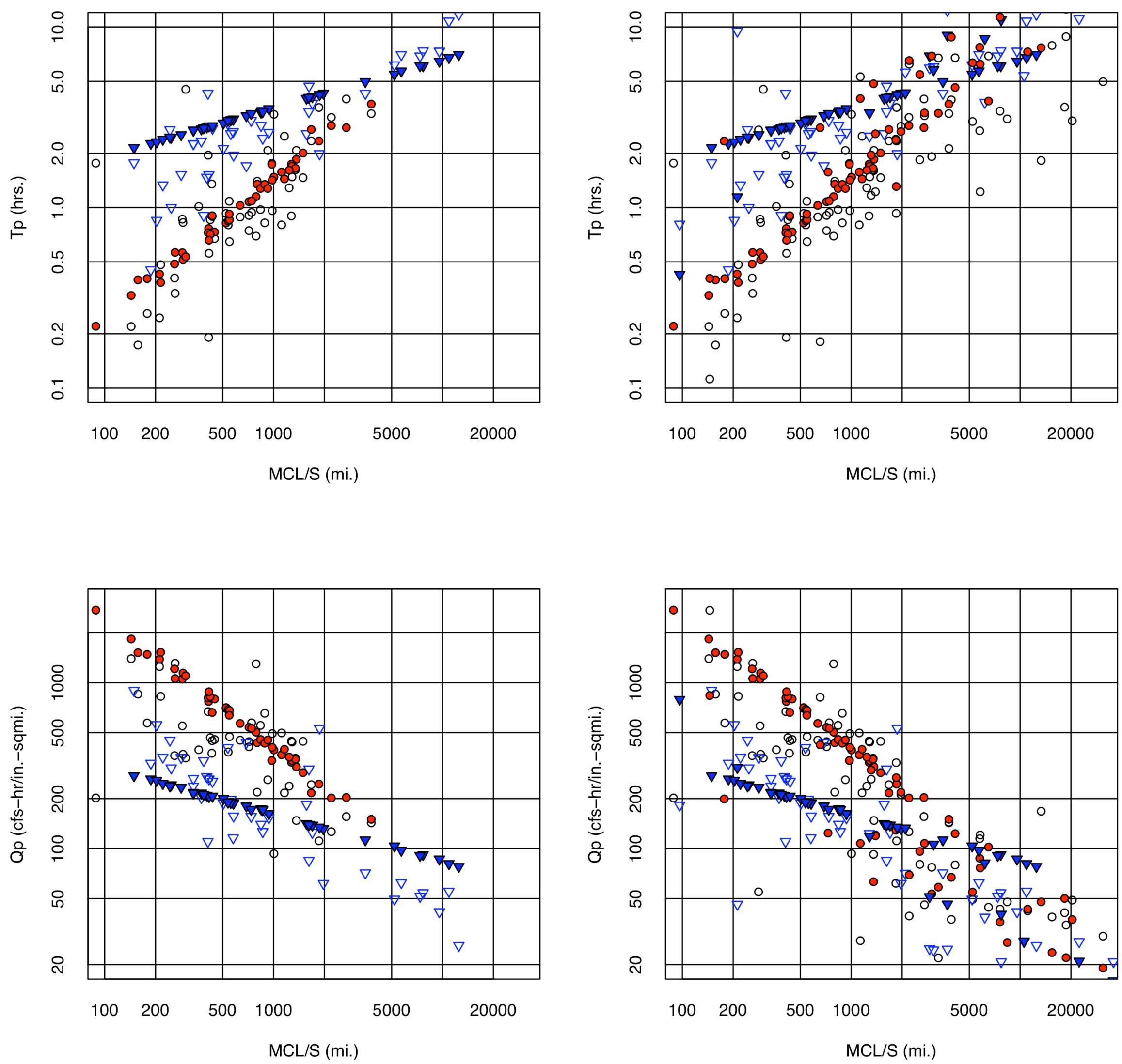

Figure 5: Relationship of T_p and Q_p for Texas watersheds. MCL/S is ratio of main channel length to slope. Left panels exclude Houston watersheds. Right panels include Houston watersheds.

○ = station median values of conventional parameters for developed watersheds.

▽ = station median values of conventional parameters for undeveloped watersheds.

● = station values of DTRM-derived parameters for developed watersheds.

▼ = station values of DTRM-derived parameters for undeveloped watersheds.

for either method when compared to each other or to the observations at a level of significance of $\alpha = 0.05$

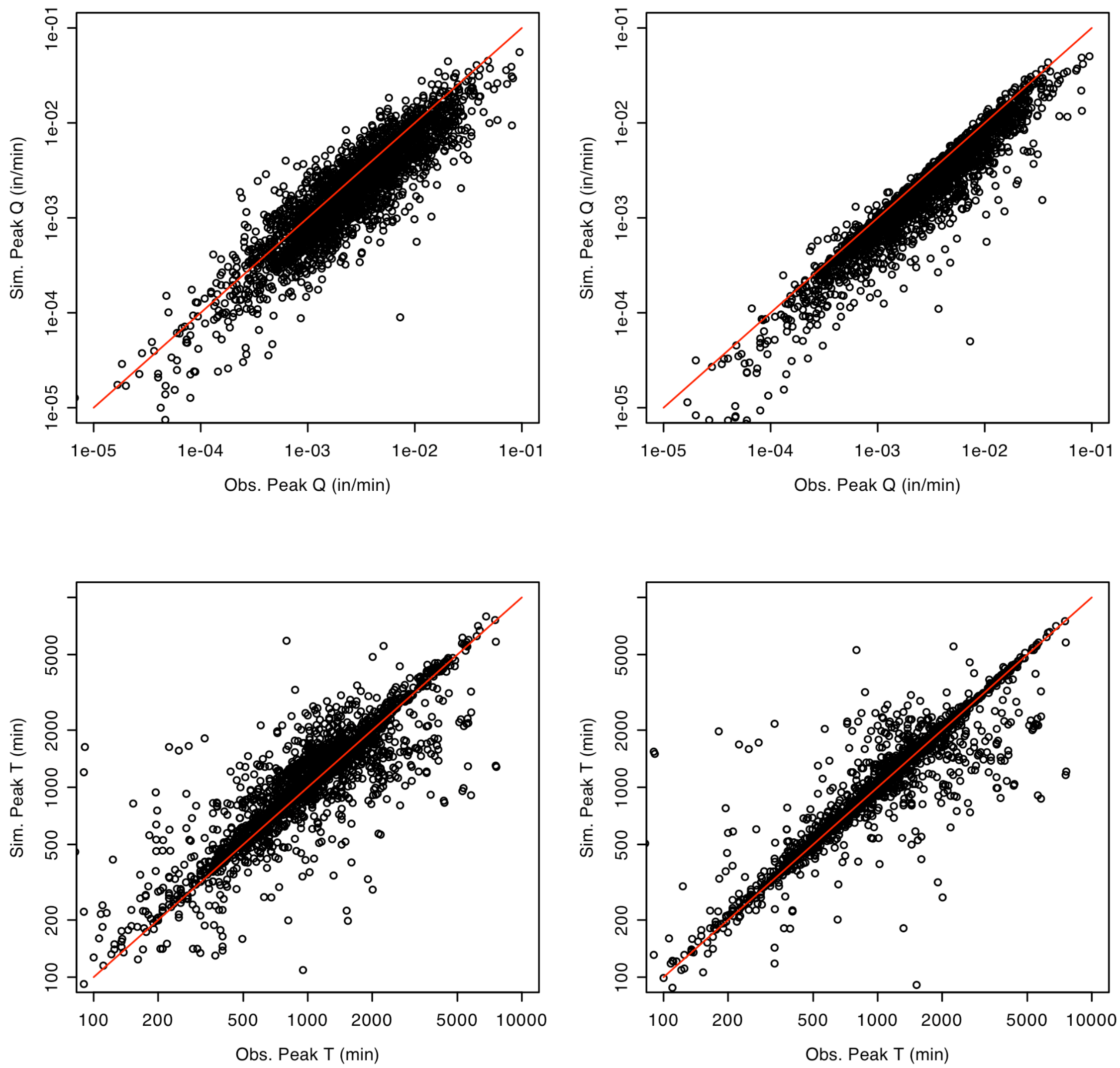

Figure 6: Relationship of simulated and observed peak flows (Q) and time of peak flows (T) for storms using particle tracking model (left images) and conventional hydrograph analysis (right images)

The watersheds were classified as undeveloped and developed. The rainfall and runoff observations across these two classifications were also analyzed to determine if there was a difference between classifications, either for rainfall or runoff.

The median and interquartile range for rainfall depth are nearly the same for either classification. A rank-sum test for difference in the median values shows that there is insufficient evidence to reject the null hypothesis that the difference in median values of rainfall depth for these two classifications is zero at a level of significance of $\alpha = 0.05$. Thus the rainfall depths are the same regardless of whether a watershed is developed or undeveloped.

The median and interquartile range for runoff depth are lower and narrower for the undeveloped watersheds as compared to the developed watersheds. The outlier portion of both classifications have similar patterns. A rank-sum test for difference in the median values shows that there is sufficient evidence to reject the null hypothesis that the difference in median values of runoff depth for these two classifications is zero at a level of significance of $\alpha = 0.05$. Thus the runoff produced by a developed watershed is different from an undeveloped watershed, and developed watersheds appear convert more rainfall to runoff than an undeveloped watershed (by a factor of roughly 2).

Conclusions

The conclusions of this study are that the DTRM procedure is a reasonable approach to estimate UH parameters from a relatively minimal description of watershed properties, in this case elevation and a classification of developed or undeveloped. The elevation data are available on the Internet, or can be prepared from paper-based maps. The classification as to developed or undeveloped can be made based on aerial imagery. The method produced UH comparable to those determined by conventional analysis as thus is a useful synthetic hydrograph approach.

Based on the review of prior work, the procedure is similar to GUH approaches, but simpler in that it in that it disregards stream order, bifurcation rules, channel flow and other measures. The procedure is also similar to existing GIS methods except instead of routing flows along a path, the travel time along a path is used to generate an arrival time distribution.

The runoff volumes are statistically different from developed watersheds as compared to undeveloped watersheds, and the difference is evident in the conventional results. No attempt was made to optimize the unit velocity terms in Equation 3 to account for different land-uses, etc., yet the approach simulated episodic behavior at about the same order of magnitude as observed behavior in terms of peak discharge and timing. The authors speculate that some variability might be reduced by such an exercise but it would greatly complicate the process.

The results in Figure 5 suggest that a lower bound of slope somewhere between 0.0002 and 0.002 exists below which kinematic-wave type equations should not be used without careful consideration.

Acknowledgments

The authors would like to acknowledge the support of the Texas Department of Transportation, program coordinator David Stolpa, P.E., project directors Jaime E. Villena, P.E., and George R. Herrmann, P.E. The authors also acknowledge contributions from their colleagues, Dr. William Asquith, Meghan Roussel, and Amanda Garcia at the U.S. Geological Survey, Austin, Texas. This study is supported through Texas Department of Transportation research projects 0-4193, 0-4194, and 0-4696. The contents of this paper reflect the views of the authors. The contents do not reflect the official view or policies of the Texas Department of Transportation (TxDOT). This paper does not constitute a standard, specification, or regulation.

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