Watershed Slope Lower Bounds for Hydrologic Methods

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ABSTRACT

Engineers design a substantial fraction portion of infrastructure that accommodates storm water drainage and conveyance. Estimation models of the response for a watershed typically contain some form of watershed slope as a principal parameter, and the response is usually inversely proportional to that slope. Therefore, as topographic slope decreases, the watershed timing parameter increases. The consequences at low enough slope, is that the timing parameters approach infinite time and the precipitation intensity that is associated with a long averaging time is so small as to be meaningless — yet low slope environments exist, are populated, and precipitation does generate runoff.

BACKGROUND

The parameters important for understanding overland flow are topography, surface roughness, soil infiltration characteristics (abstractions), and the distribution, duration, and intensity of precipitation. Of these components, the focus of this research was the impact of topographic slope on hydrologic estimates of runoff.

Why Does Slope Matter?

Topographic slope appears watershed in time equations as a consequence of simplification of the momentum equation for practical application. The slope of a surface affects the magnitude of the discharge and depth of flow at any fixed distance downstream. Slope also affects direction of flow.

Figure 1, upper panel, is a conceptual diagram that contains a surface with a finite-length intensity rainfall field similar to that used by Yu and McNown (1964). Precipitation (runoff) accumulates on the plane and produces a unidirectional flow in the downslope direction. The lower panel is a depiction of a surface with zero slope. Flow moves away from the centroid of the runoff-intensity field, either left or right depending on the location of the point of interest relative to the intensity field.

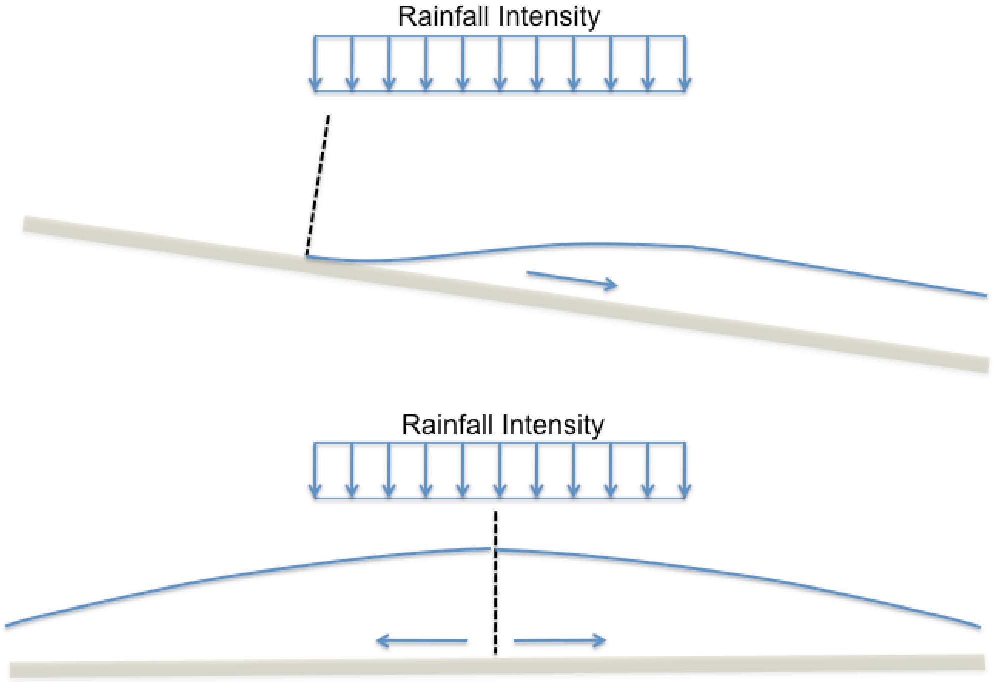

Figure 1: Conceptual diagram to illustrate importance of slope in flow models.

Slope also impacts infiltration and storage abstractions — a low slope system provides more "ponding" time for infiltration. Using Figure 1 as representative of the slope effect, the lower panel suggests greater ponding depth (driving force for infiltration) as well as lower mean velocity in the flow field, which extends the time for infiltration to occur at a location. Similarly, small irregularities in the surface are less important for watersheds with substantial topographic slope. However, such irregularities may constitute substantial storage in the zero-slope case.

What is "Low Slope?"

Equation 1 is a one dimensional equation of motion for runoff on an impervious surface adapted from Yu and McNown (1964) with reference to the upper panel of Figure 1, with the x-direction coincident with the plane of the impervious surface, and x=0 at the left edge of the precipitation field.

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}=g\left(S_0-\frac{\tau}{\rho g h}-\frac{\partial h}{\partial x}\right)-\frac{u\sigma}{h}, \tag{1}$$

where τ is the boundary stress (friction), σ is the rainfall intensity, h is the water depth, S_0 is the topographic slope, and u is the section average velocity.

The left-hand side of Equation 1 contains the convective and local acceleration terms, whereas the right-hand side are the net forces on a unit mass resulting from gravity (S_0), boundary shear (τ), and an additional retarding term to represent rainfall (σ) impact on the free surface. At equilibrium, the local acceleration term becomes zero and the convective acceleration, depth taper $\frac{\partial h}{\partial x}$, and rainfall retarding effect are relatively small compared to the gravitational term, gS_0. The time to reach this equilibrium condition is related to the time-of-concentration and other characteristic response times.

Yu and McNown (1964), following fluid mechanics convention, equate the shear stress to a friction factor (based on limited experimental data circa 1964) and substituting $\frac{\tau}{\rho} = \frac{fu^2}{8}$, develop for equilibrium conditions an equation of motion, Equation 2,

$$u = \sqrt{\frac{8gh}{f}}\sqrt{S_0}. \tag{2}$$

Equation 2 is a simplified equation of motion structurally similar to Manning's equation of motion — there is a constant associated with friction (f), a variable associated with the depth of flow (h), and a driving term associated with local topographic slope (S_0). If travel distance is considered, the relation to time is recovered from $x = ut$, as in Equation 3,

$$t = \frac{x}{\sqrt{\frac{8gh}{f}}\sqrt{S_0}}. \tag{3}$$

The consequence of this simplification is that topographic slope is a fundamental component of the equilibrium equation of motion and appears in the denominator of time equations. Later researchers, the authors included, continued this simplification.

The analysis changes for relatively small slopes because the convective terms and the depth taper are no longer negligible compared to the remaining terms (mainly because the gravitational term completely vanishes in the limit). Furthermore, at $x = 0$ in the illustration, zero depth cannot be achieved, thus the axis of symmetry will by necessity have non-zero depth, although the flow will become bidirectional in such a case.

Table 1: Dimensionless Slope (S_0) where "Low-Slope" Behavior is in Effect.

Slope (S_0)	Methods	Reference(s)
0.0005	Numerical Model Experiments	Su and Fang (2004a)
0.002	Numerical Model Experiments	van der Molen et al. (1995)
0.002	Numerical Model Experiments	Cleveland et al. (2008)
0.003	Observed Data Analysis	Current authors using Riggs (1976)
0.005	Physical Model Experiments	Li et al. (2005) and Li and Chibber (2008)
0.005	Physical Model Experiments	de Lima and Torfs (1990)
0.008	Observed Data Analysis	Sheridan and Mills (1985)

Therefore, "low slope" is postulated to begin where topographic slope is such that the convective terms are no longer negligible. A review of professional literature on the impact of slope and watershed response was reported in Cleveland et al. (2011). The authors concluded that "low slope" begins at a dimensionless slope of about 0.003 and is unequivocal at 0.0005. Table 1 (Cleveland et al., 2011) lists literature-based estimates of the threshold for onset of low-slope behavior.

PHYSICAL EXPERIMENTS

Experiments were conducted at Texas A&M University that used two small field plots and the Texas A&M rainfall simulator. The intent was to accomplish two tasks, (1) to extend an existing rainfall-runoff database and (2) to develop data for calibration of a two-dimensional hydrodynamic model. The hydrodynamic model was used to conduct parametric studies of the relation between slope, rainfall rate, travel distance, and losses to identify alternate hydrologic methods for use in low-slope situations.

Small-Plot Field Studies

Texas A&M University personnel instrumented two small field plots with recording raingages and flow measurement equipment. The pair of research watersheds are located at the Texas A&M University Riverside campus. The distance between the field plots was a few hundred feet. The surface of the first plot was concrete and the other was an open field of prairie.

The concrete-surface field plot is a portion of an abandoned airstrip taxiway. The plot area is a rectangular plot with dimensions of 100 ft by 50 ft. The outlet was located in one corner of the rectangle. The 5,000 ft^2 area is bordered by a 7 inch tall soil berm. The slope along the main diagonal is 0.0025. Figure 2 is an image of the concrete plot looking upstream from the outlet towards the far corner. The H-flume used to measure discharge from the watershed and the tipping-bucket raingage are both evident in Figure 2.

Figure 2: Photograph looking upstream from the outlet for the concrete surface watershed. Diagonal dimension from H-flume to orange cone in far corner is $\approx$ 111 feet. Dimensionless slope along the diagonal is 0.0025

The grass field plot is also located at the Texas A&M University Riverside campus about 500 ft from the concrete plot. The grass watershed is also a 100 ft by 50 ft rectangle with the outlet located in one corner of the rectangle. Similar to the concrete plot, the grass 5,000 ft^2 plot is bordered by soil berms. Site surface was maintained by mowing. The slope along the main diagonal is 0.0025. Figure 3 is an image of the research watershed looking upstream from the outlet towards the far corner. The H-flume that is used to measure discharge from the watershed and the tipping bucket raingage are also shown on Figure 3.

Figure 3: Photograph looking upstream from the outlet for the concrete surface watershed. Diagonal dimension from H-flume to the far corner (left of the orange machinery in the image background) is $\approx$ 111 feet. Dimensionless slope along the diagonal is 0.0025

The instrumentation for both sites comprised an ISCO tipping-bucket rain gage and an ISCO sampler equipped with a bubbler flow module located in a 0.75 ft H-flume. The raingage records accumulated rainfall depths in 0.01 in resolution every minute. The bubbler flow module records instantaneous depth in the H-flume at 0.001 ft resolution every minute. The instrumentation is manually connected and powered ahead of each forecasted rainfall event. The recorder is set to trigger (store data) when rainfall intensity exceeds 0.01 inches per hour or the flume discharge depth is over 0.003 ft. During the research project 26 events were recorded for the concrete study watershed and 4 events were recorded for the grass watershed.

Rainfall Simulator Studies

Texas A&M University personnel conducted a series of controlled simulator studies on an indoor artificial watershed. Water was supplied by a rainfall simulator (controlled intensity) on a bare soil bed (controlled slope). Five longitudinal slopes were examined. They were 0.00, 0.001, 0.002, 0.005, and 0.01.

The artificial watershed is a steel-framed bed 30 ft long by 6 ft wide. The bed is filled with soil to a depth of about 1 ft. Figure 4 is an image of the test-bed in the rainfall simulator enclosure. The rainfall simulator is a nozzle-type design suspended above the test bed. (The rainfall simulator is not shown in the image.) Unique to the watershed design is that the nozzle-array tilts with the bed so the distance from nozzle to the soil bed is constant regardless of the slope. Such an arrangement means that the impact velocity of a water drop onto the test bed is the same regardless of slope. The Texas A&M team filled the bed with clay and then compacted soil with a lawn roller. The bed was stored outdoors for more than a month before simulator tests were conducted[1]. The rainfall simulator can produce a sustained rainfall intensity up to 4.5 inches per hour.

Figure 4: Photograph looking upstream from the outlet for the rainfall-simulator watershed. Longitudinal dimension from the outlet (foreground) to the end of the watershed is 30 ft.

Two bubbler flow modules were used to record discharge depth and surface runoff depth near the outlet with a resolution of 0.001 ft. Values are recorded at one-minute intervals. The discharge is measured using a 22.5° V-notch weir box. Rainfall intensity is measured using an inline flowmeter connected to the rainfall simulator. A recording tipping-bucket raingage was used as an independent check of supply rate.

A total of 30 experiments were conducted at the five different slopes with two different rainfall intensities. Results were similar to those from the field studies. The collective results (from both field plots and artificial watershed) were used to fit a hydrograph for each experimental time series. These fitted results were combined with prior work to produce Figure 5, which is a plot of observed time of peak discharge (T_p) for more than 100 Texas watersheds. Based on prior research, T_p and T_c are highly correlated and convey similar information. Data from various studies of Texas watersheds conducted by the Texas Department of Transportation (0–4193, 0–4194, 0–4696) were plotted along with data collected from the Texas A&M field plots and rainfall simulator.

In general, results from the extended research database and those from the Texas A&M laboratory watershed fall near the regression line superimposed on Figure 5. The exception, which was

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[1] The month-long storage was a consequence of scheduling time in the simulator rather than any research forethought.

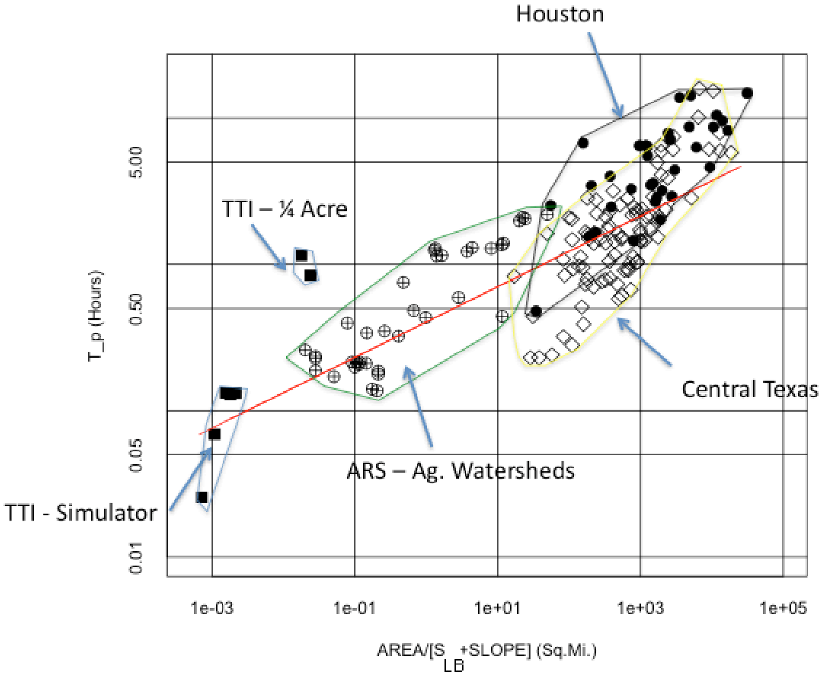

Figure 5: Relation of Time to Peak (T_p) and the ratio of area to adjusted slope for Texas watersheds.

unanticipated by the research team, are results from the two field-scale plots used by the Texas A&M team.

The earlier work of Li et al. (2005), and Li and Chibber (2008) produced findings similar to those reported herein. That is, the characteristic time of low-slope watersheds was much greater than anticipated. The work reported by Li et al. (2005) and Li and Chibber (2008) used an experimental approach and instrumentation similar to that used by the Texas A&M team. However, based on what was learned by the research team, use of H-flumes and other conventional critical-depth devices is limited for low-slope watersheds. The primary reason is that a low-slope system (by definition) has insufficient drop at the measurement location for the devices to work properly. That is, standard instrumentation requires sufficient energy (head) to develop critical depth. In a low-slope environment, the result is that the "pond" formed upstream from the device results in sufficient storage to affect the measurement. Therefore, the unusual results from the Texas A&M field sites are likely a consequence of the H-flume requirement to be at grade with the watershed itself and subtle, unmeasurable, adverse alignments of the device itself. The results are further complicated because precipitation depths from many events were insufficient to satisfy the hydrologic abstractions on the grass plot for events that occurred during the course of this research project. The simulator studies are unaffected because the runoff drops off the simulator tray into the measuring device.

PARAMETRIC MODELING

Modeling was conducted to examine the interaction of multiple parameters (slope, cover, intensity, and so forth) as a parametric study to develop alternative methods for application in low-slope settings. Two types of modeling were conducted — the first was a parametric study using a hydrodynamic model, and the second was a statistical analysis of simulated data. The hydrodynamic model used the DHM code (Hromadka and Yen, 1986) to first calibrate to field plot and simulator studies collected as part of the research reported herein, then to explore an alternative approach for low-slope settings. The data analysis used unit-hydrograph-type interpretation to search for a behavioral trend explainable by slope and other conventional variables.

The diffusion hydrodynamic model (DHM), developed by Hromadka and Yen (1986), is a two-dimensional numerical model based on the diffusion equation to approximate the horizontal momentum of vertically-averaged free-surface flow. The governing equations for DHM include the continuity equation and two equations of motion. The friction slope component in the motion equations is approximated using Manning's equation and assuming quasi-steady flow (steady within a computational time step). The precipitation input depth is converted into a volume by a product of the input depth over a time interval and the computational cell area. The rainfall volume is then added into the continuity equation as an internal source term at each computational cell. DHM was used by its original authors to model overland flow hydrographs, develop synthetic unit hydrographs in five test catchments and was compared with runoff hydrographs developed using SCS dimensionless unit hydrographs, hence the tool was suitable and has been previously used for the kind of examination in this research.

DHM on impervious surfaces

The Auburn University research team developed a utility program to construct DHM input files for different watersheds. DHM was tested using measured rainfall and runoff data collected from two lab testing impervious surfaces by Yu and McNown (1964). The slopes of impervious surfaces were 2% (500 ft long watershed) and 0.5% (252 ft long watershed). Simulated discharge hydrographs from DHM agreed well with observed discharge hydrographs for constant and variable rainfall inputs. DHM was then used to simulate discharge hydrographs from the Texas A&M field experiments. Rainfall hyetographs and runoff hydrographs were observed and recorded for 26 rainfall events between April 2009 and March 2010.

Figure 6 is a typical result of the DHM application. The simulated and observed discharges are in reasonable agreement. This result is important because DHM is characterized by watershed geometry, slope, cover type, and a grid-spacing specification and in that context constitutes a testbed for the more detailed parametric study.

Figure 7 is a plot of cumulative precipitation and discharge. The impact of different outlet boundary

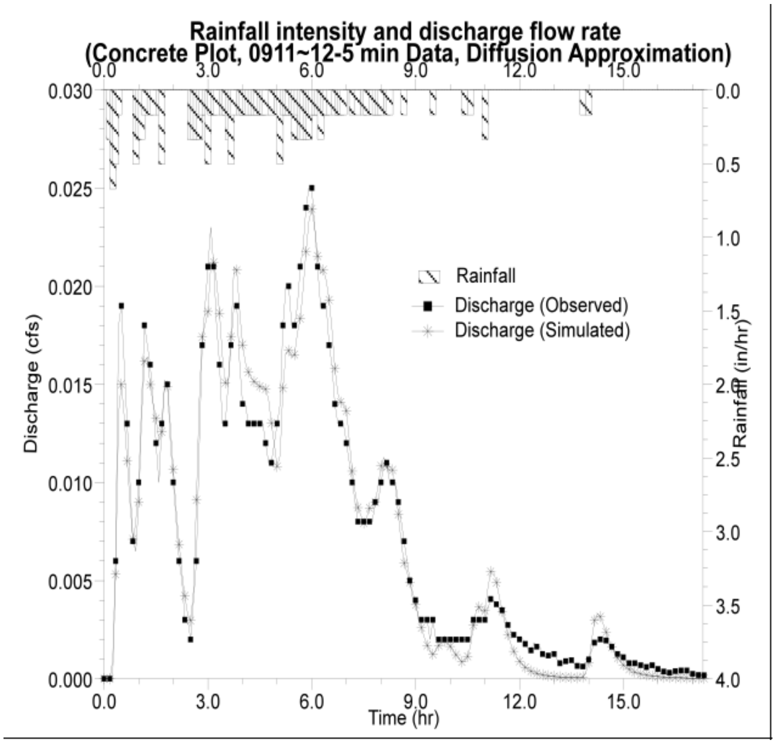

Figure 6: Typical result for DHM model applied to Texas A&M impervious (runway) experimental conditions.

conditions and grid spacing are displayed. Based on results presented in Figure 7, the effect of grid spacing is minor as is the effect of outlet boundary condition.

A set of numerical experiments was conducted using DHM to estimate the watershed time of concentration T_c. The practical determination of such a time from experimental data and numerical modeling is varied (Ben-Zvi, 1984). In this study, T_c is selected as the time when the discharge reaches 98 percent of the equilibrium flow. In the numerical experiments conducted for this study, the slope is varied and the T_c is recorded. Of particular note is that at zero slope T_c is finite (the DHM flow depth is surely reflective of the zero topographic slope, but the depth taper transports water out of the model). Figure 8 is a depiction of one set of numerical experiments.

In other studies (Izzard, 1946; Ben-Zvi, 1984; Su and Fang, 2004b; Wong, 2005), a variety of threshold values of discharge were used to identify the characteristic time (T_c). Examples are 80, 95, and 97 percent of the equilibrium flow. In each approach, a particular value was used to accommodate the asymptotic nature of the arithmetic in the model studies.

Results from the numerical experiments were used as input data for exploration using regression methods. The objective of the regression exploration was to choose a threshold topographic slope where low-slope (asymptotic) behavior begins. Figure 9 is a depiction of results from the analysis. In Figure 9, the asymptotic value for the simulated system is 100 minutes. The plot of T_c versus

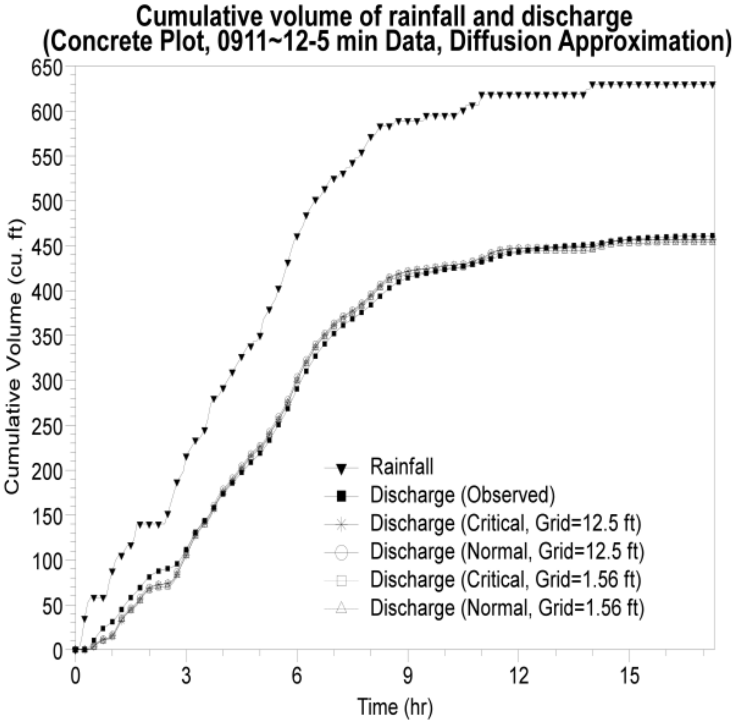

Figure 7: Typical result for DHM model applied to Texas A&M impervious (runway) experimental conditions, cumulative values. Variable computational grid size is examined in the figure — grid spacing and discharge condition have minor effect in this simulation.

slope intersects this value at a slope of about 0.0001. That is, from the numerical experiments, the threshold value for defining "low-slope" is about 0.0001. However, the estimate of the threshold derived from the literature is approximately one order of magnitude greater.

On Figure 9, the regressions were developed based on the assumption that lows-slope behavior is exhibited for watersheds slopes of 0.0005 and 0.003, respectively. Differences between the two lines are relatively minor and both intersect the asymptotic value at the same order of magnitude. Therefore, the researchers chose a dimensionless slope of 0.003 as the inception of low-slope behavior (largely based on the literature examination and the departure of the DHM values from a straight line in Figure 9 at that value. Based on these results, the researchers suggest that low-slope behavior is likely for watersheds with slopes less than about 0.003 and should be assumed for those watersheds with slopes less than 0.0005.

DHM was used to generate a suite of estimates of T_c for 90 different combinations of slope, distance, surface conditions, excess precipitation intensity, and outlet size. These values were used to develop Equation 4 to estimate T_c for functionally impervious terrains with topographic slope ranging from zero to 10 percent.

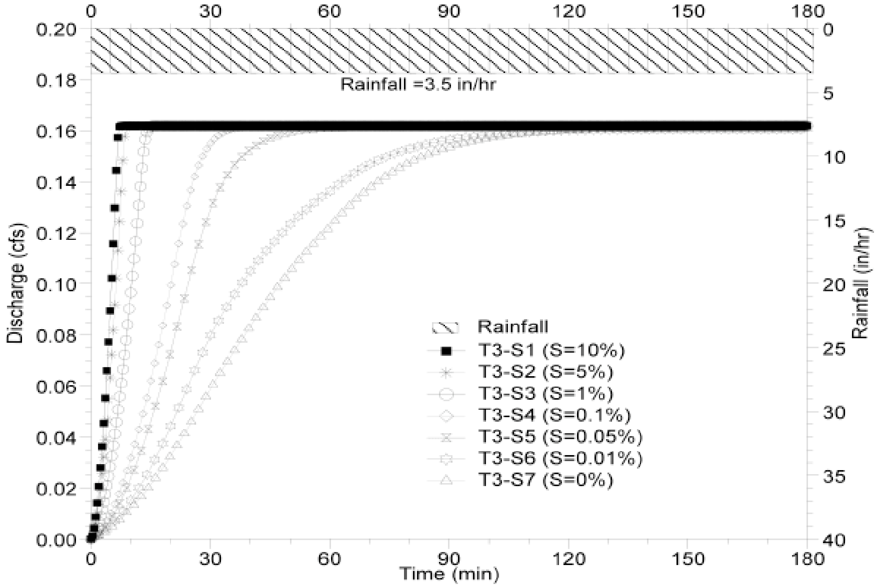

Figure 8: DHM functionally impervious surface parametric study.

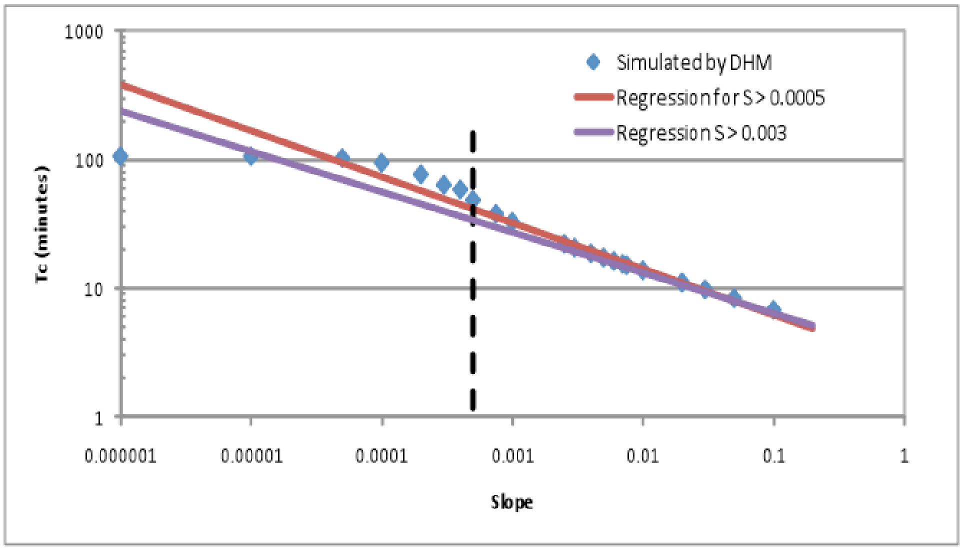

Figure 9: Regression analysis to select threshold of topographic slope to define “low-slope” behavior. The vertical dashed line is at dimensionless slope of 0.0005.

$$T_c = \frac{Cn^{0.6175} L^{0.551}}{i^{0.388} (S_{\mathrm{LB}} + S)^{0.436}} \tag{4}$$

Equation 5 is an equation developed in the same analysis for estimating the outlet factor described in Su and Fang (2004b). To obtain the outlet fact, critical depth at the watershed outlet was assumed. The researchers varied flow lengths and found that while there was some sensitivity to length, the topographic slope itself had far more impact on the numerical value of the factor.

$$C = 1.612 - 1.196 \frac{S}{S_{\mathrm{LB}} + S} \tag{5}$$

Equations 4 and 5 constitute an alternative model for estimating T_c in low-slope situations. These equations are developed without regard for functionally impervious versus pervious conditions.

DHM on pervious surfaces

The analytical approach used for impervious surfaces was also applied to results from the field and laboratory experiments conducted by Texas A&M personnel (the grass plot and the rainfall simulator studies). Figures 10 and 11 are typical results from DHM application to the grass plot studies. Only a few hydrologic events produced runoff from the grass plots and most of these produced little runoff. This was an unanticipated result. The poor agreement of DHM results and field observations is attributed to the low volume of runoff from the grass plots. Because little runoff occurred, obtaining a reasonable estimate of rainfall losses was practically impossible.

The Auburn team added a Green-Ampt model to DHM to allow estimation of surface infiltration (prior to this addition, losses were external to the program). The modified program was used to reexamine the grass-plot experiments and the rainfall simulator studies.

When DHM was applied to results from the rainfall simulator experiments, which used soil similar to the grass plot studies, results were comparable to the impervious plot studies. Therefore, the validated DHM model and the rainfall simulator results were used to develop parametric studies for low-slope alternative timing models applicable to pervious surfaces.

For development of of a dataset for use in developing the alternative timing model for pervious surfaces, soil properties were grouped into three categories — sand, loam, and clay. Each category was examined using DHM and adjusting other variables in the timing model to develop a relation between soil properties, slope, and T_c. Saturated hydraulic conductivity and the soil type assigned to the particular range of values are reported in Cleveland et al. (2011). Including non-pervious components, over 1,000 combinations of roughness (Manning's n), slope, rainfall intensity, plot length, saturated hydraulic conductivity, suction pressure, and moisture deficit were examined.

Unlike the result from DHM modeling of watersheds with impermeable surfaces, no single structure model was found. Cleveland et al. (2011) presented an adjustment procedure to account for the permeable surfaces, but the adjustment is impractical. Instead the authors' suggested that the Kerby-Kirpich method (Roussel et al., 2005) be adjusted with a slope offset of 0.0005 applied when the topographic slope was smaller than 0.003 as an interim approach.

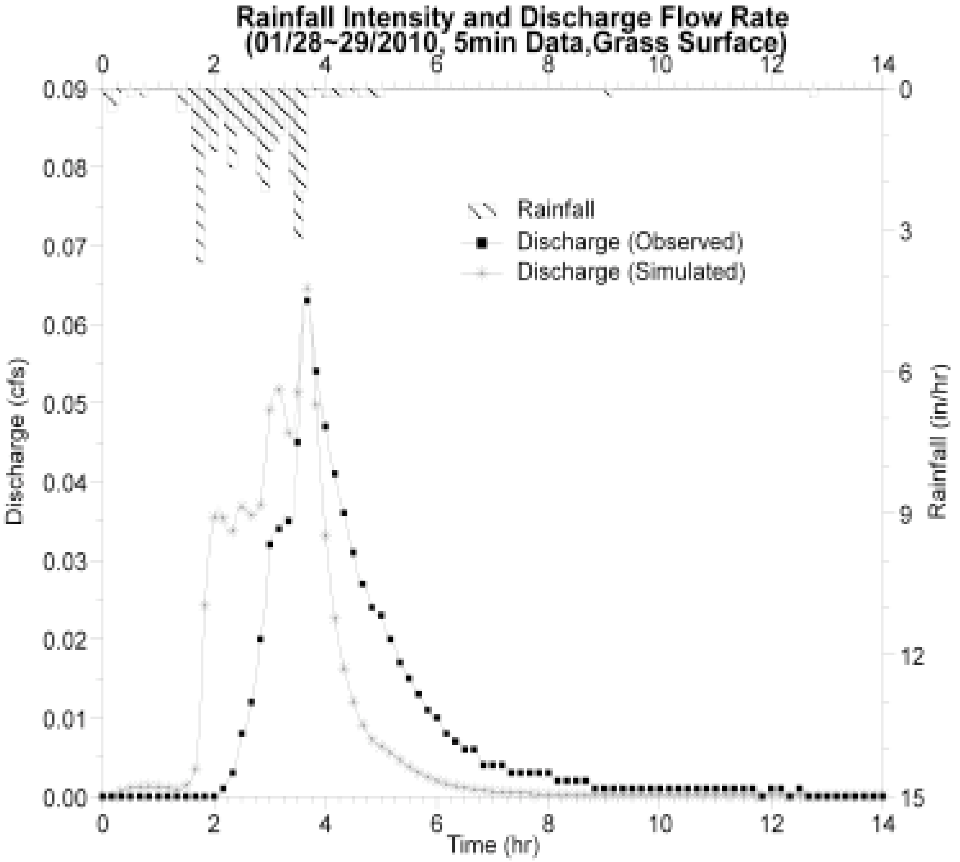

Figure 10: Typical incremental result for DHM model applied to Texas A&M grass-plot experimental conditions.

CONCLUSIONS

Methods currently used to estimate the timing parameter for a watershed contain some form of watershed slope as a principal parameter. That time response (travel time or time of concentration) is usually inversely proportional to topographic slope. The research team identified, from literature, data, modeling, and physical experiments, the dimensionless topographic slope where alternate hydrologic approaches should be considered. The dimensionless slope suggested for consideration of alternate approaches is 0.003.

The project team examined several alternate approaches from parametric studies using the Diffusion Hydrodynamic Model (DHM). Inclusion of a rainfall loss model in DHM was required to address results from field experiments and rainfall simulator experiments. Initial results retained rainfall intensity as part of the timing estimate creating an implicit estimation problem. A simple slope offset was also explored and was suggested as an adaptation of current technology. The offset value suggested is 0.0005.

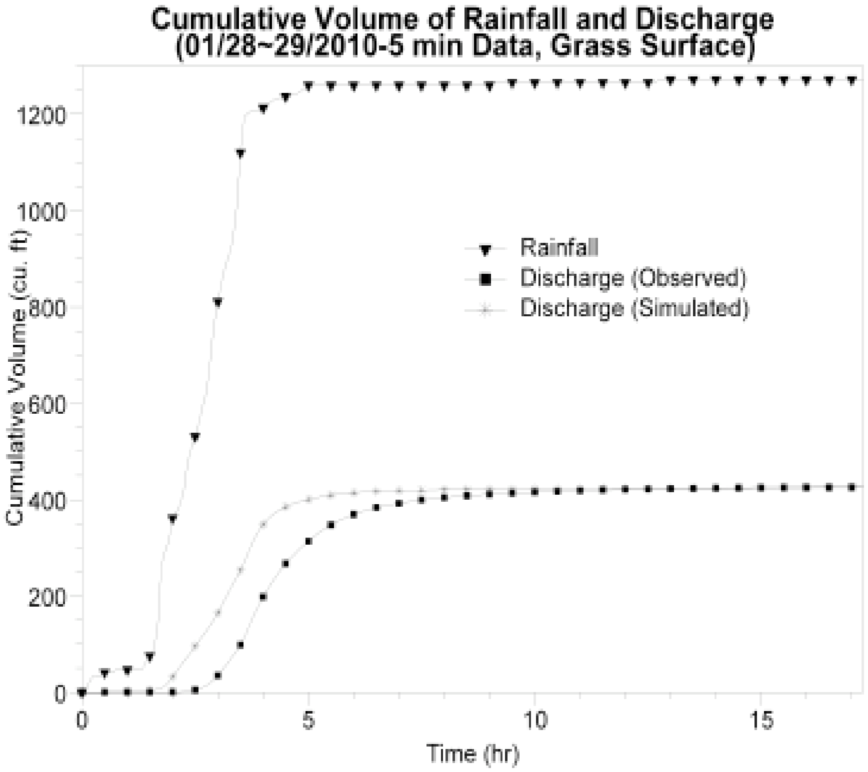

Figure 11: Typical cumulative result for DHM model applied to Texas A&M grass-plot experimental conditions.

The project team developed a research plan for more intensive study of low-slope hydrology should further work and supporting data be needed. The principal findings are that current streamgaging techniques are not well suited to low-slope hydrologic studies and different measuring methods are suggested. Several promising emerging technologies were presented as worthy of further exploration.

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