CONSIDERATION OF FUNDAMENTAL LOSS COMPONENTS, RATIONAL COEFFICIENTS, AND ARID CLIMATE

GEORGE R. HERRMANN, P. E., P. H., CFM
Transportation Engineer,
Texas Department of Transportation
4502 Knickerbocker Road
San Angelo, Texas 76904
325-947-9321
gherrma@dot.state.tx.us

and

THEODORE G. CLEVELAND, Ph.D., P.E.
Associate Professor
Department of Civil and Environmental Engineering
Texas Tech University
10th and Akron, Room 203F
Lubbock, Texas 79409
806-742-2801
theodore.cleveland@ttu.edu

ABSTRACT

A conceptual exercise is documented wherein the effect of a realistic initial abstraction on the calculation of observed runoff coefficients from simulated rainfall and runoff data series is explored. The relation between runoff coefficients and expected rainfall event depth, when considering an initial abstraction, is demonstrated in the context of increasing aridity of climate.

Key Words: Rational Method, Runoff Coefficient, Initial Abstraction, Arid Hydrology

INTRODUCTION

The association between rainfall that occurs on a watershed and the resulting runoff is very complex, and rainfall and runoff processes have been the subject of voluminous study. Put succinctly, the difference between rainfall depth and the volume of runoff (as depth) is lost rainfall, which is generically called loss. Loss is thought to be composed of a number of parts. One of those parts universally mentioned in texts is the initial abstraction (Maidment, 1993). It is a depth of rain that occurs early in a rainfall event that does not result in any runoff. There are many explanations for it. This article is not concerned with the mechanics of the initial abstraction, but with the effects it has on the assumptions underlying certain common methods.

The Rational Method is a commonly used method of estimation for very small watersheds. The method is so named because it is supposed to represent the ratio of runoff to rainfall by way of a runoff coefficient. The relation between two concepts: that of the rational coefficient and that of the initial abstraction, will be explored.

EXERCISE

Consider two cities that are similar except for climate: which we will call Drytown and Rainyville. Drytown on averages receives 26 inches of precipitation per year, whereas Rainyville on average receives 52 inches per year; twice as much as Drytown. For the sake of simplicity, assume that rain in either of them only occurs once a week, on the same day every week, thus making each event completely independent of prior events. The actual depth of rainfall for each weekly event is considered Gamma distributed about the mean event depth. A plot of the assumed rainfall probability density functions is shown in Figure 1. The cumulative distribution functions corresponding to these density functions are shown in Figure 2.

Simple arithmetic says that the constructed event mean depth is 0.5 inch for Drytown, and 1 inch for Rainyville. These two values are evident in Figure 2, which shows the cumulative distribution functions for the two cities. Rainfall events are considered to be members of the same population of events for both cities, with the standard deviations for each proportional to the mean value so that the coefficient of variation is identical for each city. We will assume that the coefficient of variation is 0.5, which means that the standard deviations are 0.25 inch and 0.5 inch respectively for the two cities.

Now consider that there is an initial abstraction of 0.5 inch at both places, independent of any external factor. Every event sees the same potential initial abstraction, regardless of which city it is in; the actual magnitude of the initial abstraction can only reach the potential 0.5 inch if rainfall equals or exceeds that depth. If rainfall does not exceed 0.5 inch, all rainfall is lost and there is no runoff. A perfect watershed in each city that returns all effective rainfall is gauged for runoff volume (which can be expressed as depth).

With these specifications, a numerical simulation exercise is performed. The exercise samples the two rainfall distributions (n=5200), removes the initial abstraction, assumes the remaining rainfall converts directly into runoff, and then summarizes the results. A Gamma distribution is assumed to be an adequate representation of the distribution of rainfall event depth for the purposes of this exercise. The Gamma distribution was chosen because the Gamma is bounded by zero on the left and is capable of reasonably approximating the observed distribution of rainfall for much of Texas (Asquith et al., 2006). In truth, the choice of a distribution for this demonstration is of little consequence, as that choice has little impact on the subsequent arithmetic under discussion. Table 1 is the result of such an experiment for these two hypothetical towns.

In Table 1 the mean, median, and standard deviation values for rainfall and resulting runoff are listed, as are the coefficient of variation computed from the mean and standard deviation. In this table, the statistics for both censored and uncensored runoff sets are shown. The sets considered censored contains only non-zero runoff values. This is the type of set that would result from watershed gaging only; no association with rainfall events is implied. The uncensored sets contain numerous zero runoff values. The zero values in these sets represent non-runoff-producing rainfall events; in effect, they are non-detect values. The importance of this distinction will become evident.

Figure 3 is a boxplot of the distribution of simulated rainfall event depths for the two cities. It is essentially a different way of displaying the information in Figure 1. Figure 4 is a similar boxplot of runoff values for each city- rainfall after the initial abstraction is removed. These sets of simulated measurements include only non-zero runoff values, hence there are no entries for events that produce no runoff. The mean values of runoff are as expected, however the standard deviation is reduced in Drytown by about 40 percent, while it is nearly unchanged in Rainyville (about 2 percent reduction). The total number of runoff events for both locations is less than the total number of rainfall events. For Rainyville, the number of runoff events is 4487 out of 5200 rainfall events For Drytown, the number of runoff events is 2294- somewhat less than half of the number of rainfall events.

However, if all rainfall events are considered, including those that produce no runoff, and compute a rational coefficient for each event, the distribution of runoff coefficients would be as shown in Figure 5, a boxplot of event-speci_c runoff coefficients. It is important to recall that if we were applying the rational method in the traditional manner to these locations, all of our simulated rainfall events would produce some runoff; it would simply be proportioned by a runoff coefficient selected from a table. Figure 4 might not appear drastically different, but Figure 5 would appear more like a re-mapping of Figure 3 or 4.

From basic arithmetic we know that, in light of the initial abstraction, had rainfall events been of uniform depth, Drytown (with 26 inches per year mean annual precipitation-MAP) would have no runoff. Runoff does occur on the watershed, though, because rainfall depth is probabilistically distributed, which results in a number of events that exceed the initial abstraction. Yet, enough events fall below the initial abstraction threshold and produce no runoff that the median runoff coefficient is zero.

Rainyville, on the other hand, would have a constant runoff coefficient of 0.5 if rainfall events were all of uniform depth. It can be observed that the median runoff coefficient is close to the expected value of 0.5. The most interesting thing evident is that the distribution of apparent runoff coefficients in Drytown is dramatically lower than it is in Rainyville- even though the only di_erence between the two cities is in the expected (mean) depth of rainfall per event. The subtraction of a constant yet substantial initial abstraction from the rainfall prior to the calculation of a rational runoff coefficient exerts a rather strong influence on the magnitude of the apparent runoff coefficient for different expected depths of precipitation.

DISCUSSION

Obviously, this conceptual exercise represents a grossly simplified example of the relationship between rainfall and runoff, the distribution of rainfall in time, and the variation of expected event depth with climate. However, the purpose of the exercise is to demonstrate that in the presence of a real, finite initial abstraction, something as simple as runoff coefficient is substantially affected by long-term rainfall depth expectation. The variation seen in this example is unrelated to any of the complicating factors often cited as cause of variation. It is related only to the association between initial abstraction and expected value of individual event rainfall depths. Variation in observed runoff coefficients has been attributed to many things, including antecedent moisture, rainfall intensity, and return period of the event.

This exercise demonstrates that if the concept of initial abstraction has merit, observed runoff coefficient can be expected to vary with event depth, and runoff coefficients over time and space can be expected to vary with mean event depth. This observation has numerous consequences. If data from numerous locations is analyzed without acknowledgement of the relative magnitudes of the initial abstraction and the expected event depth, an increase in apparent runoff coefficient easily appear to be linked with decreasing probability of exceedance. To some degree, this can be considered true, but a more proper link is to state that runoff coefficient increases with increasing event depth. At any given location, the depth-duration relationship is increasing with decreasing probability of exceedance. However, if we make the connection to depth directly, the appearance of connection to probability is presented in a different light. This understanding should improve our estimates of runoff coefficients.

When we consider nothing more complicated than an initial abstraction, observed runoff can be thought of as censored data. The censoring threshold may be unknown a priori. Runoff is a manifestation of rainfall, censored by the initial abstraction. If we compensate for what appears to be a manageable initial abstraction rather than immediately treating data as non-dimensional, understanding of the relation between rainfall and runoff should improve. Over time, the comparison of observed rainfall data and observed runoff data from specific sites should allow the identification of a censoring threshold, which is identical from a practical point of view to an initial abstraction.

The censoring process described has many implications in the statistical analysis of rainfall and runoff, and for many common concepts in practical hydrology. It is at least implicit, if not explicitly described in textbooks, that for watershed modeling methods (of which the rational method is a simple example) we assume that the frequencies of rainfall and runoff are matched. A runoff event of a given exceedance probability is assumed to be the result of rainfall event of the same exceedance probability. Statistically, such probabilities would be assigned by fitting a distribution to measured data.

When we examine our conceptual samples, we see that the assumption of frequency matching may not be universally valid. The complete sample for Rainyville consists of 5200 measured data points. After the removal of the initial abstraction, there remain 4487 data points that would correspond to runoff values. For Drytown, the number of remaining data points is much smaller- 2294 out of 5200. The concept of frequency matching would dictate that those smaller samples would be considered as representing an entire frequency curve corresponding to the rainfall frequency curve. In reality, they correspond to positions on the rainfall frequency curve of the 13.7 and 55.9 percentiles and up, respectively. This is shown in Figure 6.

In Figure 6, the gross rainfall event depths are represented by the black (Rainyville) and gray (Drytown) lines, whereas the dashed lines represent runoff values for the respective towns. Runoff values are plotted at probability values corresponding to the rainfall events that generated them. Data positions for Figure 6 were assigned by a plotting position formula. In this case, the small circles represent the rainfall values that produce the smallest runoff values from the respective rainfall series, to which the arrows point.

For the purposes of statistically analyzing an entire sample series of runoff data, the most common assumption would be that the data series represented the entire range of probability values from smallest to large and to assume that they therefore represent a range from most frequent to relatively rare. Measured data from actual runoff events are assumed to be a representative sample from a homogeneous distribution. The result of this type of analysis is the assignment of probability values to various elements of the sample. Figure 7 shows the same data represented in Figure 6, as it would be plotted without knowledge of the relative position of the rainfall events that produced the respective runoffs. The smallest values of runoff would be matched with the smallest values of rainfall.

It is immediately evident that smaller values of the simulated runoffs are assigned much different places in these graphs. The smallest values are associated with the rainfall values that produced them in Figure 6, which in the case of the Drytown data, is above the 50th percentile non-exceedance probability. However, disregarding the connection to the underlying rainfall distribution results in a bias by assigning runoff of those magnitudes to the left tail of the distribution, values close to zero non-exceedance.

From these graphs, two ideas are evident. First, that the assumption of matched frequency of rainfall and the resulting runoff is not strictly true if the frequency of runoff is computed from the censored set. Second, that the relationship between rainfall frequency and runoff frequency is not independent of expected of rainfall event depth. In humid, high rainfall areas, the assumption of matched frequency of rainfall and runoff may be reasonable, but it is likely less suitable in arid areas where the expected event depth may be small.

CONCLUSIONS

Succinctly, three important conclusions may be drawn from this exercise: For the purposes of water resource management in arid areas, many rainfall events result in no appreciable runoff, Runoff coefficients are contextual; they should be developed locally or regionally rather than being drawn from a textbook or reference manual, and Probabilistic adjustment factors for runoff coefficients are contextual; they should also be developed locally or regionally.

The numbers presented are realistic- 26 inches per year and 52 inches per year are well within the numbers found in the state of Texas- mean annual precipitation varies from about 10 to about 60 inches (Bomar, 1983). An initial abstraction if 0.5 inch is also well within the numbers found by current research in Texas (Asquith and Roussel, 2007). Regular, weekly rainfall on a specific day is obviously not realistic but it allows us to treat rainfall events as independent in effects. Rainfall events are much more randomly distributed in time.

An attempt has been made, by the use of actual rainfall statistics, to represent the highly uncertain nature of rainfall (Asquith et al, 2006). This idealized conceptual example has been contrived to illustrate how an initial abstraction tends to skew and bias estimates of runoff coefficient as compared to more uniform model assumptions. Considerable difference in the summary statistical values is attributable to the interaction of an initial abstraction and the expected event depth.

In principle, a runoff coefficient computed from an event depth and the residue after the initial abstraction represents an upper bound for a practical rational runoff coefficient. It should be a point of beginning, followed by at least once more adjustment for other losses. In considering runoff events and probabilistically matching them to rainfall, we should account for those events where there is no runoff expected- there is a practical lower bound of rainfall that would result in any appreciable runoff.

Tables of runoff coefficients are found in almost every text on small watershed hydrology (Wanielista, 1990). Most if not all such tables are virtually identical, and are presented such that no consideration is given to where they are used; runoff coefficients for areas with 100 inches of mean annual precipitation are implied to be the same as those for areas with 10 inches of mean annual precipitation.

The assumption of matched frequency that underlies the use of the rational method is also subject to considerable scrutiny. It has been recognized that runoff coefficient does not appear constant across the frequency scale for many locations (Maidment, 1993). Tables exist for the "correction" of the runoff coefficient with return period. Such a correction might be much more universal if, instead of correcting with return period, the correction process related to the expected rainfall depth. It is likely that no general, universally applicable set of corrections exists, however a procedure for local corrections should be relatively simple to develop.

For the traditional uses of the rational method such as peak-discharge design of hydraulic structures like culverts and storm sewers, the net results of the censoring effects illustrated here may be inconsequential; particularly in humid areas. However, in arid areas, there may be significant overestimation of peak rates for relatively frequent events. Storm sewers for roadway drainage are often designed for frequent events.

As population grows, arid areas become more and more important as living space. Water in such areas has tremendous social and economic value, leading to the increasing popularity of the capture and use of urban runoff; so-called water harvesting. The rational method in a volumetric form, the so-called modified rational method, is very often used to estimate the expected yield of such watersheds. In these cases, neglecting to account for the censoring of rainfall by an initial abstraction would invariably lead to overestimation of available water. In the case of Drytown, fewer than half of events produce any runoff at all. The thought experiment performed here exists in a context that is unrealistically simplified, but it is so for a purpose. Real data and real measurements are made in a world of an unknown number of complicating factors of unknown magnitude. The use of thought experiments and contrived data allow us to peel back the unknowns layer by layer and systematically account for the enormous variability we see in measured data.

The purpose of this experiment is to demonstrate the vital importance of climatic context. The results of good hydrologic work that uses data from one region may not translate simply to another region- the methodology may, but translation may depend on the use of data endemic to another region. Attempting to make broad judgments based on data from widely varying regions may require that data be "equalized" in some ways that are not immediately evident. This exercise is based on the very simplest of hydrologic concepts- that of a simple, single-valued initial abstraction, and the relationship of that abstraction to the expected depth of a rainfall event. In spite of that conceptual simplicity, we have been able to show rather wide variation when considered in the context of rational coefficients and standard practice for small watershed hydrology. An ancillary observation is that if future research is going to improve our understanding of the physical processes, it is desirable that both rainfall and runoff data be collected in an associated form. A consistent record of rainfall events that produce no runoff is important in our overall understanding of the phenomenon of runoff production, and in the improvement of our modeling techniques. These events hold the place of "nondetect" values in our overall analysis of rainfall and runoff. There is considerable information value in these nondetects that can be of use to us in many ways (Helsel, 2005).

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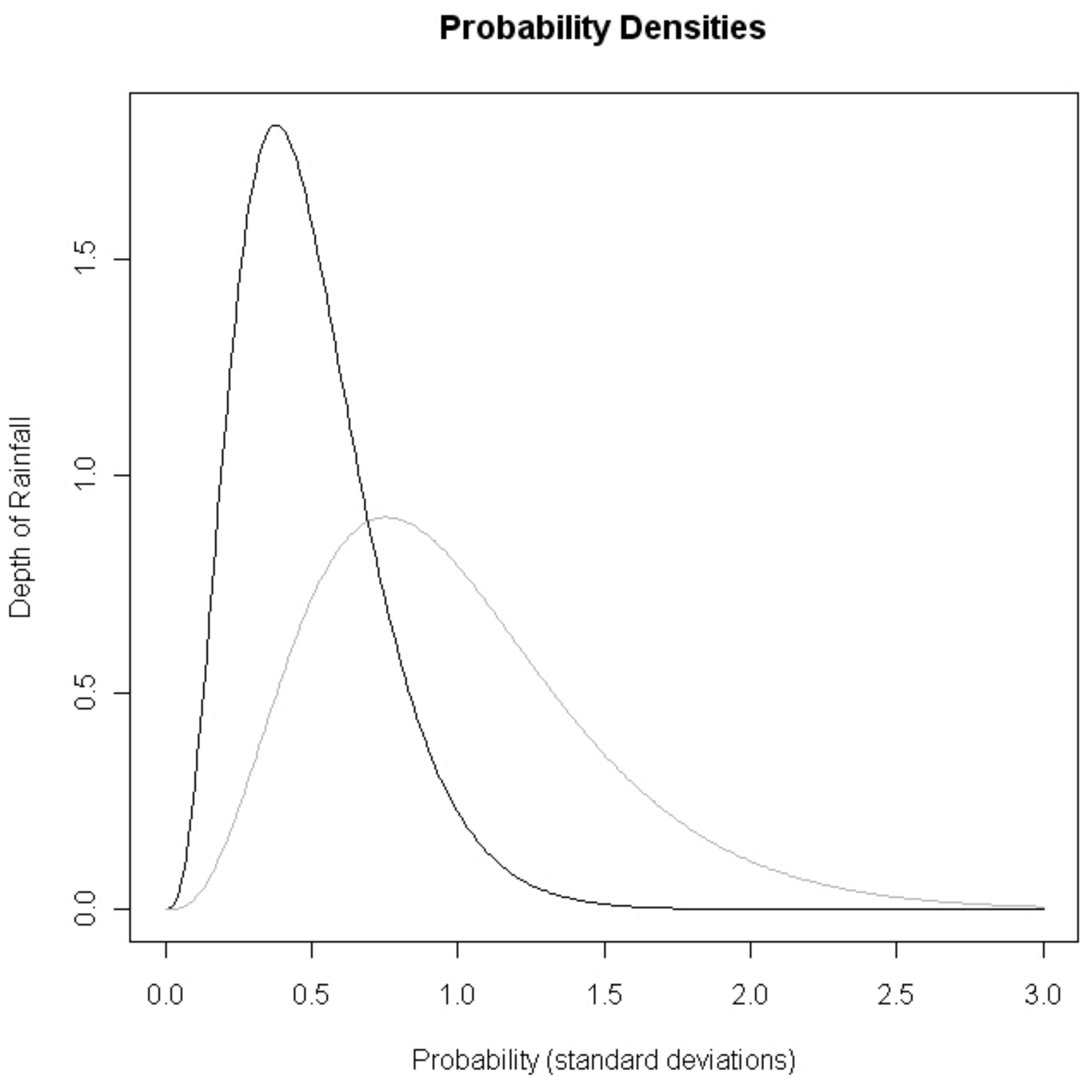

FIGURE 1: Probability Density Functions for Drytown(gray) and Rainyville (black)

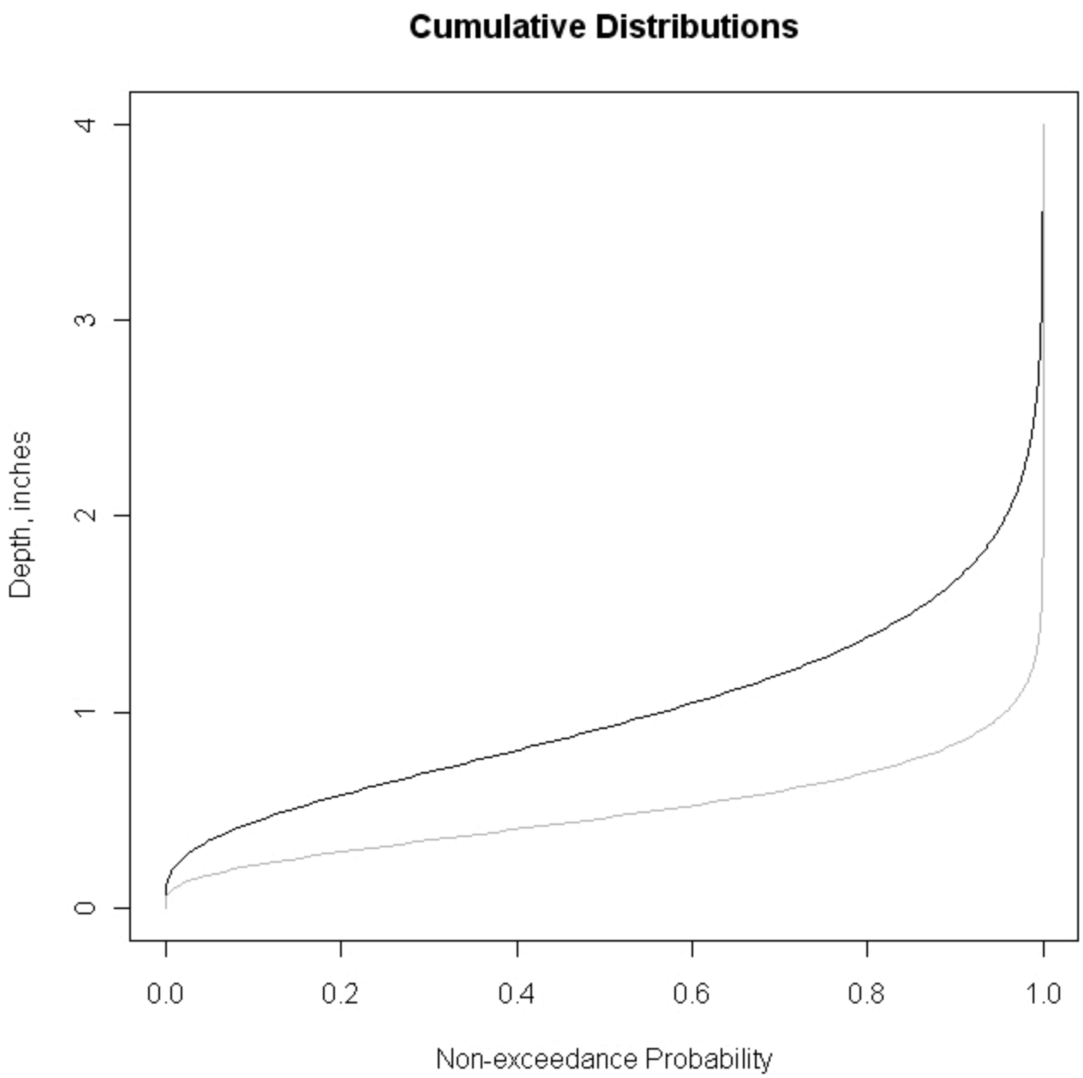

FIGURE 2: Cumulative Distribution Functions for Drytown(gray) and Rainyville (black)

TABLE 1: SUMMARY STATISTICS

	Rainyville	Drytown
Rainfall Mean	1.002	0.503
Rainfall Median	0.917	0.466
Rainfall Standard Deviation	0.499	0.250
Runoff Mean (censored)	0.603	0.225
Runoff Median(censored)	0.496	0.169
Runoff Standard Deviation(censored)	0.461	0.197
Runoff Mean (uncensored)	0.520	0.099
Runoff Median (uncensored)	0.416	0.000
Runoff Standard Deviation(uncensored)	0.476	0.172

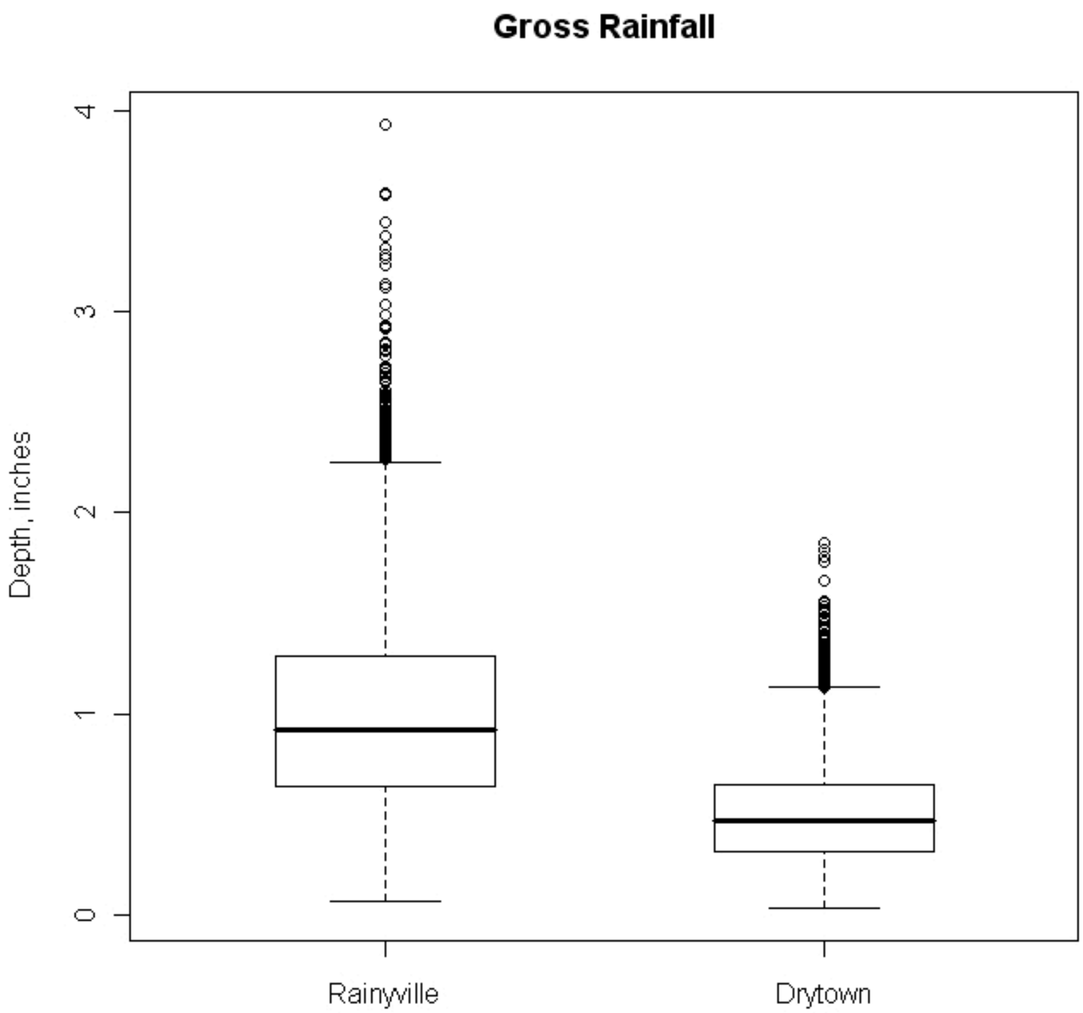

FIGURE 3: Boxplots of gross rainfall depths for Rainyville and Drytown

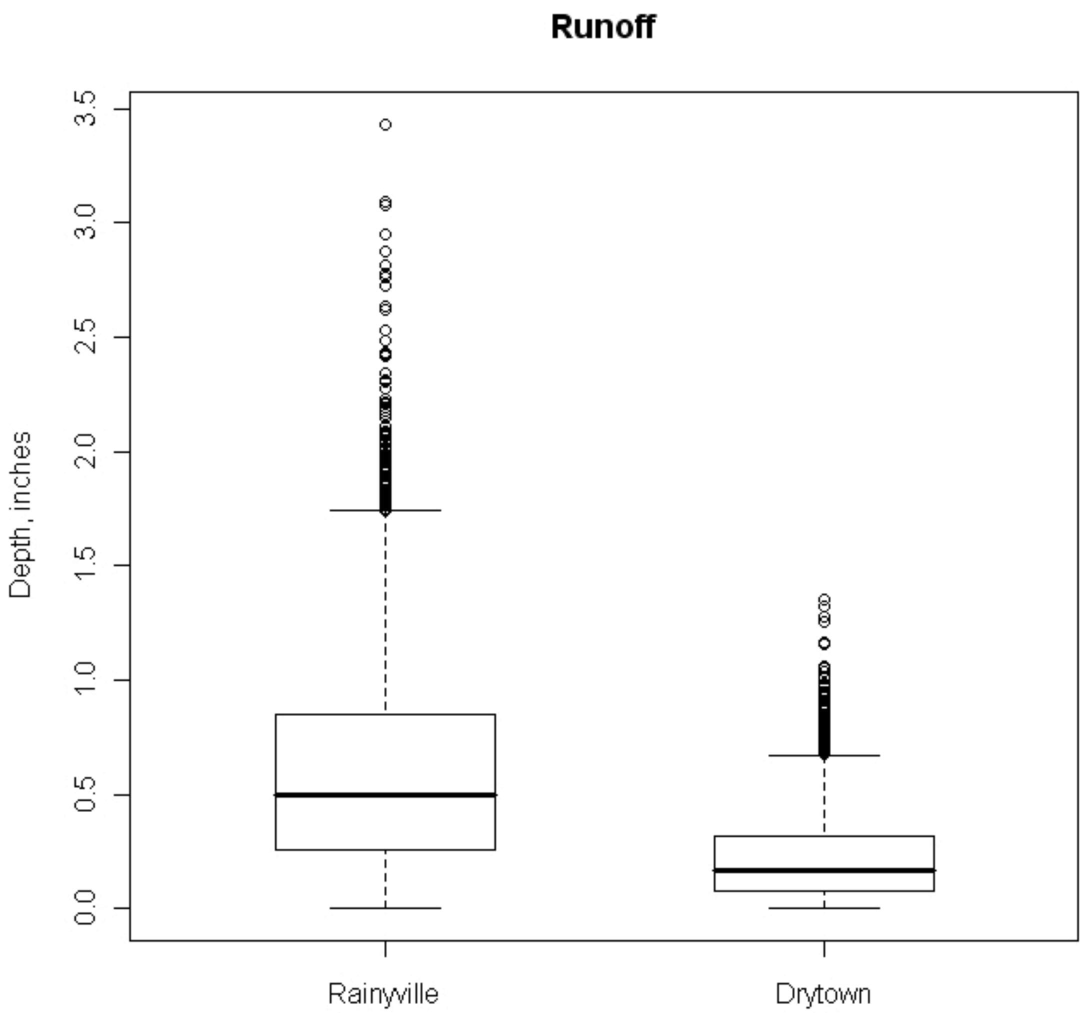

FIGURE 4: Boxplots of runoff depths for Rainyville and Drytown, 0.5 inch initial Abstraction

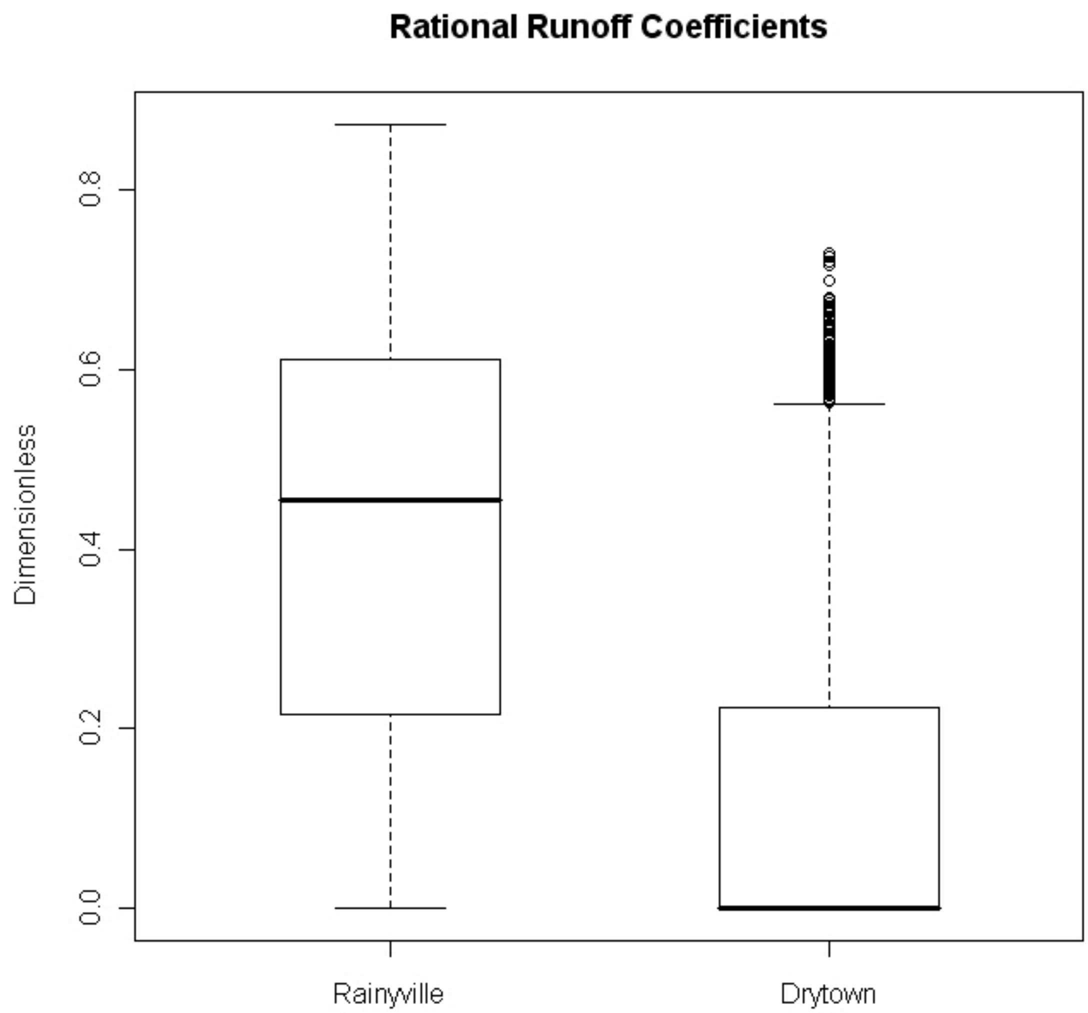

FIGURE 5: Boxplots of event specific runoff coefficients for Rainyville and Drytown, 0.5 inch initial abstraction

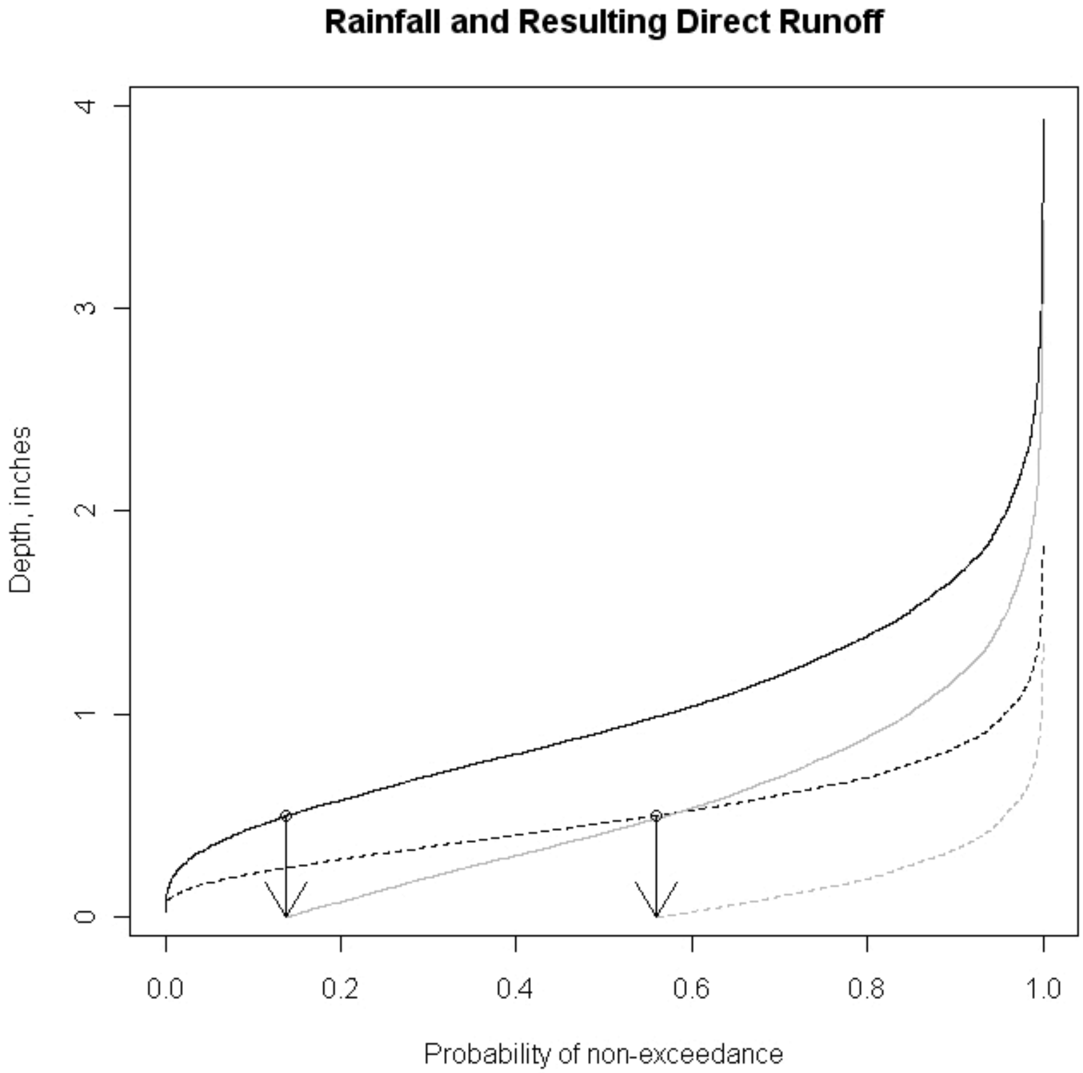

FIGURE 6: Empirical distributions (ranked sets) of rainfall and runoff for Drytown and Rainyville, with runoffs associated probabilistically with the rainfalls that produced them

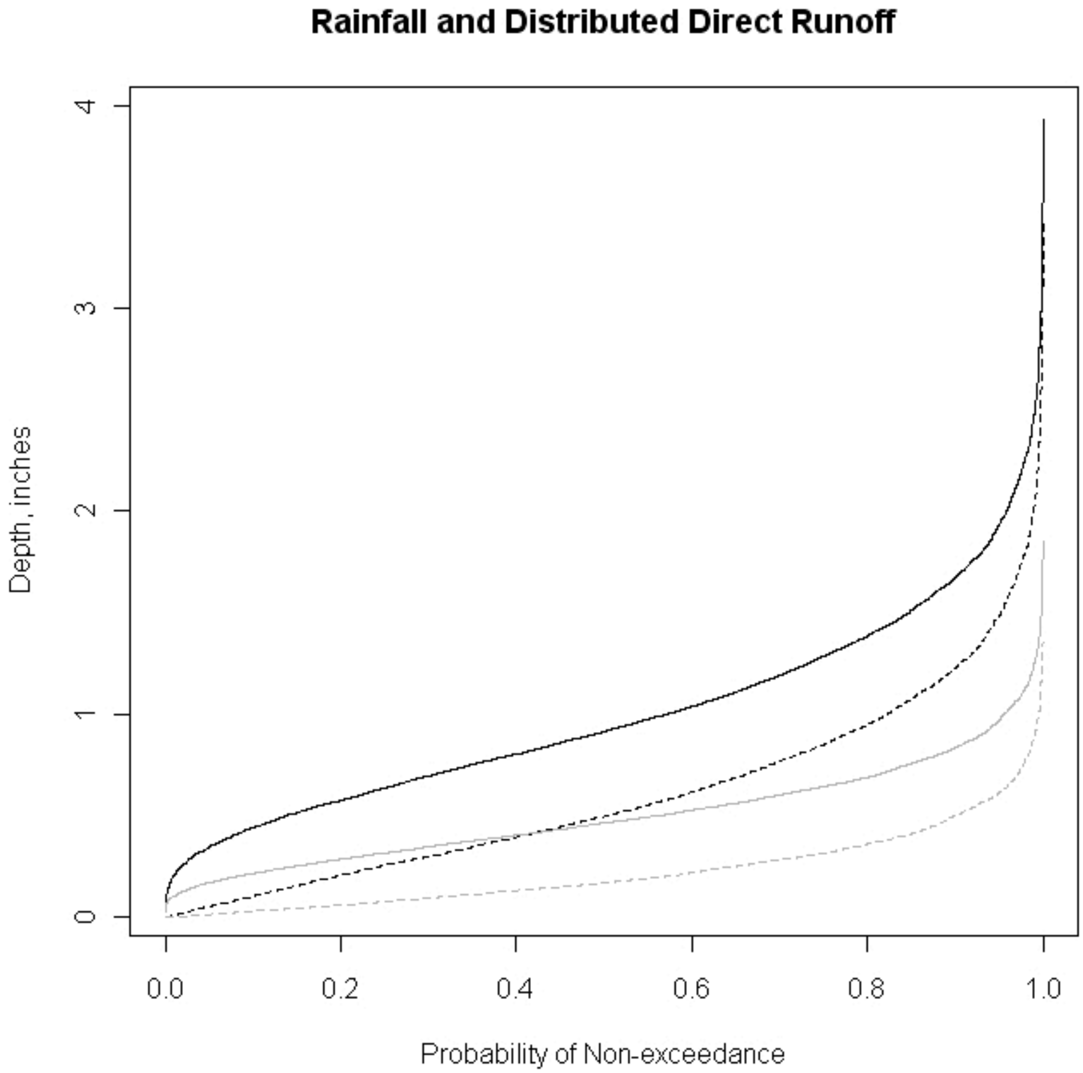

FIGURE 7: Empirical distributions (ranked sets) of rainfall and runoff for Dry- town and Rainyville, with runoffs distributed probabilistically by simple plotting positions