Rainfall Intensity in Design

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Abstract

An empirical, dimensionless-hyetograph that relates depth and duration, and thus whether a storm is front loaded, back loaded, or uniformly loaded, based on 92 gaging stations for storms known to have produced runoff is available for Texas. Statistical characteristics of storm interevent time, depth, and duration, based on analysis of hourly rainfall data for 533 rain gages are used to "dimensionalize" this hyetograph and produce a set of simulated storms. These simulated storms are analyzed to generate a set of rainfall intensities, and these intensities are compared to global maximum observed rainfalls, intensities estimated using the National Weather Service TP-40, and HY-35 publications, and a current Texas Department of Transportation design equation.

The simulated storms agree well with the other methods for rare (i.e. 90-th percentile and above) occurances and lie within the global maxima envelope. The simulated results are quite different for common (i.e. 50-th percentile) events.

Key Words: Rainfall depth; Rainfall duration; Global maximum precipitation.

Introduction

Rainfall intensity in this manuscript is the rainfall rate, $\frac{inches}{hour}$; $\frac{millimeters}{hour}$, for a specified time interval. This rate is important in hydrologic and hydraulic design in several contexts:

1. Rainfall intensity is used in the rational method for estimating peak discharges from small drainage areas without significant on-watershed (flood) storage. The numerical value used in the method is based on a response time appropriate for the drainage area, and usually lower-bound limited to 10 minutes[3].

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[3]This lower bound may vary among different agencies and jurisdictions. The value is arbitrary and is selected to prevent division of a depth by too small a time value.

2. Rainfall intensities also appear either directly or indirectly in unit hydrograph techniques and in design hyetographs. In this context, the methods are used to model the temporal distribution of rainfall and discharge to recover more complex responses that is possible in the rational method and over larger spatial areas than are appropriate for the rational method.

3. In BMP design, a specified rainfall intensity influences such design, and determines in some sense how long a BMP can perform its water quality function.

4. Actual rainfall intensities certainly influence the peak discharge rate from any watershed as well as determine the release of on-watershed storage in any given rainfall event. This interpretation is in part why a drainage system that can handle a 1-inch excess depth distributed over one hour, might be inadequate if that same depth is applied in a single minute, and is probably the most scientifically interesting reason to re-examine rainfall intensities for design.

This manuscript examines global maximum observed rainfalls from several historical sources and presents them in an intensity relationship. These rainfalls are compared to recent Texas hydrologic research as well as current recommended approaches to intensity. The manuscript goal is to address two questions:

1. Are estimated intensities consistent with global observed values?

2. Are recent studies producing different estimates as compared to older technology?

Data Sources

Asquith and others (2006) analyzed rainfall data for Texas, Oklahoma, and New Mexico and provide quantile functions and tables of L-moments that can be used to estimate the storm durations and storm depths for several hundred stations in this geographic region. A set of "estimated" intensities are available from depths and durations charted in Hershfield (1961) and Fredrick, and others (1977). Global maximum observed precipitation for different time intervals are available from various sources (Barcelo and others, 1997; Jennings, 1950; Paulhus, 1965; Smith,and others, 2001). Sether-Williams and others (2004) analyzed rainfall data for 92 stations in Texas for storms known to have produced runoff, and produced dimensionless hyetographs that relate cumulative storm depth to cumulative storm duration. These four sources comprise the underlying database (or estimates) used in this manuscript to examine intensities.

Instantaneous and Average Rainfall Intensities

Sether-Williams and others (2004) analyzed rainfall data for 92 stations in Texas for storms known to have produced runoff, and produced dimensionless hyetographs that relate cumulative storm depth to cumulative storm duration. Figure 1 is the set of dimensionless hyetographs.

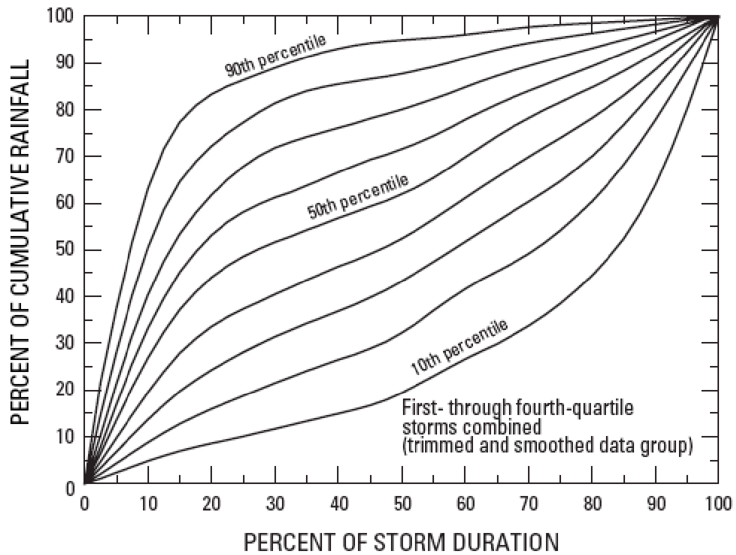

Figure 1: Empirical, Dimensionless Rainfall Hyetograph for Texas. From Sether-Williams and others (2004).

The slope at any point on these dimensionless curves would represent the instantaneous intensity for a particular storm. The maximum slopes are on the 90-th percentile hyetograph near the beginning of the storm or 10-percentile hyetograph late in the storm, and it is at these locations in a storm the maximum intensities are logically anticipated. More importantly, rather than where in a storm the maximum intensity might occur, is that the maximum intensity (slope) is larger than an average intensity[4].

The average or uniform rainfall intensity, Equation 1, is the ratio of total rainfall depth, P for a storm and the length, D (or duration) of that storm.

$$\bar{I} = \frac{P}{D} \tag{1}$$

[4] Albeit for a much shorter time interval.

The global maximum data in this paper are takes from sources that report the data as total rainfall depth and total storm duration, thus the global maximum intensities in this paper are all average intensities. Likewise, the National Weather Service TP-40 and HY-35 values reported in this paper are also average intensities in that the values from these sources are computed as the ratio of depth to duration. The average intensity, by definition, does not account for temporal variations of rainfall within a storm.

A design intensity equation, Equation 2, that relates duration and intensity appears in many hydrology contexts (TxDOT, 2002; Hann and others, 1994), and is commonly used to constructed intensities for specified quantiles. The value $\bar{I}_{\%}$ represents the specified percentile intensity at a given duration, t_c,[5], and the values of the estimated depth, b, time shift, d, and exponent, e, are usually determined from a tabulation (TxDOT 2002) or from local depth-duration-frequency data.

$$\bar{I}_{\%} = \frac{b}{(t_c + d)^e} \tag{2}$$

A global envelope curve (Paulhus,1965), Equation 3, that relates depth, P, in inches, and duration, D, in hours, also appears in many hydrology contexts and provides a useful upper boundary for rainfall depth and average intensity estimates.

$$P = 16.6D^{0.475} \tag{3}$$

If the global curve is expressed as an average intensity using Equation 1, the result is structurally identical to the design intensity equation (provided the time shift, d, is zero). This relationship is expressed in Equation 4.

$$\bar{I}_{GMax} = \frac{16.6}{(D + 0.0)^{0.525}} \tag{4}$$

A dimensionless average intensity relationship is sketched on Figure 2 as a straight line segment joining the corners of the plot. All of the empirical curves have significant portions of their dimensionless time history with slopes different from (non-parallel) to this average intensity dimensionless hyetograph, thus in computations where the magnitude of instantaneous intensity is important the average intensity is inadequate. However, these dimensionless hyetographs need to be made dimensional to be of any practical value for estimating intensities.

[5] The duration D and t_c have analogous meanings in this context. They differ in that t_c is specified by the designer, usually as a function of a drainage area, while D is intended to apply to an observed storm duration.

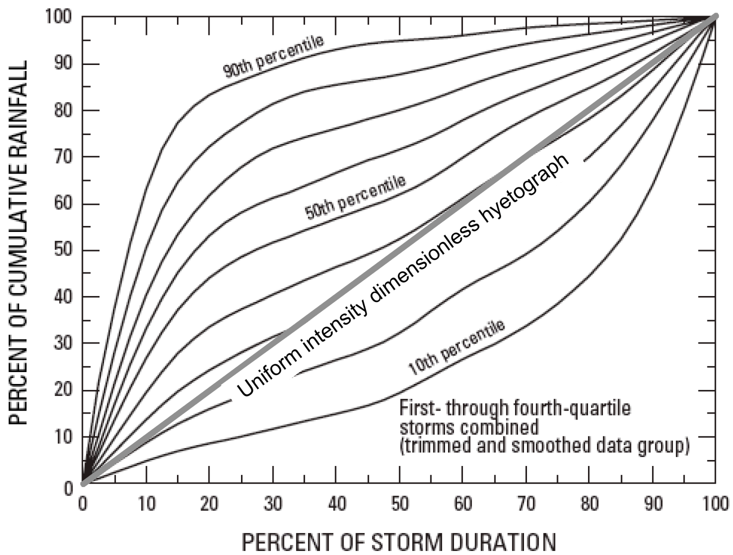

Figure 2: Empirical, Dimensionless Rainfall Hyetograph for Texas, with a uniform intensity dimensionless hyetograph. Adapted from Sether-Williams and others (2004).

Asquith and others (2006), presented the techniques required to dimensionalize these hyetographs although they did not explicitly do so. In their report, they presented an example (Example 5, pg 43) that illustrated how an analyst might statistically estimate average rainfall intensity for design purposes using the tabulated L-moments for a station and the Kappa distribution. In this manuscript the author's extend the example to illustrate how to combine these two concepts (dimensionless hyetographs and depth-duration quantiles) to generate intensity estimates.

At the time of the Asquith and others (2006) report, the authors did not provide the algorithm to fit the Kappa distribution to the tabulated L-moments. Since that time, Asquith (2007) built the *lmomco* package that runs in the **R** (R Core Development Team, 2007) statistical software environment, and these computations are now readily available. Both the R environment and Lmomco are available without financial charge from the Comprehensive R Archive Network (CRAN), and are distributed in versions that will run on a Windows, Macintosh, or Linux/UNIX operating systems.

Statistical Simulation of Instantaneous Rainfall Intensities

Figure 3 is a map that locates the selected rainfall sites used in this manuscript. These sites are selected, in part because at least two global maximum events have been observed in or near these stations in Harris County, Texas.

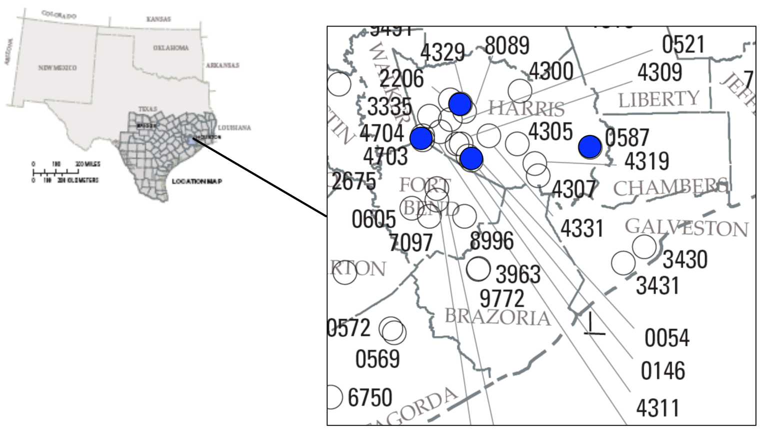

Figure 3: Location map of Harris County in relation to Texas, and the four study locations, Station Numbers: 0587, 4311, 4704, 4329

Asquith and others (2006) tabulated the L-moments for the stations depicted in Figure 3 for both the depth and duration [6] for different minimum interevent (times without rainfall) arrival times. This paper considered the 72-hour minimum interevent arrival times. This choice is based on the assumption that the shorter interevent tabulations represent behavior of a series of smaller storm systems (bands of storms training back-to-back across the area), and the 72-hour interevent values would represent more distinct, separate weather systems. While the smaller storm systems can produce large storm depths, exploratory analysis showed that they produces similar average intensities as the longer intervals, so the initial assumption was supported.

The L-moments are taken from the table and the Kappa distribution is fitted to these moments [7] Figure 4 is a portion of the tabulation of moments for the Texas study. In the figure, the values that are needed for fitting the Kappa distribution

[6] Different tables.

[7] More precisely, parameters of the Kappa distribution are adjusted so its L-moments are identical to the tabulated values.

for storm depth to a particular station are indicated. There is a similar table for duration (not pictured) that is also required for the intensity calculation.

Appendix 4–2.1. L-moments of storm depth defined by 6-hour minimum interevent time for hourly rainfall stations in Texas.

[–, not available]

Station no.	Depth mean (inches)	Depth L-scale (inches)	Depth L-CV (dimensionless)	Depth L-skew (dimensionless)	Depth L-kurtosis (dimensionless)	Depth Tau5 (dimensionless)	Station no.	Depth mean (inches)	Depth L-scale (inches)	Depth L-CV (dimensionless)	Depth L-skew (dimensionless)	Depth L-kurtosis (dimensionless)	Depth Tau5 (dimensionless)
0015	0.10273	0.07309	0.71150	0.64511	0.37189	0.20108	1154	0.37295	0.25986	0.69676	0.57203	0.31963	0.19463
0016	.38511	.25319	.65743	.50821	.25779	.16041	1165	.41702	.25491	.61127	.47291	.23266	.13521
0050	.50593	.29615	.58536	.43998	.21025	.12812	1185	.38435	.21807	.56738	.44470	.21001	.11434
0054	.31767	.18918	.59551	.47295	.22546	.11673	1186	.47148	.30674	.65059	.54254	.32113	.20723
0120	.60333	.34811	.57697	.41632	.18128	.08660	1188	.36091	.22691	.62872	.52724	.29420	.22636
0145	.37637	.27471	.72989	.65386	.45179	.33752	1245	.50585	.32884	.65007	.56565	.34435	.24622
0146	.35231	.20630	.58558	.38256	.10211	.01005	1246	.51250	.29959	.58457	.54484	.27126	.18238
0174	.32717	.17482	.53434	.53045	.29287	.17264	1267	.38735	.25338	.65413	.56782	.36857	.27911
0178	.26120	.17420	.66692	.59557	.34420	.17655	1304	.48273	.30605	.63399	.52854	.31054	.21522
0179	.29057	.16757	.57667	.47202	.22912	.11305	1325	.57612	.36736	.63765	.52144	.28187	.17563
0202	.48328	.26817	.55490	.51610	.24559	.14331	1429	.51873	.31486	.60698	.50041	.27546	.17399
0206	.54830	.30667	.55931	.47264	.23593	.15685	1431	.55718	.34261	.61490	.49416	.25140	.14443
0208	--	--	--	--	--	--	1432	.56381	.34775	.61678	.47764	.23891	.14393
0211	.30687	.20486	.66758	.53862	.29593	.19045	1433	.56333	.34151	.60623	.49279	.27261	.17319
0244	.45937	.24944	.54300	.32092	.08509	.03929	1434	.55992	.33985	.60696	.48783	.25455	.15369
0248	.35557	.20144	.56654	.51825	.28829	.15970	1435	.57954	.35543	.61330	.47738	.23958	.13883
0262	.58471	.34641	.59244	.46963	.24353	.15408	1436	.57380	.34692	.60460	.48197	.25623	.16255
0271	.69897	.43081	.61636	.44289	.21929	.18320	1437	.45071	.30677	.68064	.51465	.19012	.04387
0380	.62676	.40903	.65261	.55519	.33996	.23983	1438	.55409	.33903	.61186	.48286	.24756	.14680
0394	.46000	.24564	.53399	.34123	.16605	.05181	1462	--	--	--	--	--	--
0408	.86676	.46666	.53839	.35275	.20296	.17076	1492	.48235	.28420	.58919	.52638	.27591	.16216
0427	.43151	.25342	.58730	.58740	.28600	.15341	1500	.53606	.30894	.57631	.35829	.04138	-.06899
0428	.40801	.28272	.69293	.55708	.30334	.18985	1528	.47542	.29252	.61529	.55852	.31362	.18292
0429	.47570	.32658	.68653	.54493	.30187	.20464	1541	.65571	.37994	.57943	.47134	.18501	.11444
0463	.47370	.27083	.57173	.50067	.30770	.20084	1569	.50392	.34179	.67827	.57943	.37849	.29311
0493	.70842	.28749	.40581	.32264	.23370	.12487	1632	.47857	.26095	.54527	.12044	-.25730	.08212
0495	.32531	.18685	.57438	.46009	.25757	.17510	1641	.41386	.23517	.56824	.43397	.21010	.13376
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.37862	.21198	.55987	.53120	.28375	.15603
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.78113	.49390	.63229	.53790	.26175	.18446
0509	.52306	.32295	.61742	.53368	[illegible]	.18730	1671	.53025	.32321	.60955	.51905	.27902	.16730
0518	.55096	[illegible]	[illegible]	.49440	[illegible]	[illegible]	1680	.53631	.32269	.60169	.48079	.26018	.17040
0521	.42575	[illegible]	[illegible]	.50048	[illegible]	[illegible]	1694	.44124	.23819	.53982	.47688	.16994	.09024
0556	.50496	[illegible]	[illegible]	.47864	.26507	[illegible]	1696	.42951	.24908	.57991	.44753	.22765	.14778
0569	.61755	.39031	.63203	.54491	.30312	.18429	1697	.43084	.25071	.58191	.46878	.23303	.12625
0572	.55580	.35213	.63355	.52475	.29810	.19635	1698	.40740	.23556	.57820	.51632	.28599	.16120
0576	[illegible]	[illegible]	.70234	[illegible]	.35602	.24110	1720	.45870	.27593	.60153	.59269	.28396	.13711
0580	[illegible]	[illegible]	.66393	[illegible]	[illegible]	.22153	1761	.27129	.16508	.60850	.43922	.19379	.12601
0587	.57882	.37118	.64128	.51392	.27750	.18293	1773	.63171	.36447	.57695	.47188	.23508	.15098
0605	.60080	.30497	.50760	.39297	.20415	.13276	1810	.40231	.24425	.60711	.52907	.33370	.23100

```
lmrain<-vec2lmom(c(0.57882, 0.37118, 0.51392, 0.2775 ))
```

Figure 4: Table of L-moments for stations in Texas (from Asquith and others, 2006). The highlighted values, for study station 0587, are the arguments that are used by *lmomco* to fit the Kappa distribution.

Once the tabulations are assembled, the R program listed in Figure 5, performs the necessary computations to generate a set of intensities, each corresponding to some random probability.

In these simulations we have assumed that the depth and duration are independent random variables, thus we are not a-priori specifying if a high-percentile depth is associated with a low-percentile duration [8]. Once these depths and durations are

[8]This particular relationship should result in the most extreme intensities, however we chose not

```
# R Code to simulate Harris County Intensities (6-hour)
# Load the L-moment package from CRAN and attach as a library
 library(lmomco)
# Quantile Functions for Depth and Duration.
# Asquith and others, 2006, Eqns 13, and 14.
q_func<-function(f,p1,p2,p3,p4){(p1+(p2/p3)*(1-((1-f^p4)/p4)^p3))}
# L-moments for each station from Appendix 4, Asquith and others, 2006
# Station 0587, 6-hour inter-event arrival time
 lmdep<-vec2lmom(c(0.57882, 0.37118, 0.51392, 0.2775 ))
 lmdur<-vec2lmom(c(6.3865, 3.1849, 0.43733, 0.2504 ))
# get Kappa parameters from L-moments
 pardep<-lmom2par(lmdep,type="kap")
 pardur<-lmom2par(lmdur,type="kap")
 # generate 2500 random probabilities
 fdep<-runif(2500,0,1); fdur<-runif(2500,0,1)
# generate depths and durations associated with probabilities
dep<-q_func(fdep,pardep$para[1],pardep$para[2],pardep$para[3],pardep$para[4])
dur<-q_func(fdur,pardur$para[1],pardur$para[2],pardur$para[3],pardur$para[4])
# calculate intensities
avg_intensity<-dep/dur
```

Figure 5: Portion of R source that produces statistical simulation of intensity for a tabulated station.

computed, then for each pair, the dimensionless unit hyetograph is used to generate intensities within a storm (using 2.5-percentile increments).

Figure 6 is a plot of the simulated average intensities using the methods just described for 72-hour minimum interevent arrival times. These values are the cloud of blue markers. The solid red markers are the global maximum average intensities values inferred from various sources (Barcelo and others, 1997; Jennings, 1950; Paulhus, 1965; Smith,and others, 2001), and the open red markers are average intensities inferred from the U.S. National Weather Service publications TP-40 and HY-35 for Harris County, Texas, both sets of intensities computed using Equation 1.

The global envelope line, Equation 4, indeed is greater than the simulated cloud, TP-40, HY-35, or the design intensity equation (parameterized by b,d, and e values specified in the TxDOT hydraulics manual) - an anticipated result. The design intensity equation follows the shape of the TP-40, HY-35 markers, and lies along the 99-th percentile markers from those sources.

Using the simulated values as a “population” of possible values, the design inten-

to force such a relationship.

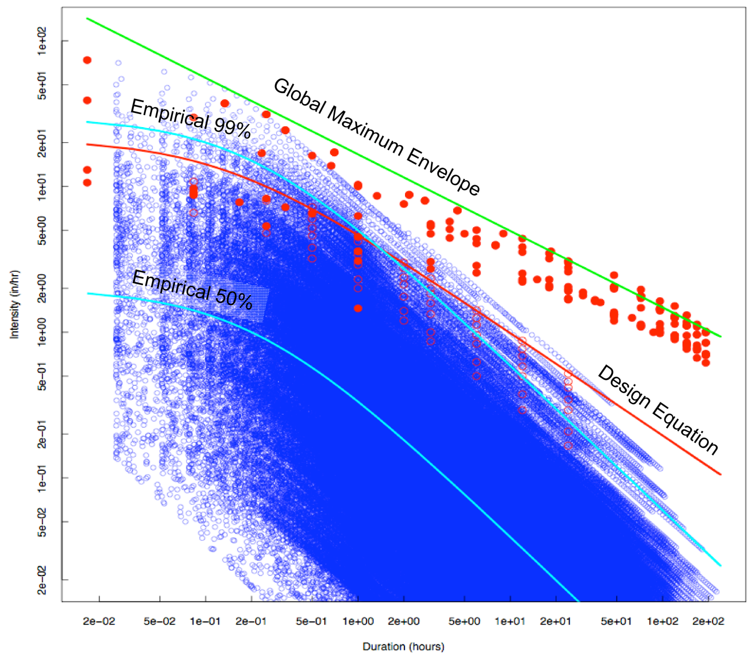

Figure 6: Statistical simulated instantaneous rainfall intensity using L-moments for 72-hour interevent arrival time for selected Harris County, Texas locations and the 90-th percentile dimensionless hyetograph.
Red line is the design intensity equation using TxDOT 99-th percentile value parameters, Eqn. 2;
Green line is global maximum average intensity envelope, Eqn. 4;
Solid red markers are intensities from global maximum events;
Open red markers are intensities estimated from NWS HY-35 and NWS TP-40;
Light blue lines are empirical 99-th and 50-th percentile envelopes, Eqn. 5

sity line represents about an 98-th percentile division, certainly close enough to the expected 99-th percentile that the design intensity equation is developed to model. However, the design equation is achieving this value by estimating intensities in the 1-hour or greater durations in excess of the simulation cloud, while a considerable number of simulated intensities lie above the design equation at very short durations. The usual limit of 10-minutes for the design equation is quite reasonable in this context as intensities increase by about one-log cycle as the time interval is decreased at the same rate.

The statistical simulation approach does permit the analyst to develop different empirical design equations. For example, Equation 5, with the value $D = 6$ is a model that encompasses the 99-th percentile of the simulation cloud, and preserves the shape of the simulation cloud at higher durations while preserving some of the desired asymptotic behavior at short durations. A similar model for the 50-th percentile is the same equation with the value $D = 0.4$. These two curves are also plotted on Figure 6.

$$I_{HC\_Max} = \frac{D}{t_c + 0.2} \tag{5}$$

The global envelope line, Equation 4, is useful as an upper bound, but it represents extremely rare events, that if used for routine design would result in unnecessarily large accomodations. In this example, there are $195,000$ markers in the plot, representing 5000 simulated "storms". Of these storms, only two come relatively close to the global envelope, indicating the rarity of such events[9].

Of greater importance than the rarity of the global maximum events is that at larger durations the global envelope, TP-40, HY-35, and TxDOT design equation has distinctly different slope that does the Texas simulated "storms." This difference, assuming the simulations are an accurate representation of actual behavior, has some important implications.

Figure **??** is the same two plots with Houston, Texas intensity-duration-frequency curves plotted as an overlay. The scales of the Houston plot are adjusted to be approximately correct with respect to the simulated storms. This figure illustrates that the design equation, Equation 2 parameterized for Harris County, Texas and the Houston curves are nearly identical except for the shorter durations. Equation 2 parameterized for a 50-th percentile when compared the the simulated storms is in effect a 97-th percentile curve[10]. Thus, the TP-40, HY-35, and the TxDOT design equation all represent the rare (90-th percentile and above) events well and there is little need for change in the methodology. However, in representing less rare events, there is a significant difference that should be explored.

Part of the difference is how the simulations are interpreted. In the present work these simulations represent storms that occur no less than every 3-days, while the TP-40, HY-35, and the design equation attempt to represent the largest storms in any given year, thus these two populations should be quite different. What is reassuring

[9] Such events, assuming simulations are accurate, are a $\frac{2}{5000} = 0.04\%$ chance event.

[10] This curve is not plotted, but would be parallel to the current design curve, but pass through the lowest set of open red markers.

is that the rarest events are the same regardless of underlying interpretation, while the common events behave quite differently.

Design models are relatively straightforward to postulate and determine their empirical probability levels using the statistical simulation approach by a straightforward partitioning of the simulated intensity-duration results. Whether such models are necessary is beyond the scope of this paper.

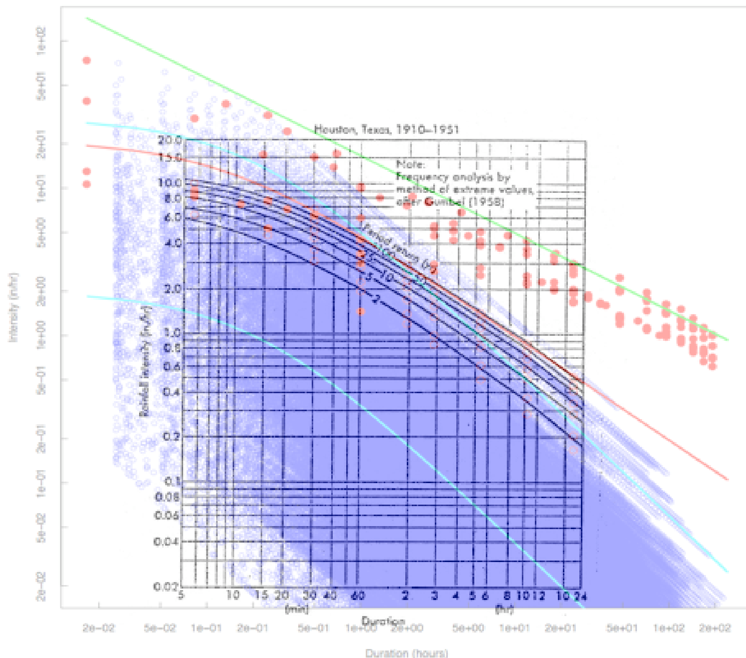

Figure 7: Statistical simulated instantaneous rainfall intensity using L-moments for 72-hour interevent arrival time for selected Harris County, Texas locations and the 90-th percentile dimensionless hyetograph, with Houston, Texas intensity-duration-frequency curves as an overlay.

Summary and Conclusions

This paper presented examples using recent technology developed for Texas to estimate rainfall intensities in a design context using L-moments and Kappa distributions to estimate depths and durations for simulated "storms". These depth and duration estimates are extended to time histories using empirical dimensionless hyetographs.

The approach produces results that are consistent with and contained within the global maximum observed values for rainfall and duration, agree well with intensities estimated using TP-40 and HY-35, and existing design methods for rare events (i.e. 90-th percentile and greater).

The results are different for more common events because of how the "storms" are created[11]. Example envelopes for Harris County for different probabilities are illustrated

The largest assumption in this paper is that depth and duration are independent – the data used suggest that this assumption may not be poor, but the author's believe that these two variables are highly coupled and the conditional dependence should be studied.

Finally, to answer the two questions posed at the beginning:

1. Are estimated intensities consistent with global observed values? Yes, the simulated storms using L-moments, the Kappa distribution, and the 90-th percentile dimensionless hyetograph fall at of below the global observed intensities.

2. Are recent studies producing different estimates as compared to older technology? A qualified yes. For rare events, 90-th percentile and above the estimated intensities are about the same. For common events, the results are quite different, in part because of what the simulated storms represent. These differences certainly need further study.

[11]The simulated storms are not necessarily the largest storms in any given year, they are just storms with known depth and duration probabilities. In contrast the TP-40, HY-35 and design methods all are based on largest storm within a year analysis.

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