Unit Hydrographs for Small Watersheds in Texas Using Particle-Tracking

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Abstract

The unit hydrograph is a well-documented method for predicting the runoff hydrograph of a watershed given an excess rainfall hyetograph. Watershed-specific unit hydrographs are parameterized by some conceptual time such as time to peak runoff or time of concentration. Estimation of time parameters when paired rainfall and runoff observations are available is reasonably straightforward. In the absence of paired observations, the parameters are estimated from regression equations that relate unit hydrograph behavior to watershed physical characteristics such as area, slope, and land use. This study performed an exploratory assessment of a particle tracking (PT) method to estimate unit hydrograph timing parameters directly from a digital elevation model (DEM) instead of the regression approach.

The PT method was used to estimate the unit-hydrograph timing parameters of 135 selected small watersheds in Texas using a DEM of each watershed, and an assumed characteristic velocity to generate a watershed-specific unit-hydrograph. The study watersheds have drainage areas ranging from approximately 0.25 to 150 square miles, main channel lengths ranging from approximately 1 to 50 miles, and dimensionless main channel slopes between approximately 0.0002 and 0.02. The selected watersheds include coastal plains as well as inland hilly regions. Paired rainfall-runoff data
for over 2,600 storms were compiled; these provide actual observations to test the performance of the PT model.

Key Words: digital terrain models, unit hydrograph, time to peak, time of concentration, rainfall-runoff

Introduction

Estimation of the time-response characteristics of a watershed is fundamental in hydrologic analysis and rainfall-runoff response modeling. Responses to real or design rainfall, such as peak discharge, hydrograph recession, and the time evolution of cumulative runoff, are greatly influenced by time characteristics. Rainfall-runoff models that incorporate timing parameter variables are used by engineers and others for

hydrologic design purposes, including the design of bridges, culverts, and detention facilities. Therefore, during 2004-2005, a consortium of researchers in cooperation with the Texas Department of Transportation (TxDOT), investigated timing parameter estimation approaches for selected Texas watersheds (TxDOT Research Project 0-4696).

The time-response characteristics of the watershed frequently are represented by two conceptual time parameters, time of concentration (T_c) and time to peak (T_p). T_c is typically defined as the time required for runoff to travel from the most distant point along a hydraulic pathline in the watershed to the outlet. T_p is defined as the time from the beginning of direct runoff to the peak discharge value of a unit runoff hydrograph. Conversion between the two is required as most hydrograph models use T_p but hydrologic engineers often first consider T_c.

For this study, 92 watersheds with USGS streamflow-gaging stations were selected for estimation. The necessary rainfall and runoff data (Asquith and others, 2004) for investigation are available for these watersheds. For the 92 watersheds selected for study, drainage areas are between approximately 0.25 to 150 square miles, main channel lengths are between approximately 1 to 50 miles, and dimensionless main channel slopes range approximately from 0.002 to 0.02.

This paper presents an independent estimation of T_c and T_p for comparison to the several other methods examined in the research project and reported in Roussel and others (2005). This paper was documented separately because is substantially different from the empirical approaches considered in the Roussel report and is not likely to be directly used by applications-oriented personnel. However, the results are important because they support estimates derived using empirical approaches and those estimates backwards-extracted from the measured rainfall-runoff responses. Furthermore, the work in this paper was not considered to be sufficiently refined enough for generalization. Despite these limitations the work constitutes an independent consideration of timing parameters consistent with the timing parameters reported in Rousseau and others (2005).

Prior Studies

Traditionally as suggested by Sherman (1932) and explained in many references, the UH of a watershed is derived from observed runoff and rainfall records. For ungaged watersheds, such data are unavailable, and synthetic methods are used to infer the unit hydrograph. These methods vary in how the geomorphic information from the watershed is incorporated to produce estimates of the unit hydrograph.

Clark (1945) developed a method for generating unit hydrographs for a watershed based on routing a time-area relationship through a linear reservoir. Excess rainfall covering a watershed to some unit depth is released instantly and allowed to traverse the watershed and the time-area relation represents the translation hydrograph. The time-area relationships are usually inferred from a topographic map. The linear reservoir is added to reflect storage effects of the watershed. Clark's method clearly attempts to relate geomorphic properties to watershed response.

Leinhard (1964) derived a unit hydrograph model using a statistical-mechanical analogy and two important assumptions. The first is that the travel time taken by an excess raindrop landing on the watershed to the outlet is proportional to the path-line distance the raindrop must travel. The second assumption is that the area swept by any characteristic distance is proportional to some power of that characteristic distance. Dimensionally, the ratio of the travel time to path length would be a characteristic velocity. Lienhard's derivation did not attempt to relate watershed properties that might appear on a map to the hydrologic response, but the connection was implied.

Rodriguez-Iturbe and Valdez (1979) and Gupta and others (1980) examined the structure of unit hydrographs conceptualized as residence time distributions from a geomorphic perspective and provided guidance to parameterize the hydrographs in terms of Horton's bifurcation ratio, stream length ratio, and stream area ratio and an independently specified basin lag time. In these works the result was called a geomorphic unit hydrograph (GUH). Like Leinhard's derivation the relationships of path, path length, and travel time are fundamental in the development of the unit hydrographs. Furthermore, all these derivations rely on the concept of representing the excess rainfall as an ensemble of particles distributed on the watershed.

Jin (1992) developed a GUH based on a gamma-distribution and suggested a way to parameterize the distribution based on path types and a streamflow velocity. Like the prior work, the concept of distance, velocity and time was crucial. In Jin's GUH the initial estimate of velocity was based on a peak observed discharge for a basin, thus some kind of streamflow record was required, or some estimate of bankflow discharge would be required.

Maidment (1993) developed a GIS-based approach using the classical time-area method and GIS scripts. Muzik (1996) approached the time-area modeling in a similar fashion. These works used flow routing based on a constant velocity or subjectively predetermined velocity map independently incorporating concepts of a GUH.

Kull and Feldman (1998) assumed that travel time for each cell in the watershed was simply proportional to the time of concentration scaled by the ratio of travel length of the cell over the maximum travel length. Thus the velocity from any point to the outlet is uniform and constant. Each cellÕs excess rainfall is lagged to the outlet based on the travel distance from the cell. Travel time in overland and channel flow are determined beforehand. This approach is essentially a version of ClarkÕs (1945) methodology and is implemented in HEC-GEOHMS (HEC 2000).

Saghafian and Julien (1995) derived a GIS-based time-to-equilibrium approach for any location on a watershed based on a uniform overland flow model, that incorporated elevation information. Saghafian and others (2002) used this concept to develop a time-variable isochrone GIS technique to generate runoff hydrographs for non-uniform hyetographs (non-uniform in space and time).

Olivera and Maidment (1999) developed a raster-based, spatially distributed routing technique based on a first-passage-time response function (a gamma-type unit hydrograph at the cell scale).

Lee and Yen (1997) recognized that a kinematic-wave model could be used to estimate travel times over a path to the watershed outlet and developed a procedure to parameterize a GUH by relating slope along a set of planes defined by stream order that are linked to each other an the watershed outlet.

Additionally numerous methods in the literature relate properties such as selected lengths, slopes, fraction of sewer served areas, etc. to unit hydrograph timing parameters. A selection of the more common methods appears in Rousseau and others (2005). The methods in the Rousseau report generally use a handful of measures to estimate the timing parameters and were developed prior to common availability of digital elevation data.

The significance of all these studies is that the concepts of distance, velocity and time need to be linked to physical characteristics of the watershed to parameterize a unit hydrograph in the absence of observed runoff and rainfall data. Additionally, the GIS studies appeared to have evolved in parallel to the GUH theory incorporating similar ideas while implicitly parameterizing the underlying GUH by various methods. Shamseldin and Nash (1998) argue that GUH theory is equivalent to the assumption of a generalized UH equation described by a distribution whose parameters must subsequently be related by regression (or otherwise) to appropriate catchment characteristics.

This paper presents the results of a hybrid approach to estimate the parameters of a GUH by analysis of an arrival time distribution of rainfall particles, whose travel speeds and paths are determined by local watershed slope. A particle-tracking program was used to generate the arrival time distribution, and 30-meter digital elevation model (DEM) data were used to compute local slopes and travel paths. A UH equation was then fit to the arrival time distribution to extract a timing parameter and a shape parameter, unique to each watershed; an approach similar to Shamsheldin and Nash's (1998) argument.

Because a rainfall-runoff database exists for the study watersheds, the resulting unit hydrographs can be used to generate runoff hydrographs for the historical rainfalls and these modeled hydrographs compared to the observed hydographs to evaluate the performance of the particle tracking approach. UH were also generated by a conventional analysis (of the observed data) approach to provide a performance comparison.

Rainfall-Runoff Database

A digital database of rainfall and runoff values for over 1,600 storms from 92 developed and undeveloped watersheds in Texas was used for the research. The database is described and tabulated in Asquith and others (2004). A watershed properties database was developed from 30-meter digital elevation models. The watershed properties

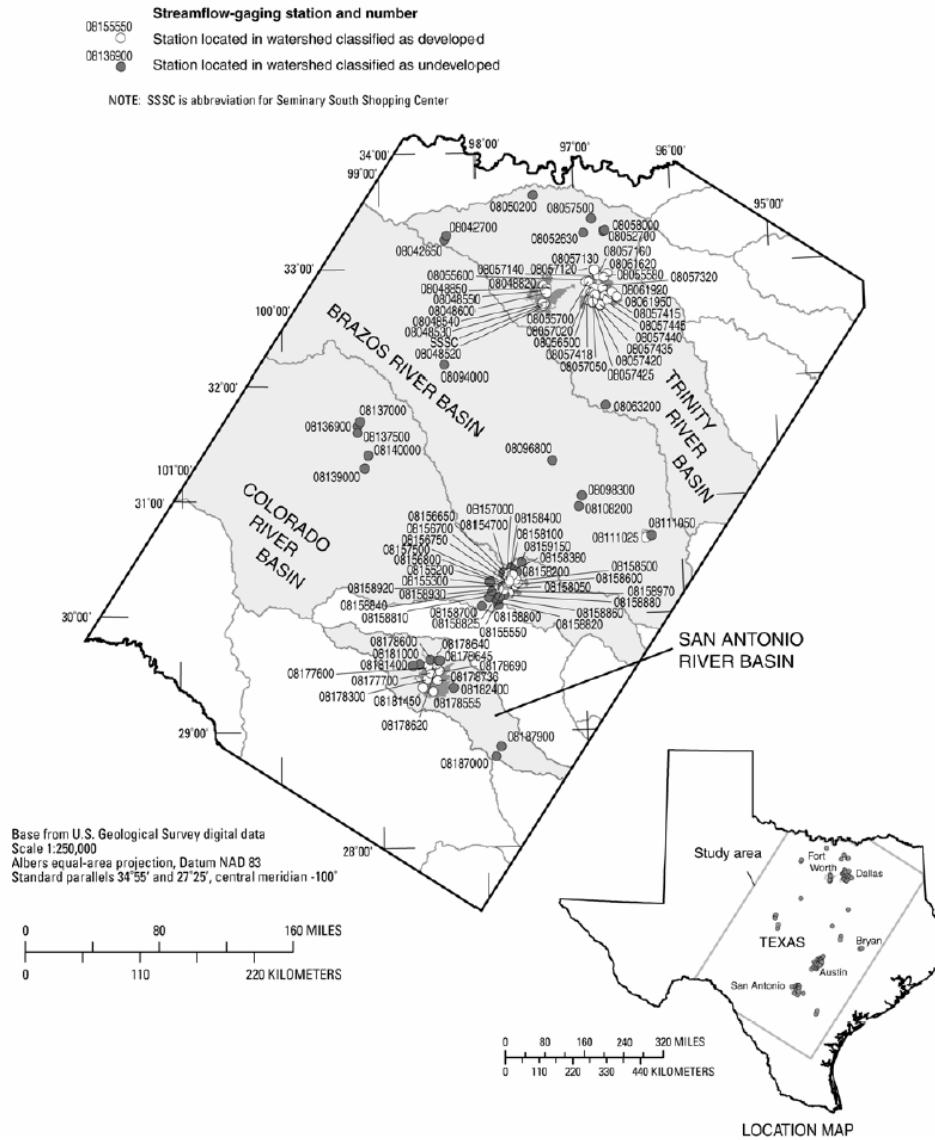database is described in Roussel and others (2005). Figure 1 is a map of the study watershed locations.

Figure 1. Locations of USGS streamflow-gaging stations represented in the database. (from Roussel and others, 2005)

Included in this paper are results from an additional 33 watersheds in the Houston area. Although these constitute additional data as compared to the Roussel (2005) report, the conclusions are unchanged.

Methodology

Leinhard's unit hydrograph model is a generalized gamma distribution (Leinhard, 1964; 1967) and is expressed as

$$f(t) = \frac{\beta}{\Gamma(n/\beta)} \left(\frac{n}{\beta}\right)^{n/\beta} \frac{1}{t_{rm\beta}} \left(\frac{t}{t_{rm\beta}}\right)^{n-1} \exp\left[-\frac{n}{\beta}\left(\frac{t}{t_{rm\beta}}\right)^{\beta}\right]$$

[1]

The distribution parameters n and $t_{rm\beta}$ have physical significance in that $t_{rm\beta}$, is a mean residence time of a raindrop on the watershed, and n, is an accessibility number, roughly equal to the exponent on the distance-area relationship (a shape parameter). β, is the degree of the moment of the residence time; $\beta=1$ would be the arithmetic mean, while for $\beta=2$ the residence time is a root-mean-square time. $\beta=2$ is used throughout this work. Equation 1 can also be expressed as a dimensionless hydrograph using the following transformations (Leinhard, 1972) to express the distribution in conventional dimensionless form.

$$t_{rm\beta} = \left(\frac{n}{n-1}\right)^{1/\beta} T_p$$

[2]

$$Q_p = f(T_p)$$

[3]

Expressed as a dimensionless hydrograph distribution equation 1 becomes

$$\frac{Q}{Q_p} = \left(\frac{t}{T_p}\right)^{n-1} \exp\left[-\frac{n-1}{\beta}\left(\left(\frac{t}{T_p}\right)^{\beta} - 1\right)\right]$$

[4]

The challenge is how to determine values of n and $t_{rm\beta}$ from a terrain model, which is conceptually equivalent to determining unit hydrograph parameters from physical watershed characteristics (for example: main channel length, slope, etc.), except this work examines an ensemble of characteristics (all the potential flow paths, slope along these paths, etc.). Specifically this research addressed this task by placing a computational parcel on each cell of a DEM grid, computing the direction this parcel would move from an 8-cell pour point model (O'Calligan and Mark, 1984), and computed the speed of the parcel according to a uniform flow equation. The entire ensemble of particles is moved contemporaneously and the arrival times of individual particles at the watershed outlet are recorded. The cumulative arrival time distribution of the particle ensemble is a residence time distribution of excess rainfall on the watershed and thus this distribution must contain information equivalent to an S-curve hydrograph. By fitting the Leinhard hydrograph (Equation 1 or 4) to this empirical S-curve, unit hydrograph parameters are recovered. Specification of how the particles move in response to their position on the watershed elevation grid determines the specific shape of the S-curve and ultimately the estimates for T_p and T_c.

Figure 2 depicts the watershed that drains past the U.S. Geological Survey (USGS) gaging station 08057320, also called the Ash Creek watershed in this research. In the figure the solid curve represents the path that a excess raindrop would follow from the northeast part of the watershed to the outlet located in the southwest corner of the watershed. This curve is called the pathline. Any point in the watershed can be represented by its Cartesian coordinates, x and y. A particle at any point in the

watershed will lie on its pathline. The particle at any position will have an x-component and y-component of velocity, and these two components can be resolved into a pathline component of velocity. The relationship between the pathline system and the Cartesian system is depicted in Figure 2 as the two velocity vector systems on the Eastern side of the figure, near the peak of a hill.

Over a short time interval, the particle will move a distance determined by the product of the appropriate component velocities and the time interval. Either a Cartesian coordinate system or a path-line coordinate system can be used. In the present case a pathline system is used. The work assumed the square of velocity is proportional to watershed slope, and therefore the velocity field depends on the particle positions. Equation 3 represents the formula in a path line coordinate system used to determine the velocity at any location in the watershed.

$$u(\xi) \cdot |u(\xi)| = k^2 * \frac{dz}{d\xi}\bigg|_{(\xi)} \qquad [5]$$

The value of k^2 represents the square of velocity of the particle on a unit slope. The absolute value formulation is used so that the numerical method preserves correct directional information (flow is always downslope). This approach is similar to existing methods, but makes no distinction between channel and overland flow. All results presented in this paper are based on this velocity model.

In the present work we have adopted a Manning's-type structure for k resulting in the following kinematics model:

$$u(\xi) = \frac{1.5}{n_f} d^{\frac{2}{3}} (\frac{dz}{d\xi}\bigg|_{(\xi)})^{\frac{1}{2}} \qquad [6]$$

where n_f is a frictional term (an adjustable parameter) that is conceptually analogous but not numerically equal to Manning's n, and d is a mean flow depth (an adjustable parameter). Equation 6 is intended to look like a Manning's equation (the last term is the local slope of the watershed at the particle location). These kinematics are identical to Wooding's (1965) kinematic wave analysis for overland flow and similar to the isochrone derivation technique of Sagafian and Julien (1995) who adapted the kinematic wave theory for distributed rainfall-runoff modeling and presented an example (Saghafian and others, 2002) for a single watershed in West Africa.

Figure 3 is the normalized arrival time distribution for the watershed in Figure 2 along with the subsequent fitted curvilinear hydrograph models. The markers in the figure represent the empirical S-curve hydrograph for the watershed. The program that makes these computations in referred to herein as the Digital Terrain Runoff Model (DTRM). The solid S-curve is the cumulative distribution function based on the unit hydrograph function (Equation 1 or 4). The cumulative distribution is "fit" to the empirical S-curve hydrograph using a least square error minimization criterion. Once the distribution parameters, n and $t_{rm\beta}$ are recovered, they are then converted into conventional hydrograph parameters using Equations 2 and 3.
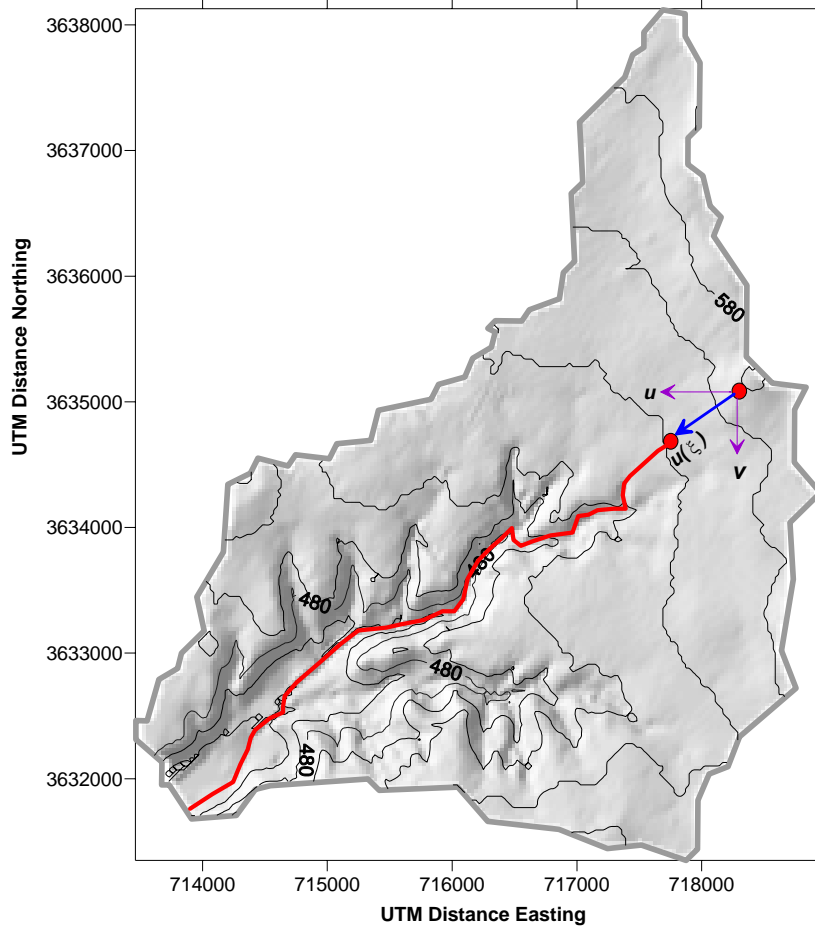
Figure 2. Shaded relief map of Ash Creek watershed

In principle, the time of concentration, T_c, should be the time at which the cumulative hydrograph is unity, but the cumulative hydrograph approaches unity asymptotically, so the authors' selected T_c as the time when the cumulative hydrograph obtained a value 0.98, a fraction of the total distribution. The choice of the value 0.98 is strictly ad hoc.

Application and Results

The DTRM was applied to the entire set of watersheds using 30-meter digital elevation data. The values used in equation 5 for generating the cumulative hydrographs are *n=0.04* and *d=0.2* and were determined by trial-and-error using the Ash Creek watershed and the June 3, 1973 storm to "calibrate" the particle-tracking model. Then these two values were applied to all watersheds regardless of size and location.
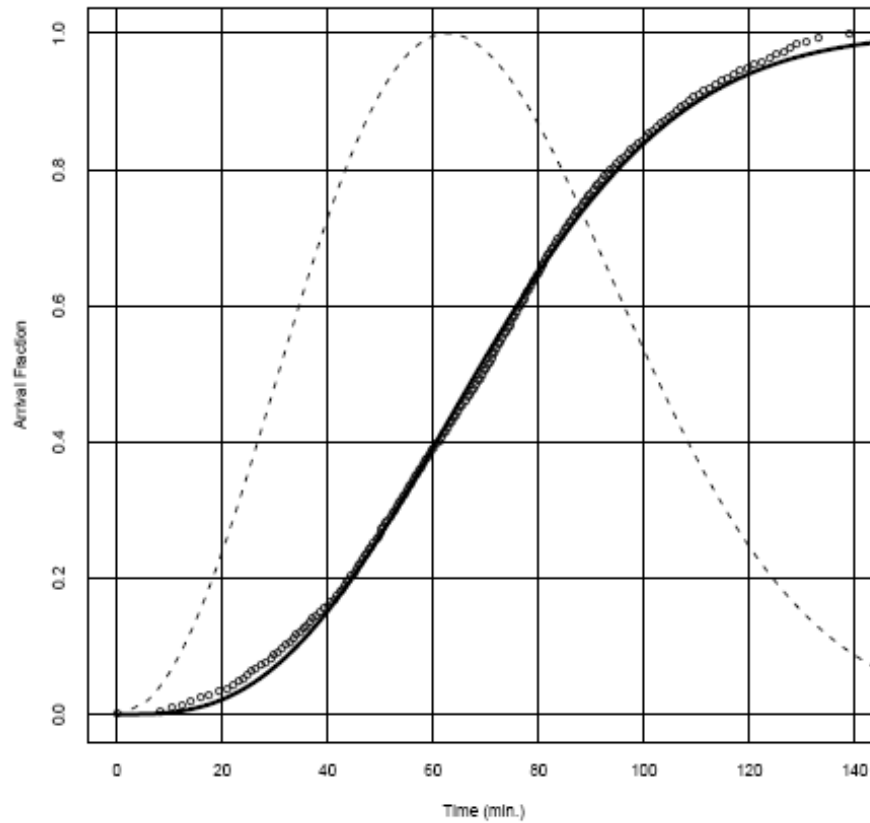
Figure 3. Empirical cumulative arrival time distribution (open circles) and fitted cumulative unit hydrograph distribution (solid line). The cumulative unit hydrograph is the integral of Equation 1. The dashed line is the dimensionless unit hydrograph for this watershed (Equation 4).

Convolving the historical rainfall hyetographs through the unit hydrograph parameterized by the DTRM values and comparing the model hydrograph with the historical hydrograph tested the performance of the method. For each watershed, DRTM was run once and a single Leinhard hydrograph, with two parameters is generated for each watershed. These two values are determined entirely from topographic data and the assumed n_f and d; no actual rainfall-runoff data is used by the DTRM.

Figure 4 is a representative example of output from this testing using observed data from the authors' database. The observed hydrograph is the dashed line with the step-wise changes in value, while the smooth curve is the model result using the same hyetograph (input rainfall) and convolving this rainfall with the Lienhard unit hydrograph using the watershed values for n and $t_{rm\beta}$. The plot in Figure 4 is typical, but not all storms were reproduced equally well.
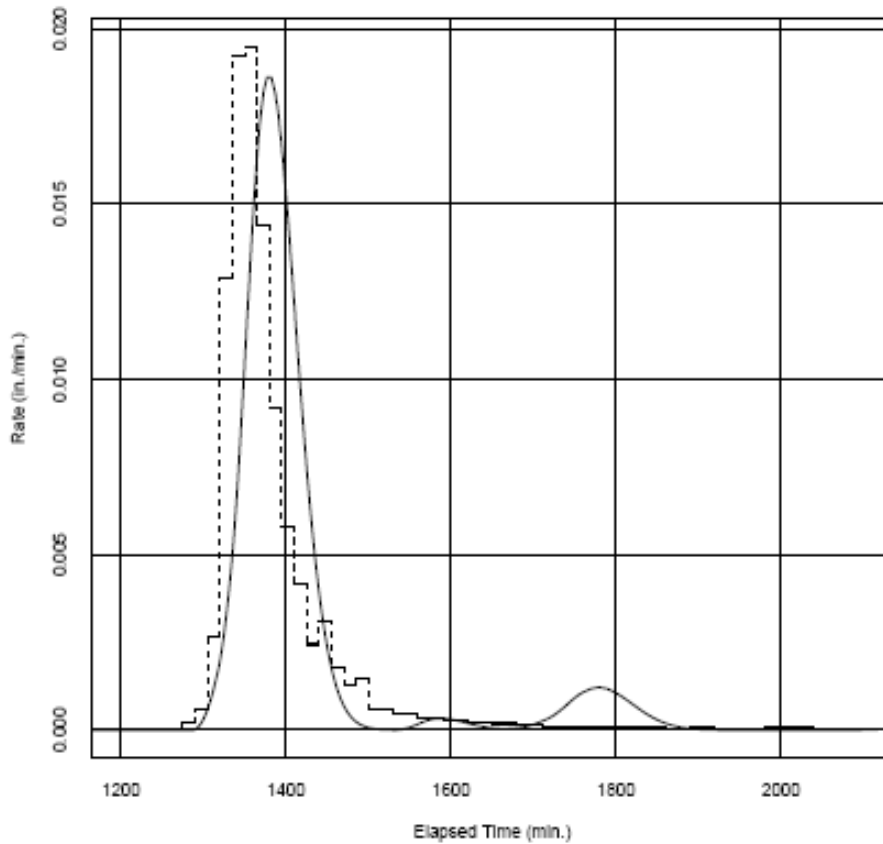
Figure 4. Observed (dashed) and Simulated (solid) runoff hydrograph for Ash Creek Watershed, May 27, 1975 storm.

Figure 5 is a plot of the relationship of main channel length (a characteristic dimension) and the time to peak, T_p for the study watersheds. Open circles are station median values of conventional parameters for developed watersheds obtained from a storm-by-storm inverse analysis where the hydrograph parameters are extracted from paired rainfall-runoff data for undeveloped and developed watersheds. Open triangles are station median values of conventional parameters for undeveloped watersheds also obtained from a storm-by-storm inverse analysis. Details of the inverse analysis are reported in Cleveland and others (2006b). Closed circles are station values of DTRM-derived parameters for developed watersheds. Closed triangles are station values of DTRM-derived parameters for undeveloped watersheds. The entirely synthetic results have a similar trend to the results obtained by analysis of actual storm data. Figure 5 and the typical results in Figure 4 suggest that topographic information alone with a single calibration event is sufficient to produce qualitatively acceptable response hydrographs for the study watersheds.
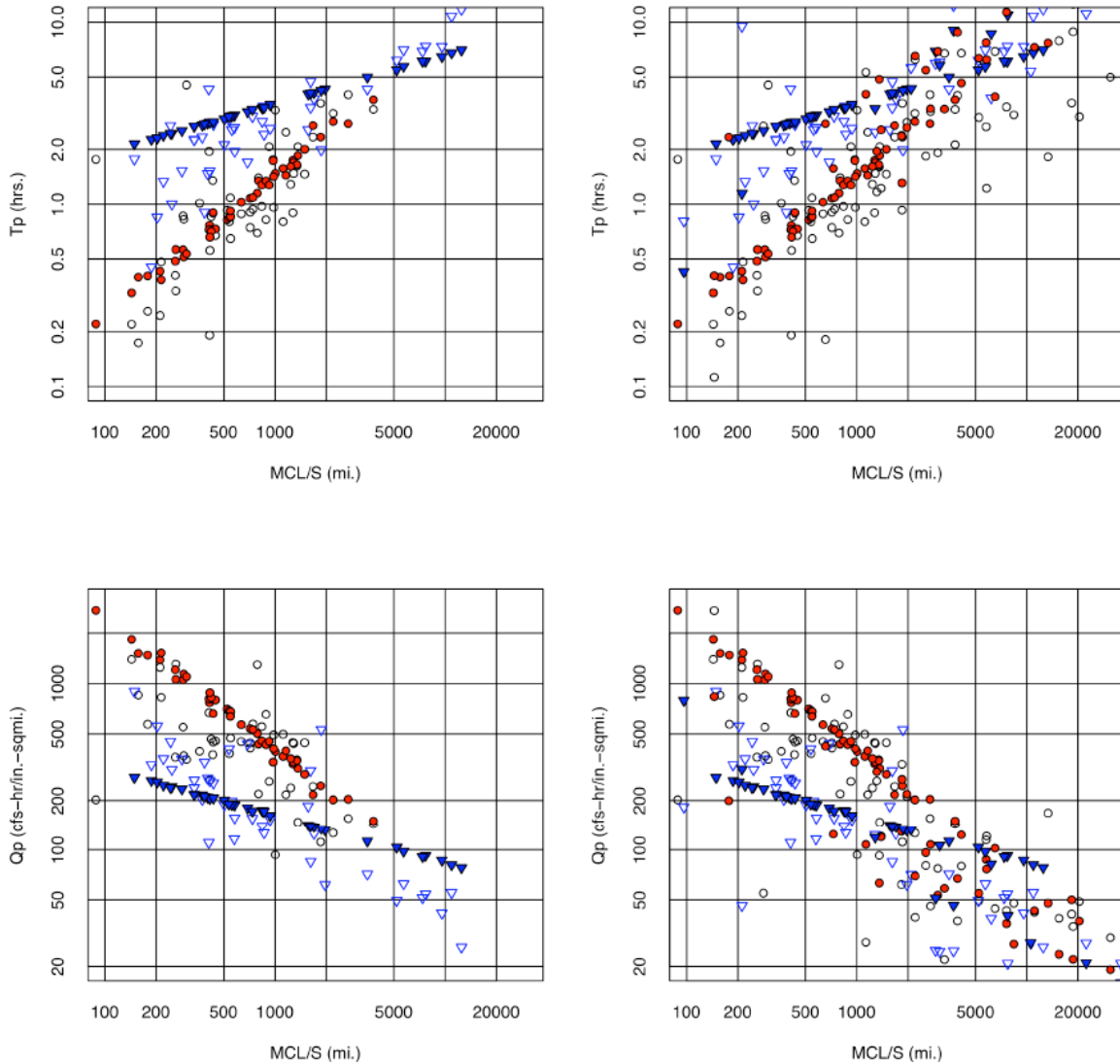
Figure 5. Relationship of T_p and Q_p for Texas watersheds. MCL/S is ratio of main channel length to slope. Left panels exclude Houston watersheds. Right panels include Houston watersheds.

A quantitative comparison is performed by comparison of the results to timing parameters obtained by other methods (Roussel and others, 2005). Several empirical methods are included in the comparison. These estimates are independent from DTRM, sharing only the same underlying digital elevation data, watershed boundary, and outlet location. Figure 6 is a plot of T_c computed by selected methods in Roussel and others (2005) versus watershed area. The T_{98} values (the DTRM surrogate for T_c) are plotted as shaded hexagons. The values plot with the same general trend as the other formulas. The error bars on Figure 6 represent +/- two standard deviations about a mean value for T_c for all the methods plotted except for the DTRM. The DTRM results all plot within the error bars for the other methods indicating that the DTRM results are within +/- two standard deviations of an average value computed by the other methods. The dashed line on Figure 6 is an ordinary least squares log-linear regression through the DTRM

results. This line also falls within the error bars for the other methods further reinforcing that the DTRM results are comparable.
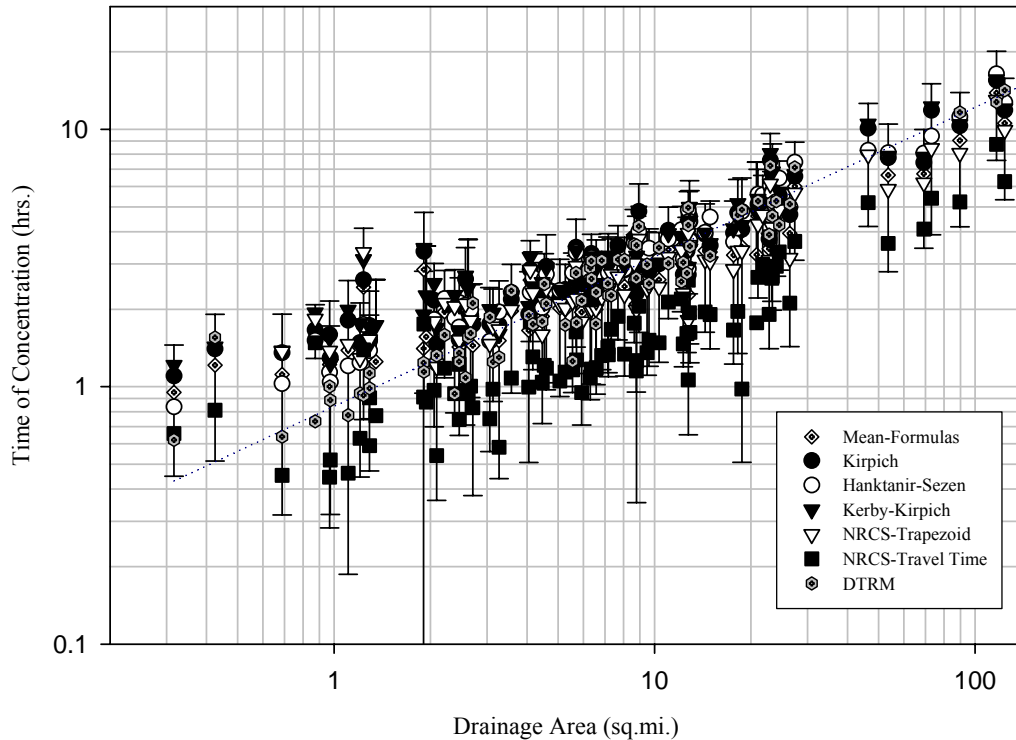

Figure 6. T_c versus watershed area for various methods.

Conclusions

The terrain-based runoff model generates qualitatively acceptable direct runoff hydrographs from minimal physical detail of the watershed. The elevation data are available on the Internet, or can be prepared from paper-based maps.

No attempt was made to optimize the friction terms in the DTRM model to account for different land-uses, etc., yet the approach simulated episodic behavior at about the same order of magnitude as observed behavior in terms of peak discharge and timing. Thus, for the small watersheds studied in this research, topography is a significant factor controlling runoff behavior and consequently the timing parameters common in all hydrologic models.

The DTRM ignored distinctions between channel flow and overland flow yet produced estimates within +/- two standard deviations of other methods.   The method ignored distinctions between undeveloped and developed watersheds, and this distinction is a subject of on-going research by the authors. The similarity of the results to the other methods increases confidence in the other methods, and indicates that the other methods, while overtly empirical, incorporate similar simplified-physics as does DTRM.

T_{98} is a reasonable surrogate for T_c when compared to T_c computed using the same topographic data. The terrain-based procedure described here requires considerable effort to construct the input data files from the elevation data and to guess values at the friction term and flow depths.

References

Asquith, W. H., Thompson, D. B., Cleveland, T. G., and Fang, X., 2004, Synthesis of rainfall and runoff data used for Texas Department of Transportation Research Projects 0–4193 and 0–4194: U.S. Geological Survey Open-File Report 2004–1035, 1,049 p.

Clark, C. O. 1945, Storage and the unit hydrograph, Trans. Am. Soc. Civil Engng 110, p. 1419-1446.

Cleveland, T.G., Fang, X., and Thompson, D.B., 2006a. Timing Parameter Estimation Using a Particle Tracking Method, Texas Department of Transportation Research Report 0-4696-3.

Cleveland, T.G., X. He., Asquith, W.H., Fang, X., and Thompson, D.B., 2006b. Instant-aneous unit hydrograph selection for rainfall-runoff modeling of small watersheds in North and South Central Texas. J. Irrigation and Drainage Engineering, Vol 132, No 5, pp 479-485.

Chutha, P., and J. C. I. Dooge, The shape parameters of the geomorphologic unit hydrograph, J. of Hydrology, 117, 81-97.

Hydrologic Engineering Center (HEC), 2000, Hydrologic modeling system, HEC-HMS, Technical Reference Manual, 149 p.

Jin C., A deterministic gamma-type geomorphologic instantaneous unit hydrograph based on path types, Wat. Resour. Res., 28(2), 479-486, 1992.

Kull, D. W., and Feldman, A. D., 1998, Evolution of Clark's unit graph method to spatially distributed runoff." J. Hydrologic Engineering, ASCE 3(1), p. 9-19.

Leinhard, J. H. 1972, Prediction of the dimensionless unit hydrograph, Nordic Hydrology, 3, p. 107-109

Leinhard, J. H., 1964, A statistical mechanical prediction of the dimensionless unit hydrograph, Journal of Geophysical Research, Vol 69. No. 24., p. 5231-5238.

Lienhard, J. H., and Meyer P. L., 1967, A physical basis for the generalized gamma distribution, Quarterly of Applied Math. Vol 25, No. 3. p. 330-334.

Maidment, D. R. 1993, Developing a spatially distributed unit hydrograph by using GIS, Proceedings of HydroHIS'93, IAHS Pub. No. 211.

Muzik, I. 1996, Lumped modeling and GIS in flood predictions, In Singh, V.P. (Ed.) Geographical Information Systems in Hydrology, Kulwer Academic Publishers, The Netherlands.

O' Calligan, J. F., and Mark, D. M. 1984, Extraction of drainage networks from digital elevation data, Computer Vision, Graphics, and Image Processing, No. 28, p. 323-344.

Olivera, F. and Maidment, D., 1999, Geographic information systems (GIS)-based spatially distributed model for runoff routing, Water Resour. Res. 35(4), p. 1135-1164.

Rodriquez-Iturbe, I., and J. B. Valdes, The geomorphologic structure of hydrologic response, Wat. Resour. Res., 15(6), 1409-1420, 1979.

Roussel, M. C., Thompson, D. B., Fang, X., Cleveland, T. G., and Garcia, A. C., 2005, Timing parameter estimation for applicable Texas watersheds, Texas Department of Transportation Research Report 0-4696-2.

Shamseldin, A. Y., and J. E. Nash, The geomorphological unit hydrograph – a critical review, Hydrology and Earth Syst. Sci., 2(1), 1-8, 1998.

Saghafian, B. and Julien, P. Y., 1995, Time to equilibrium for spatially variable watersheds, J. of Hydrology, 172, p. 231-245.

Saghafian, B., Julien, P. Y., and Rajaie, H., 2002, Runoff hydrograph simulation based on time variable isochrone technique, J. of Hydrology. 261, p. 193-203.

Wooding, R. A. 1965, A hydraulic model for the catchment-stream problem, J. of Hydrology. 3, p. 254-267.