Revisit of NRCS Unit Hydrograph Procedures

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Abstract:

The Natural Resources Conservation Service (NRCS or formally SCS) dimensionless hydrograph (DH) procedure was derived from a large number of natural unit hydrographs from watersheds varying widely in size and geographical locations. This DH has been widely used for many hydrological design practices. This DH has two key parameters: time to peak and peak discharge, which are estimated from empirical relationships by given watershed area and time of concentration. The standard peak rate factor is 484 but ranges from nearly 600 for steep mountainous conditions to 300 for flat swampy conditions. Should the shape of DH be changed if a peak rate factor other than 484 is used? If yes, how should the shape of DH be changed with the peak rate factor? What are possible peak rate factors for some Texas watersheds? A comprehensive discussion on existing literature and results from a study on 90 watersheds in Texas will be presented. General use of the NRCS procedure without consideration of actual regional or site characteristics can result in poor correlation with statistical expectation, inadequate design or over-designed structures.

Introduction and Background

The dimensionless hydrograph (DH) developed by the Natural Resources Conservation Service (NRCS, formerly SCS – Soil Conservation Service) has been widely used to construct synthetic unit hydrographs (SUH) for hydrological design and analysis by many agencies and consulting firms. This dimensionless hydrograph developed by Victor Mockus (NRCS, 1972) was derived from a large number of natural unit hydrographs from watersheds varying widely in size and geographical locations and is given in Figure 1. This DH has a point of inflection approximately 1.7 times the time to peak (T_p) and the time to peak about 0.2 of the time base (T_b). T_p is equal to the watershed lag time (T_L, defined as the time from the centroid of rainfall excess to the peak discharge of hydrograph) plus the half of the rainfall excess duration or the duration of unit hydrograph (D in Fig. 1).

$$T_p = T_L + D/2 \tag{1}$$

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This dimensionless curvilinear hydrograph has 37.5 percent of the total volume in the rising side, which is represented by one unit of time and one unit of discharge. These characteristics of the NRCS DH represent values that have been adopted for an "average" watershed. This DH can also be represented by an equivalent triangular hydrograph (Fig. 1). The peak discharge (Q_p) is given as

$$Q_p = K_o A / T_p \qquad (2)$$

where Q_p is in cfs (cubic feet per second), A is drainage area in mile^2, and T_p is in hours. K_o is the peak rate (discharge) factor (PRF) and considered equal to 484 (English units) assuming a triangular hydrograph with a time base being 8/3 T_p (Fig. 1). "K_o" is related to the internal storage characteristic of a basin and may vary considerably depending on watershed characteristics and scale (size) of a basin (NRCS, 1972). NRCS suggests that K_o may vary from a value of nearly 600 for steep mountainous conditions to a value nearly to 300 in the flat (swampy country) coastal plains of the state (NRCS, 1972). For a very flat, high-water-table watershed, the NRCS peak rate factor of 484 even 300 likely is too large. The University of Florida (Capece et al., 1986) has found that a peak rate factor of 75–100 is appropriate for Flatwoods watersheds. Its value can be as low as 50.

The time base of triangular hydrograph is 8/3 T_p and is based on empirical values for average rural experimental watersheds, and should be reduced (causing increased peak flow) for steep conditions or increased (causing decreased peak flow) for flat conditions. In addition, the empirical relationship for average lag time is assumed to be 0.6 T_c, where T_c is the time of concentration.

$$T_L = 0.6\, Tc \qquad (3)$$

T_c is the time it takes a water parcel to travel from the hydraulically most distant part of the watershed to the outlet. In hydrograph analysis (Fig. 1) T_c is defined as the time difference from the end of excess rainfall to the inflection point of the unit hydrograph (T_{in}).

$$T_c = T_{in} - D = 1.7T_p - D \qquad (4)$$

Using the equations (1), (3) and (4), the duration of unit hydrograph is recommended as

$$D \cong 0.133T_c \ and\ D < 0.17T_c \text{ , or} \qquad (5)$$

$$D \cong 0.2T_p \ and\ D < 0.25T_p \qquad (6)$$

Based on criteria in equations (5) and (6), the duration D is typically selected as approximately equal to the rainfall data interval (for example, use 10 minutes as D for T_c = 1.2 hour and $0.133T_c$ = 9.6 minutes). Figure 2 shows four unit hydrographs developed from NRCS DH for watersheds with T_c ranging from 1.2 to 3.0 hours (hr) and when the peak rate factor of 484 is used for all watersheds. These four watersheds having the same drainage area (4.6 mile^2) but different T_c are hypothetical watersheds used to demonstrate sensitivity of unit hydrographs on the change of T_c value. The peak discharge decreases from 4217 cfs to 1687 cfs as T_c increases from 1.2 hr to 3.0 hr. The duration of unitgraph changes from 10 minutes to 25 minutes. With numerical integration, it was found that all four unit hydrographs have 1" of total direct runoff.

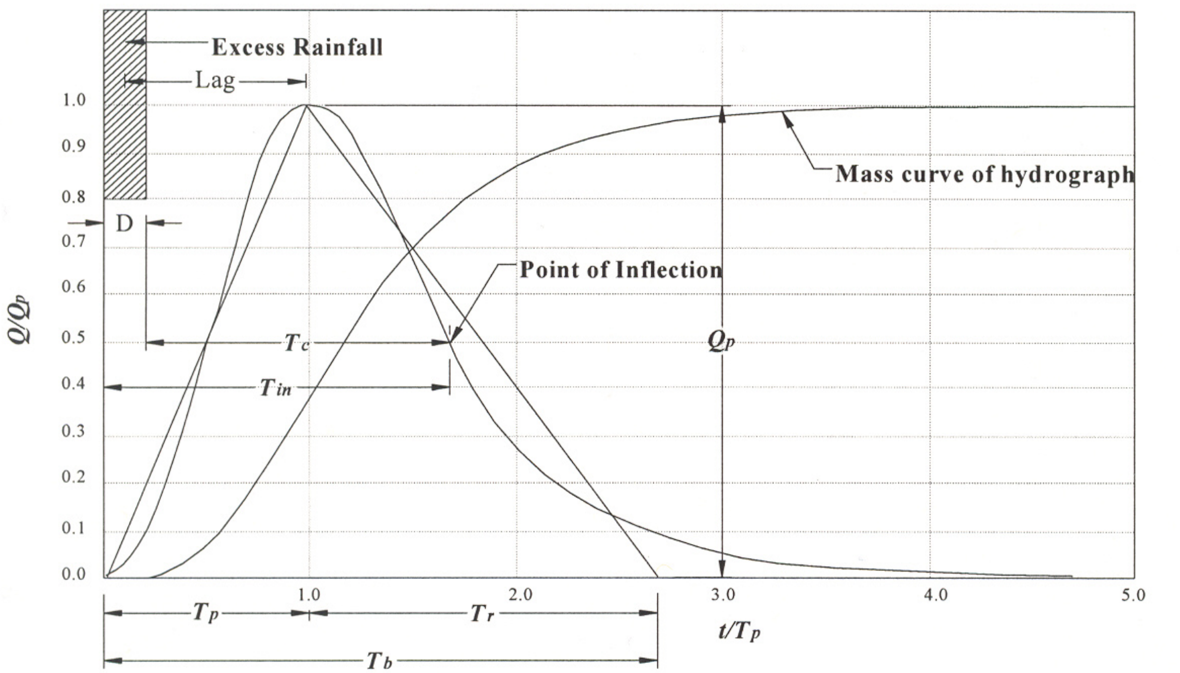

Figure 1. NRCS dimensionless hydrograph (DH) and triangular Hydrograph.

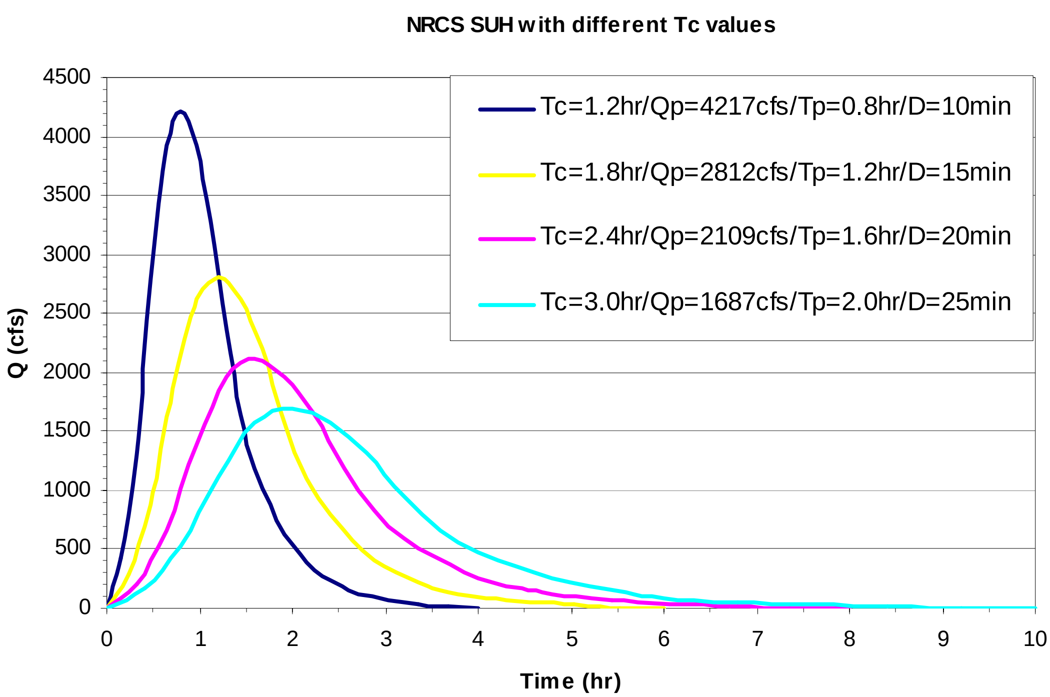

Figure 2. Synthetic unit hydrographs developed from NRCS DH with four T_c values.

When one applies the NRCS DH procedure to develop SUH for a watershed, there is an interesting question none has asked for a long time. Does one need change the shape (ordinates) of DH if a different peak rate factor instead of 484 is used? NRCS (1972) and none of current hydrology related books (e.g., Chow et al., 1988; Maidment, 1993; Viessman and Lewis, 2002; and McCuen, 2005) have addressed the question. This may not be a problem (question) at all. First of all, a simple example is used to demonstrate whether or not this is a problem. In the section 4, Hydrology of the National Engineering Handbook (NRCS, 1972), there is an example which was used to demonstrate NRCS DH procedure to develop SUH. In the example, the watershed has drainage area of 4.6 $mile^2$ and time of concentration of 2.3 hours, unit hydrograph was developed from NRCS DH procedure by using PRF of 484. Using equation (5) to compute the duration of unit hydrograph:

$$D = 0.133x2.3 = 0.306 \text{ hr, use } D = 0.30 \text{ hr } (<0.17T_c) \quad (7)$$

Using equations (1) and (3) to compute time to peak (T_p):

$$T_p = 0.3/2 + 0.6*2.3 = 1.53 \text{ hr} \quad (8)$$

Using equation (2) to compute peak discharge (Q_p):

$$Q_p = (484*4.6)/1.53 = 1455.2 \text{ cfs} \quad (9)$$

Ordinates for NRCS DH are given in Table 1. Using a time step of 0.3 hr and Table 1 with necessary linear interpolation, unit hydrograph developed by NRCS procedure is given in Fig. 3. Figure 3 also includes “unit hydrographs” developed by varying the peak rate factor to 370 and 600 and without changing the shape of DH. This means that only peak discharges (Q_p) were recalculated by equation (2) and the same time and discharge ratios in Table 1 were used again to calculate “unit hydrograph” ordinates for PRF of 370 and 600. Figure 3 shows a problem for basic unit hydrograph concept (Sherman, 1932). By performing numerical integration, it was found that the total direct runoff volume for “unit hydrographs” with PRF of 600, 484 and 370 are 1.26”, 1.0”, and 0.77”, respectively. Therefore, those hydrographs developed by only varying PRF and no changes on DH shape are on longer unit hydrographs. Someone may suggest that ordinates of hydrographs for PRF of 600 and 370 in Fig. 3 should be divided by the total direct runoff, and this will result a unit hydrograph almost the same as unit hydrograph for PRF of 484, then there is no meaning at all to change the PRF factor.

Someone may question the pervious example and state that you should reduce time of concentration for PRF = 600 since higher PRF is for steep mountainous area, and increase time of concentration for PRF = 370 since lower PRF is for swampy area. Figure 4 shows new “unit hydrographs” developed from NRCS DH for PRF = 600 with T_c = 1.2 hr and for PRF = 370 with T_c = 3.5 hr. Change of T_c leads change of duration for unit hydrographs and peak discharges also. After performing numerical integration, it was found again that the total direct runoff volume for “unit hydrographs” are 1.26” for PRF = 600 with T_c = 1.2hr and 0.77” for PRF = 370 with T_c =3.5hr. This again demonstrates that change of T_c but no change on DH shape will still violate unit hydrograph concept. Above two applications of NRCS DH lead us to conclude that the shape of DH should be changed if a different PRF is used. What should the shape of DH for other PRF values be?

Table 1. Ratios for dimensional unit hydrograph and mass curve (NRCS, 1972).

Time Ratios (t/T_p)	Discharge Ratios (q/Q_p)	Mass Curve Ratios (Q_a/Q)	Time Ratios (t/T_p)	Discharge Ratios (q/Q_p)	Mass Curve Ratios (Q_a/Q)
0	0.000	0.000	1.7	0.460	0.790
0.1	0.030	0.001	1.8	0.390	0.822
0.2	0.100	0.006	1.9	0.330	0.849
0.3	0.190	0.017	2.0	0.280	0.871
0.4	0.310	0.035	2.2	0.207	0.908
0.5	0.470	0.065	2.4	0.147	0.934
0.6	0.660	0.107	2.6	0.107	0.953
0.7	0.820	0.163	2.8	0.077	0.967
0.8	0.930	0.228	3.0	0.055	0.977
0.9	0.990	0.300	3.2	0.040	0.984
1.0	1.000	0.375	3.4	0.029	0.989
1.1	0.990	0.450	3.6	0.021	0.993
1.2	0.930	0.522	3.8	0.015	0.995
1.3	0.860	0.589	4.0	0.011	0.997
1.4	0.780	0.650	4.5	0.005	0.999
1.5	0.680	0.705	5.0	0.000	1.000
1.6	0.560	0.751			

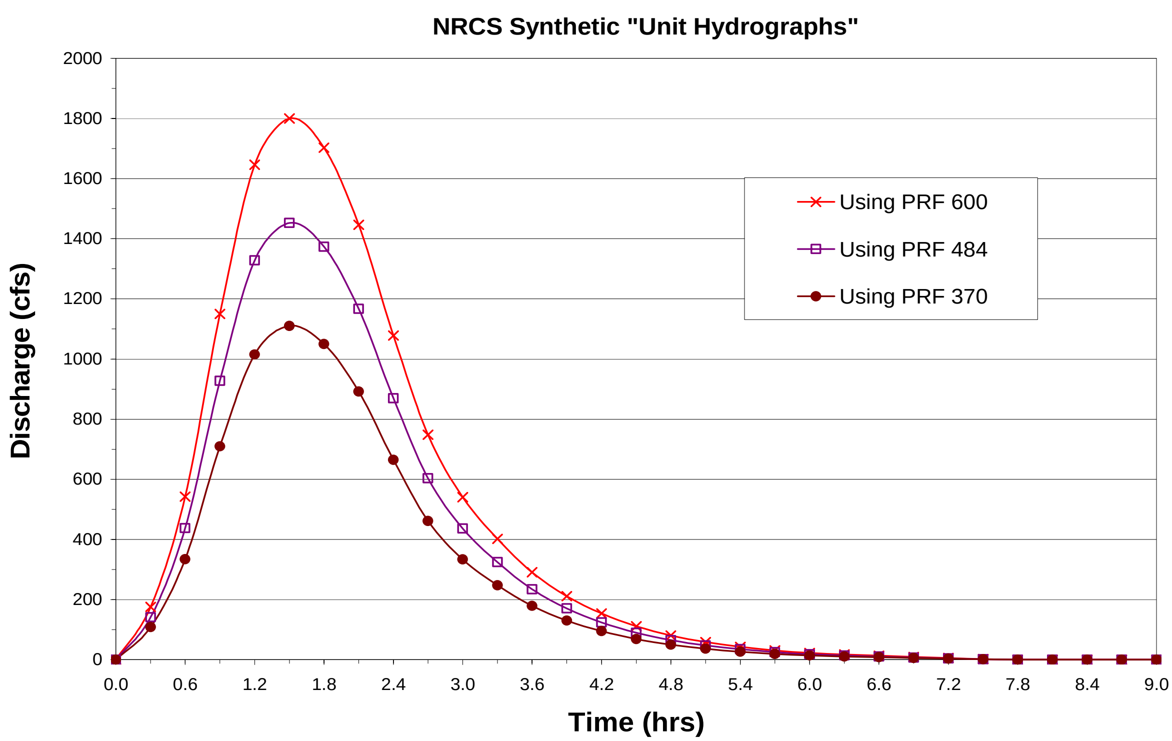

Figure 3. Synthetic unit hydrographs developed by NRCS procedures with PRF of 370, 484 and 600 with same time of concentration.

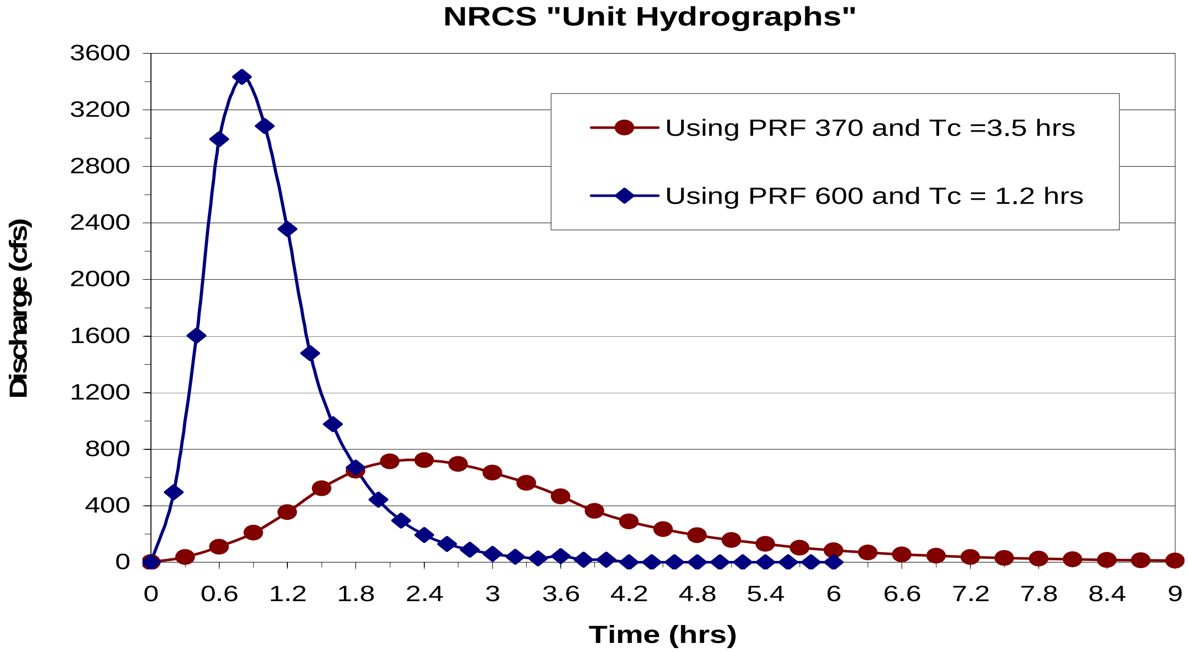

Figure 4. Synthetic unit hydrographs developed by NRCS procedures with PRF of 370 and 600 for different time of concentrations.

Revisit of NRCS Unit Hydrograph Procedures

NRCS SUH procedure is the current method used by many agencies for hydrological designs. NRCS method only requires the time of concentration and watershed area as input to develop unit hydrograph. The peak discharge can be estimated by the simple Equation (2) with a peak rate factor (PRF) of 484 (NRCS, 1972). The PRF equals the product of a unit conversion factor of 645.33 and a K factor of 0.75 derived from the triangular unit hydrograph (Fig. 1) (NRCS, 1972). Let's revisit a unit hydrograph developed from NRCS DH to examine the direct runoff volume (V_D) from 1 inch of rainfall excess for a watershed with area of $\boldsymbol{A}$ mile^2:

$$V_D = 1 \text{ in} * A \text{ mile}^2 \text{ x } 1/12 \text{ ft/in x } 640 \text{ x } 43560 \text{ ft}^2/\text{mile}^2 = 2323200\, A \text{ (ft}^3) \qquad (10)$$

To compute the direct runoff volume under NRCS synthetic unit hydrograph as

$$V_D = \int_0^\infty Q(t)\, dt = Q_p T_p \int_0^\infty (Q/Q_p)\, d(t/T_p) = Q_P T_p \times 1.336 \times 3600\ (ft^3) \qquad (11)$$

where peak discharge Q_p and time to peak T_p are in cfs and hr, respectively, and 1.336 is the numerical integration of NRCS dimensionless hydrograph ordinates given in Table 1. SF = 1.336 means the shape factor for DH. To make both direct runoffs to be equal, one can get

$$Q_p = \frac{2323200A}{1.336 \times 3600\, T_p} = \frac{645.33A}{SF \times T_p} = \frac{483A}{T_p} = \frac{PRF \times A}{T_p} \qquad (12)$$

or $$PRF = \frac{645.33}{SF} = \frac{Q_p T_p}{A(mile^2)} \tag{13}$$

If the volume (numerical integration) under NRCS DH is exact 1/0.75 = 1.333, the peak rate factor will be 484. The equation (13) indicates that the peak rate factor PRF is equal to the conversion factor 645.33 divided by the shape factor SF of NRCS DH. It means that, if the peak rate factor varies from 300 to 600, the shape factor SF for NRCS DH must change also, which implies that the shape of NRCS DH must change with the peak factor. NRCS DH ordinates given in Table 1 and published in the National Engineering Handbook (NRCS, 1972) and in numerous hydrology books are **only** for the peak rate factor of 484. The shape of DH determines the peak rate factor and vice versa. The higher the peak rate factor will result the higher the peak discharge from the watershed.

What could be ordinates for modified NRCS DH under different peak factors? The standard 484 DH was not originally developed from an equation; however, the Gamma function fits the shape fairly well. Use of a two-parameter Gamma distribution for representing unit hydrograph (UH) has a long hydrologic history starting with Edson (1950) who presented a theoretical expression for the unit hydrograph assuming Q to be proportional to $t^x e^{-yt}$.

$$Q(t) = \frac{cAy(yt)^x e^{-yt}}{\Gamma(x+1)} \tag{14}$$

where *Q(t)* is the discharge in *cfs* at time *t*, *A* is the drainage area in square miles, and *x* and *y* are the parameters that can be presented in terms of peak discharge. $\Gamma(x+1)$ is the Gamma function of *x* + 1. Nash (1959) and Dooge (1959) expressed the instantaneous unit hydrograph (IUH) in the form of Gamma distribution based on the concept of *n* linear reservoirs with equal storage coefficient *K*,

$$q = \frac{1}{K\Gamma(n)}\left(\frac{t}{K}\right)^{n-1} e^{-\frac{t}{K}} \tag{15}$$

where *n* and *K* are the parameters defining the shape of IUH and *q* is the runoff depth per unit time per unit effective rainfall [q having a dimension of 1/T]. Aron and White (1982) and Viessman and Lewis (2002) also gave a two-parameter Gamma function f(t) to represent the runoff depth per unit time per unit effective rainfall in equation (16):

$$f(t) = \frac{t^{\alpha} e^{(-t/\beta)}}{\beta^{\alpha+1}\Gamma(\alpha+1)} = \frac{Q(t)}{C_v * A_a} \tag{16}$$

where $0 < t < \infty$, $\alpha > -1$, $\beta > 0$, discharge Q(t) for unit hydrograph is in cfs, drainage area A_a is in acres, and C_v = 1.008 is a unit conversion factor. The most useful feature of the Gamma distribution function is that it guarantees a unit area (i.e., 1" total direct runoff) under the curve for any shape factors $\alpha > -1$ (Viessman and Lewis, 2004). By comparing equations (15) and (16), one can find that

$$n = \alpha + 1 \text{ and } K = \beta \tag{17}$$

for two forms of Gamma functions. Hann et al. (1994) gave n = 4.77 and Singh (2000) gave n = 4.7 for two-parameter Gamma function as good approximation of NRCS DH with PRF = 484 as shown in Fig. 5. Therefore, it is proposed to use Gamma function as a regional dimensionless hydrograph for Texas watersheds to examine relationship between the peak rate factor (PRF) and the shape of dimensionless hydrograph.

Gamma parameter α is a dimensionless shape factor, and β is a positive scale factor with same unit as t axis (time) and controlling base length of unitgraph. The product $\alpha\beta$ gives the value t corresponding to the apex or maximum value (i.e., peak discharge) of *f(t)*, that is

$$T_p = \alpha \beta \tag{18a}$$

$$\frac{Q_p}{C_v A_a} = \frac{Q(T_p)}{C_v A_a} = \frac{T_p^{\alpha} e^{(-T_p/\beta)}}{\beta^{\alpha+1}\Gamma(\alpha+1)} = \frac{(\alpha\beta)^{\alpha} e^{-\alpha}}{\beta^{\alpha+1}\Gamma(\alpha+1)} = \frac{\alpha^{\alpha+1} e^{-\alpha}}{\alpha\beta\Gamma(\alpha+1)} \tag{18b}$$

$$Q_p = \frac{C_v A_a}{T_p} \frac{\alpha^{\alpha+1}}{e^{\alpha}\Gamma(\alpha+1)} = \frac{C_v A_a}{T_p}\phi(\alpha) \tag{18c}$$

$$\phi(\alpha) = \frac{Q_p T_p}{C_v A_a} = \frac{\alpha^{\alpha+1}}{e^{\alpha}\Gamma(\alpha+1)} \tag{18d}$$

where $\phi(\alpha)$ is a dimensionless parameter as function of α. After utilizing above relationships, ordinates of Gamma dimensionless unit hydrograph (DUH) can be given as

$$Q(t)/Q_p = (t/T_p)^{\alpha} e^{(1-t/T_p)\alpha} \tag{19}$$

Equation (19) is the simplest form to develop two-parameter Gamma unit hydrograph. By combining equations (13) and (18), one can get

$$\phi(\alpha) = \frac{Q_p T_p}{C_v A_a} = \frac{645.33}{1.008 \times 640 \times SF} = \frac{1}{SF} = \frac{PRF}{645.33} \tag{20}$$

Singh (2000) also connected the peak rate factor to the Gamma function parameter as

$$\beta_o = PRF/645 = q_p T_p = \frac{Q_p}{C_v A_a} T_p \tag{21}$$

where β_o is a dimensionless parameter, q_p is the peak runoff depth per unit time per unit effective rainfall. Actually β_o is the same as $\phi(\alpha) = Q_p T_p/(C_v A_a)$ used by Aron and White (1982) and Viessman and Lewis (2004). Equations (20) and (21) shows that both Singh's (2003) and current study developed the same relationship of the peak rate factor and Gamma unit hydrograph parameter α. The next logic question would be how one can determine α from $\phi(\alpha)$ in Equation (18d) and vice versa. If this can be done, then one can develop different corresponding Gamma dimensionless unit hydrographs for any given PRF based on equations (19) and (20).

Through in-depth literature review, it was found that many attempts to compute Gamma parameter *n* or *α* from *ϕ(α)* or β_o and vice versa were done by Aron and White (1982), Collins (1983), Hann et al., (1994), Singh (1982, 2003), and Bhunya et al. (2003) as summarized in Table 2. Figure 6 shows that these equations give almost the same results when α >1, which is typical range of value for α of Gamma dimensionless unit hydrographs developed from rainfall and runoff data for Texas watersheds (Khanal, 2004). Two equations from Bhunya et al. (2003) are considered to be the most accurate ones since they were developed by error minimization procedures. Table 3 shows that given peak rate factors can be used to compute $\phi(\alpha)$ by using equation (20), then Gamma function parameter α can be determined from Aron and White's equation. Table 4 demonstrates that given n or α can be used to directly compute $\phi(\alpha)$ or β from equations developed by Bhunya et al. (2003), and then to determine the peak rate factor.

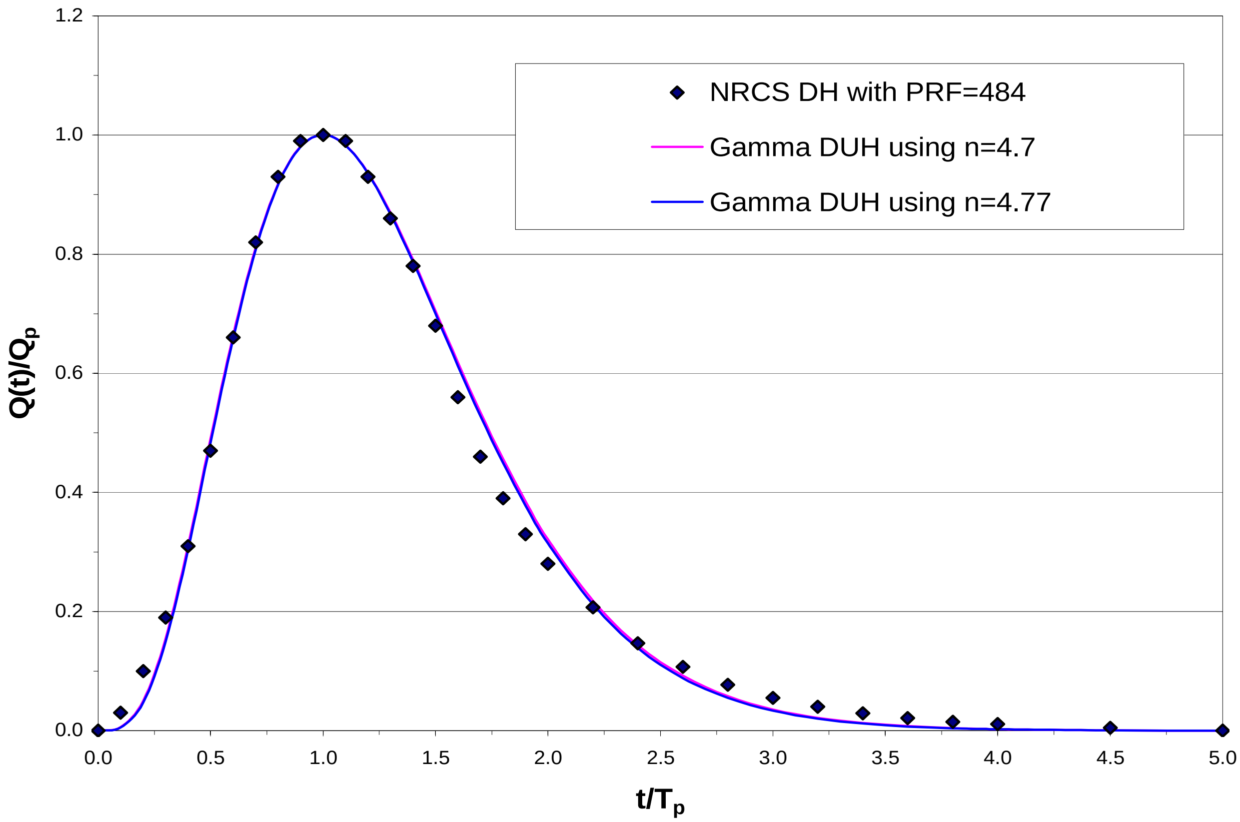

Figure 5. Fitting of Gamma DUH to NRCS DH with PRF=484.

Figure 7a shows Gamma DUHs for different peak factors in Table 3 as modified NRCS dimensionless hydrograph and includes NRCS DH for comparison. Table 3 and Fig. 7a also highlight results and curves for PRFs used in above examples (370, 484, and 660) for easy comparison. Figure 7b shows unit hydrographs developed from Gamma dimensionless unit hydrographs in Fig. 7a for PRF of 370, 484, and 600, respectively. Figure 7a in comparison to Fig. 3 shows how different shapes of UH should be for different peak rate factors. Both Table 3 and Fig. 7a also include Gamma parameters and a curve for a very low PRF of 150, which was found for watersheds in Florida, and could also be true for watersheds in coastal areas in Texas (further study is necessary in the future). From above discussion and information given in Table 3 and Fig. 7, it means that one shape for NRCS DH is not acceptable at all for other peak rate factors, and is only true for the peak rate factor of 484. One can clearly see if the peak rate factors are different for various watersheds or regions, then the shape of the dimensionless hydrograph has to be changed.

Table 2. List of equations to compute Gamma function parameters.

Investigators	Equation for Gamma Parameters	Note
Aron and White (1982)	$\alpha = 0.045 + 0.5\phi + 5.6\phi^2 + 0.3\phi^3$	$\phi = Q_p T_p /(C_v A)$
Collins (1983)	$\alpha = 0.5\phi + 5.9\phi^2$	$1 < \alpha < 8$
Hann et al. (1994)	$n = 1 + 6.5(Q_p T_p / V)^{1.92}$	V is total volume of effective rainfall
Singh (1998)	$n = 1.09 + 0.164\beta_o + 6.19\beta_o{}^2$	$\beta_o = q_p T_p$
Singh (2000)	$n = 7/6 + 2\pi\beta_o{}^2$	
Singh (2000)	$\beta_o = C_p(1 + 0.5t_r / t_l) = 12/(11C_p)$	For Snyder UH
Singh (2000)	$\beta_o = D_f / 645$	D_f is the peak rate factor
Bhunya et al. (2003)	$n = 5.53\beta_o{}^{1.75} + 1.04$	$0.01 < \beta_o < 0.35$
Bhunya et al. (2003)	$n = 6.29\beta_o{}^{1.998} + 1.157$	$\beta_o \geq 0.35$
Bhunya et al. (2003)	$n = 0.158/\lambda^2 + 0.831$	$\lambda = q_p K < 0.27$
Bhunya et al. (2003)	$n = 21.834\lambda^2 - 23.58\lambda + 7.716$	$\lambda \geq 0.27$

Note: [1] $n = \alpha + 1$ and $K = \beta$; [2] q_p is the peak runoff depth per unit time per unit effective rainfall.

Table 3. Computation of Gamma parameters from peak rate factors.

Peak Rate Factor -(PRF)	$\phi(\alpha)$ = PRF/645.33 (Equation 20)	α from Aron and White's Equation	SF -Shape Factor for DH (Equation 20)
300	0.465	1.52	2.127
370	**0.573**	**2.23**	**1.744**
400	0.620	2.58	1.613
450	0.697	3.22	1.435
484	**0.750**	**3.70**	**1.336**
500	0.775	3.94	1.291
550	0.852	4.73	1.173
600	**0.930**	**4.60**	**1.076**
150	0.233	0.47	4.292

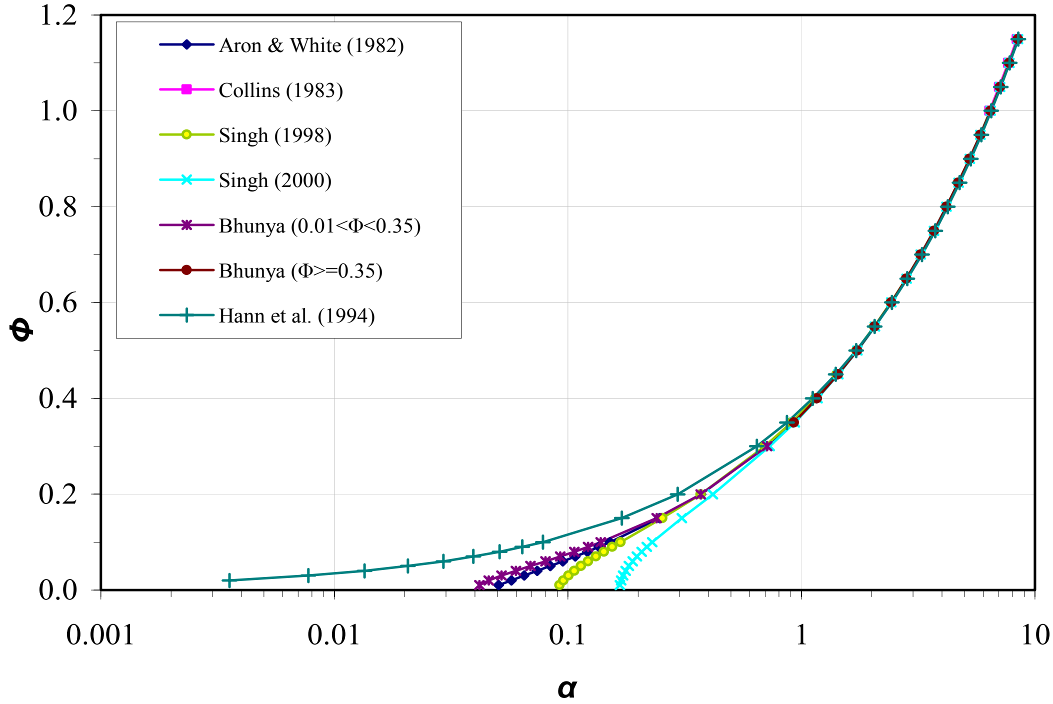

Figure 6. Comparison of equations for the relationships between α and Φ.

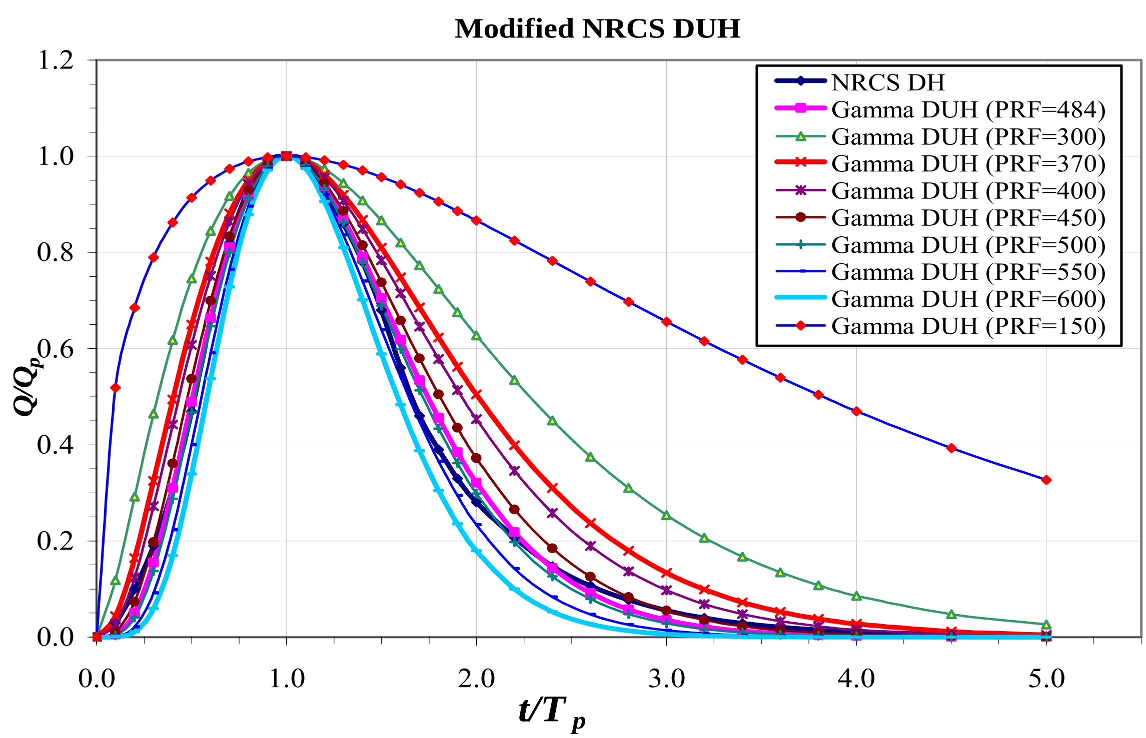

Figure 7a. NRCS DH and Gamma function DUH for various peak rate factors**.**

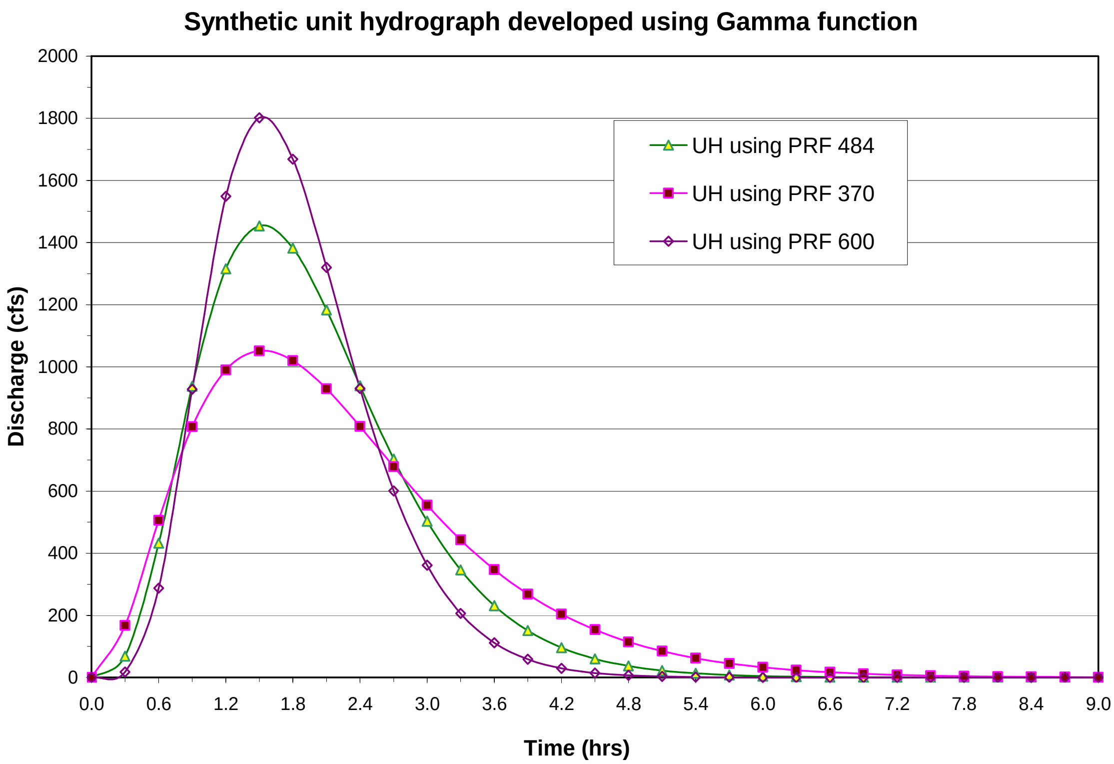

Figure 7b. Synthetic unit hydrographs developed from Gamma function for PRF of 370, 484, and 600, respectively.

Table 4. Gamma parameters versus peak rate factors from NRCS document.

Gamma parameter $\alpha = n\text{-}1$	β_o from equation developed by Bhunya et al. (2003)	Peak Rate Factor -*(PRF)* (Equation 20)
0.26	0.157	101
1	0.369	238
2	0.541	349
3	0.671	433
3.7	0.750	484
4	0.781	504
5	0.878	566

Regional Unit Hydrograph for Texas Watersheds

For above discussion, one can see that Gamma dimensionless unit hydrograph [Equation (19)] can be a good option for regional unit hydrograph. Equation 19 shows that it needs three basic parameters: Q_p, T_p, and α. After considering relationships in Equations (18) and (20) and in Table 2, one can see that only two parameters are necessary for regional UH parameterization and there are six possible combinations for regional parameterization. (1) To parameterize α or *n* and T_p (e.g., develop regression equations for α and T_p with respect to watershed parameters). NRCS unit hydrograph procedure belongs to this type of regional parameterization. It fixes α = 3.7 and has one shape for dimensionless hydrograph. Another alternative is to vary α value, which will change the shape of DH, and then change PRF and Q_p as discussed above. (2) To parameterize α and Q_p, then find time to peak T_p from $\phi(\alpha)$ using equations in Table 2 and equation (20). (3) To parameterize Q_p and T_p, then find α from $\phi(\alpha)$. (4) To parameterize PRF and Q_p, then find α and T_p from $\phi(\alpha)$ =645.33/PRF. (5) To parameterize PRF and T_p, then find α and Q_p from $\phi(\alpha)$ =645.33/PRF. (6) To parameterize β and T_p, then find α as T_p/β and Q_p from $\phi(\alpha)$. Gamma function parameter β is the same as reservoir storage coefficient K for IUH development (Nash and Dooge, 1959; Wu, 1964). It relates to the storage coefficient K_1, which Wu (1964) used to develop regional unit hydrographs (Gamma function) for small watersheds in Indiana. Wu (1964) developed two regression equations for K_1 and T_p as function of watershed parameters – drainage area (mile^2), main channel length (mile) and mean slope of stream. In comparison to Wu's method, to parameterize β and T_p is much easily for development of Gamma UH because of relationships among α, β, $\phi(\alpha)$ and T_p given in equations (18) and (20) and equations in Table 2.

A multiple-institution research project was funded by the Texas Department of Transportation to develop the regional unit hydrograph for Texas watersheds. One of the methods adapted was to develop the unit hydrographs by linear programming from 1600 recorded rainfall-runoff data sets for 90 USGS (US Geological Survey) gage stations in central Texas watersheds (Fig. 8). Unit hydrograph development by linear programming was previously summarized in a paper (Fang et al., 2003) for 2003 Texas ASCE spring meeting in Corpus Christi. The unit hydrographs for each individual rainfall-runoff events were developed by linear programming and were synthesized to develop them into regional dimensional or dimensionless unit hydrographs for watersheds without rainfall-runoff data, termed as regionalization. As discussed above, mathematical Gamma function as DUH was used. The unit hydrograph developed by linear programming was fitted to a two-parameter Gamma UH based on Q_p and T_p of UH by using equation (20) and equation by Aron and White (1982) in Table 2. Gamma unit hydrographs for 5-minute duration (Khanal, 2004) were developed for Texas watersheds, and obviously, unit hydrographs for other durations (e.g., 10 minutes, 30 minutes and etc.) can be developed from 5-minute UH by standard S-hydrograph method (Viessman and Lewis, 2002).

After the Gamma unit hydrographs were developed for all rainfall-runoff events for all watersheds, mean values of Gamma UH parameters were determined for each watershed: mean Q_p, mean T_p, mean α, and mean β. Figure 9 as an example shows peak discharges of Gamma UHs for individual events (small open circles) and watershed mean Q_p for Texas watersheds. To develop regional regression equations of Gamma UH parameters, watershed parameters including drainage area (A in mile^2), main channel length (L in mile), and main channel slope (S in ft/mile) were determined from 30-meter DEM. Table 5 gives watershed parameters for Texas watersheds studied and includes USGS 8-digit ID for gauge stations. The relationship between individual parameters was determined with correlation matrices developed from PlotIT software (Scientific Programming Enterprises, 1996). The correlation matrix gives the possible correlation coefficients between any two parameters. The correlation matrices show that the shape factor α has very weak correlation to any of watershed parameters (Khanal, 2004), and it was not considered further to develop regression equation. Therefore, the options (1) and (2) of parameterization/regionalization discussed above are not good candidates. It was decided to develop regression equations for watershed mean Q_p and T_p [the option (3)] for regional unit hydrograph parameterization, and other options for parameterization will be studied in the future. The regression equations for Tp were developed for watersheds for drainage area less than 10 mile^2 and greater than 10 mile^2, respectively.

$$T_p = 2.65 A^{0.134} L^{-0.089} S^{-0.317} \quad \text{for A < 10 mile}^2 \qquad (22a)$$

$$T_p = 34.82 A^{0.431} L^{-0.491} S^{-0.970} \quad \text{for A >10 mile}^2 \qquad (22b)$$

$$Q_p = 46.99 A^{0.910} L^{-0.219} S^{0.707} \quad \text{for all areas} \qquad (22c)$$

From the above analysis, regional unit hydrograph for central Texas watersheds will be a Gamma synthetic UH [Equation (19)]. The key parameters for regional unit hydrograph are time to peak T_p (hr) and peak discharge Q_p (cfs). These two parameters will allow us to compute the dimensionless parameter $\phi(\alpha)$ for DUH. This parameter can be used to compute peak rate factor *PRF* directly [Equation (20), PRF = 645.33 $\phi(\alpha)$] and compute Gamma function parameter α by Bhunya's Equation given in Table 2. Let's list these key equations again:

$$\phi(\alpha) = \frac{T_p Q_p}{1.008 A_a} = \frac{T_p Q_p}{645.33 A} \qquad (23a)$$

$$\alpha = 5.53 \phi(\alpha)^{1.75} + 0.04 \quad \text{for } 0.01 < \phi(\alpha) < 0.35 \qquad (23b)$$

$$\alpha = 6.29 \phi(\alpha)^{1.998} + 0.157 \quad \text{for } \phi(\alpha) > 0.35 \qquad (23c)$$

where A_a is drainage area in acres and A is in mile^2. Table 5 also gives time to peak T_p and peak discharge Q_p computed from regression equations, dimensionless parameter $\phi(\alpha)$, Gamma UH shape factor α, and peak rate factor PRF for Texas watersheds studied. Figure 10 shows predicted PRF from regression equations (PRF in Table 5) and PRF derived from original watershed mean T_p and Q_p values. Average PRF for watersheds in central Texas is 370 with standard deviation of 76 (blue line in Fig. 10), which is lower than 484 (red line in Fig. 10) for standard NRCS dimensionless hydrograph. Dimensionless Gamma unit hydrograph for PRF = 370 is given in Fig. 7a and dimensional Gamma unit hydrograph for PRF = 370 and T_c = 3.5 hr was given in Fig. 7b as an example.

Current Status of NRCS DH

When above understanding on NRCS DH has reached, at the same time, another important document was discovered. NRCS (2003) was updating the Chapter 16, Hydrographs of the National Engineering Handbook. The draft dated on March 2003 was obtained from Internet. This version of Chapter 16 was prepared by the Natural Resources Conservation Service (NRCS) under guidance of Donald E. Woodward, national hydraulic engineer (retired),

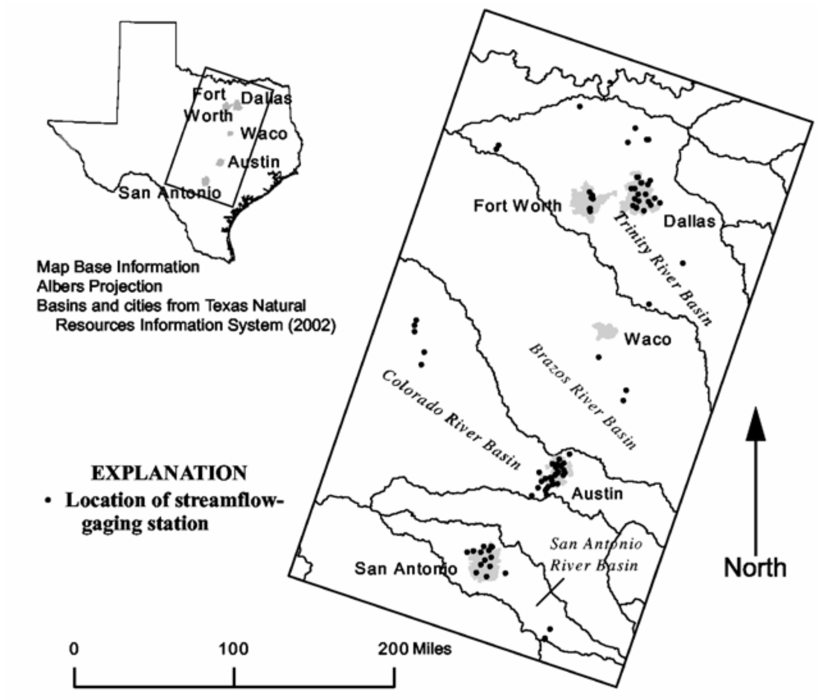

Figure 8. Map showing locations of USGS streamflow gaging stations in rainfall-runoff data base (from Asquith, 2003).

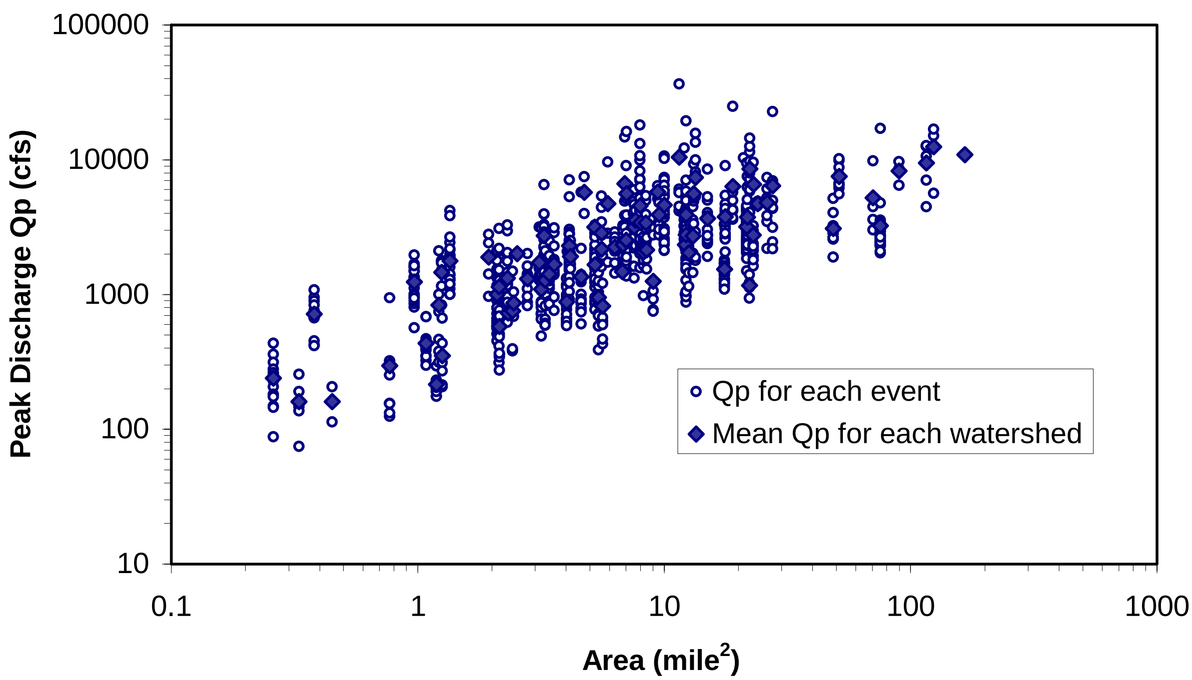

Figure 9. Peak discharges (cfs) for Gamma unit hydrographs in Texas watersheds developed from linear programming of recorded rainfall-runoff data.

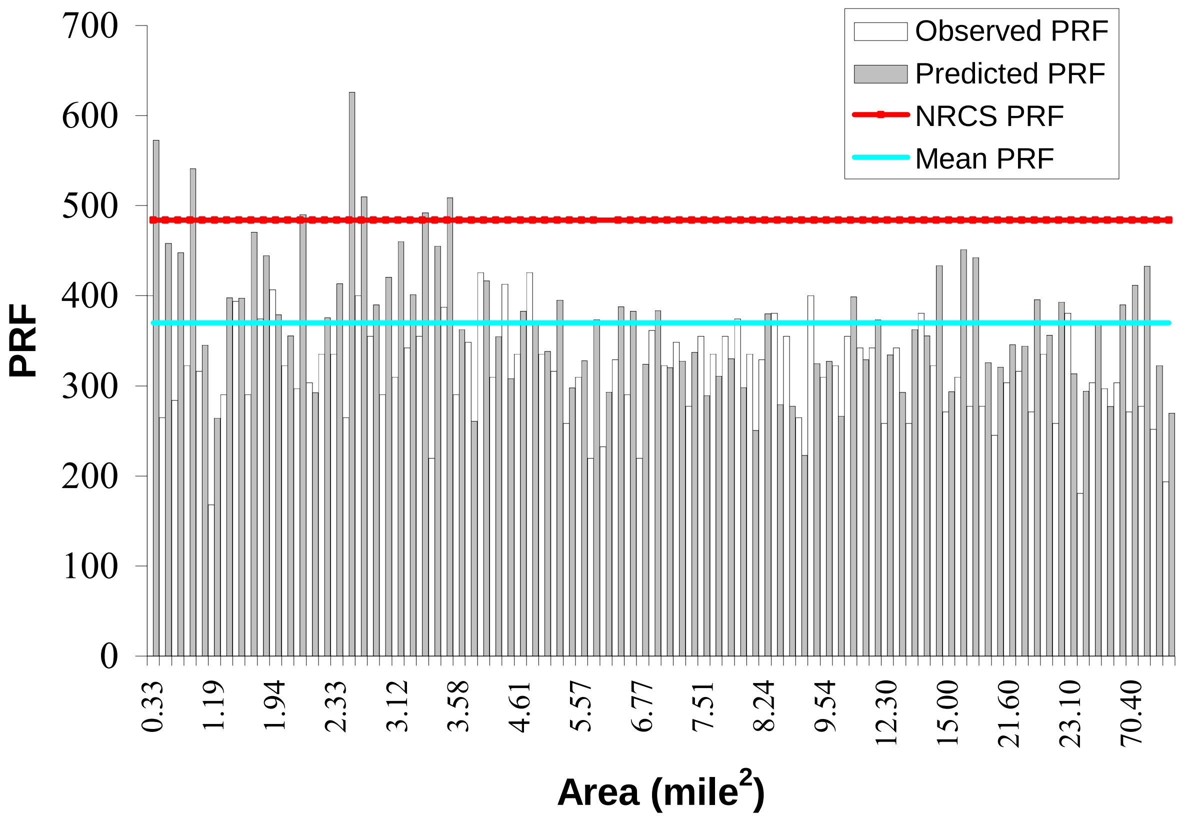

Figure 10. Peak rate factors of synthetic unit hydrographs for central Texas watersheds.

Washington, DC. William H. Merkel, hydraulic engineer, prepared the section dealing with unit hydrograph development on gauged watersheds. Katherine E. Chaison developed the dimensionless unit hydrograph tables and plots in appendix, and Helen Fox Moody edited and reviewed the chapter and developed the tables and figures for an example. The document is **still currently not available** on official NRCS web site yet. In the document, NRCS is no longer suggesting one shape for NRCS DH, and provided DH shape as Gamma function UH for the peak rate factor from 100 to 600 with an increment of 50 in PRF. The equation used by NRCS is the same as Equation (19) proposed by Aron and White in 1982 and used for the current study. The document also provided a step-by-step procedure to estimate the peak rate factor from measured rainfall-runoff data by using TR-20 model. This method is similar to the method used in GUGAS developed by Dr. Asquith at USGS – Austin: adjust parameters for Gamma UH to compare/match computed direct runoff hydrograph (DRH) with observed DRH. The current study is to develop UH first from linear programming and then fit it as two-parameter Gamma unit hydrograph. The procedure used from NRCS document is briefly summarized below.

Table 5a. Watershed parameters and predicted parameters for regional Gamma unit hydrograph.

USGS Station ID	A ($mile^2$)	L (mile)	S (ft/mile)	T_p (hour)	Q_p (cfs)	$\Phi(\alpha)$	α	PRF
8177600	0.33	1.30	69.70	0.58	325	0.89	5.11	573
8178736	0.45	1.67	46.30	0.67	306	0.71	3.33	458
8050200	0.77	2.64	58.95	0.64	535	0.69	3.19	448
8048530	0.97	1.70	65.92	0.67	786	0.84	4.58	541
8048550	1.08	2.02	23.62	0.92	404	0.54	1.96	345
8181450	1.19	3.13	16.62	1.01	313	0.41	1.21	264
8057130	1.22	2.63	41.20	0.77	631	0.62	2.55	398
8057415	1.25	1.88	31.42	0.87	574	0.62	2.54	397
8058000	1.26	2.09	52.71	0.73	814	0.73	3.51	471
8048540	1.35	2.37	49.89	0.74	811	0.69	3.14	445
8055580	1.94	3.00	38.29	0.83	888	0.59	2.33	379
8052630	2.10	3.30	34.75	0.85	873	0.55	2.07	356
8057500	2.14	2.07	54.66	0.77	1355	0.76	3.79	490
8048600	2.15	3.85	23.70	0.95	658	0.45	1.45	292
8157000	2.31	4.12	47.20	0.77	1126	0.58	2.29	376
8178645	2.33	3.96	58.40	0.72	1331	0.64	2.74	413
8178555	2.43	0.46	30.78	1.08	1409	0.97	6.08	626
8178640	2.45	3.04	80.60	0.67	1854	0.79	4.08	510
8057440	2.53	3.52	45.31	0.80	1230	0.60	2.45	390
8156650	2.79	3.00	48.01	0.81	1451	0.65	2.83	420
8155550	3.12	3.66	69.95	0.72	2007	0.71	3.36	460
8094000	3.18	3.35	45.81	0.83	1544	0.62	2.59	401
8178300	3.26	3.58	81.14	0.69	2331	0.76	3.82	492
8187000	3.29	2.62	51.92	0.82	1836	0.71	3.29	455
8139000	3.42	3.36	83.64	0.69	2523	0.79	4.07	509
8158880	3.58	4.40	43.18	0.84	1553	0.56	2.14	363
8137000	4.02	4.40	18.28	1.11	940	0.40	1.18	261
8178620	4.05	3.61	51.95	0.82	2068	0.65	2.78	416
8157500	4.13	5.16	45.56	0.83	1774	0.55	2.06	355
8057160	4.17	5.34	32.55	0.92	1401	0.48	1.59	308
8159150	4.61	3.74	42.39	0.88	2000	0.59	2.37	383
8057020	4.75	5.09	49.36	0.82	2139	0.57	2.22	370
8158380	5.22	4.01	32.21	0.97	1816	0.52	1.89	338
8096800	5.25	4.49	52.45	0.83	2514	0.61	2.52	395
8140000	5.41	5.91	31.42	0.95	1693	0.46	1.50	298
8158400	5.57	4.48	32.22	0.97	1881	0.51	1.78	328
8181000	5.57	5.42	52.32	0.82	2541	0.58	2.27	374
8048820	5.64	6.03	30.53	0.96	1716	0.45	1.46	293
8057435	5.91	4.12	45.98	0.88	2600	0.60	2.43	388
8158920	6.30	4.97	51.27	0.84	2856	0.59	2.37	383
8057120	6.77	5.19	34.32	0.96	2274	0.50	1.74	324
8042650	6.82	4.63	48.24	0.88	2986	0.59	2.38	383
8057320	6.92	5.42	34.40	0.96	2302	0.50	1.71	320
8182400	7.01	4.87	33.45	0.98	2338	0.51	1.78	328
8156700	7.03	4.53	34.00	0.98	2409	0.52	1.88	337
8055600	7.51	6.74	31.12	0.99	2203	0.45	1.42	289
8156750	7.56	5.13	30.17	1.02	2301	0.48	1.62	311

Table 5b. Watershed parameters and predicted parameters for regional Gamma unit hydrograph.

USGS Station ID	A ($mile^2$)	L (mile)	S (ft/mile)	T_p (hour)	Q_p (cfs)	$\Phi(\alpha)$	α	PRF
8057418	7.65	5.65	38.01	0.94	2682	0.51	1.81	330
8056500	7.98	6.37	31.98	0.99	2403	0.46	1.50	298
8061620	8.05	5.52	18.28	1.20	1683	0.39	1.11	251
8158840	8.24	4.96	48.71	0.89	3518	0.59	2.34	380
8187900	8.43	4.87	21.71	1.15	2037	0.43	1.34	279
8057140	8.50	7.47	29.96	1.00	2347	0.43	1.32	277
8057445	9.03	8.42	18.62	1.17	1725	0.35	0.90	223
8057050	9.42	6.21	38.34	0.96	3194	0.50	1.75	325
8178600	9.54	7.05	43.19	0.91	3419	0.51	1.78	327
8055700	10.00	7.77	27.36	1.05	2530	0.41	1.23	266
8057425	11.50	6.16	37.37	1.22	3768	0.62	2.57	399
8158500	12.10	8.59	33.81	1.17	3418	0.51	1.80	330
8158810	12.20	6.29	49.11	0.95	4801	0.58	2.27	374
8048850	12.30	9.40	25.45	1.48	2783	0.52	1.85	335
8156800	12.30	10.58	30.54	1.17	3085	0.45	1.46	294
8158050	13.10	7.36	39.30	1.13	4228	0.56	2.15	363
8057420	13.20	8.33	30.63	1.35	3474	0.55	2.07	356
8061920	13.40	7.64	18.52	2.31	2515	0.67	3.01	434
8181400	15.00	9.82	48.06	0.85	5176	0.46	1.46	294
8063200	17.60	8.73	15.79	2.85	2796	0.70	3.25	452
8048520	17.70	7.53	25.61	1.92	4087	0.69	3.13	443
8158930	19.00	10.40	37.49	1.17	5317	0.51	1.77	327
8158825	21.00	12.53	27.36	1.51	4475	0.50	1.72	322
8042700	21.60	11.57	26.52	1.64	4570	0.54	1.97	346
8136900	21.80	12.42	22.44	1.87	4032	0.54	1.96	345
8098300	22.20	13.73	10.35	3.79	2321	0.61	2.54	397
8154700	22.30	10.04	36.29	1.31	6058	0.55	2.08	357
8061950	23.00	12.65	13.86	3.02	2999	0.61	2.50	394
8158860	23.10	12.79	31.97	1.34	5424	0.49	1.65	314
8158820	24.00	14.85	28.52	1.41	5014	0.46	1.47	295
8158200	26.20	10.92	30.11	1.62	6036	0.58	2.26	373
8158970	27.60	17.61	27.12	1.45	5294	0.43	1.33	278
8137500	48.60	19.96	11.01	4.17	4557	0.61	2.47	391
8155200	70.40	19.38	15.69	3.52	8254	0.64	2.73	413
8155300	75.50	23.23	8.67	5.90	5558	0.67	3.01	434
8158700	89.70	28.50	19.12	2.67	10875	0.50	1.74	324
8158800	116.00	45.07	15.27	2.96	10603	0.42	1.26	271

Step 1—Derive input to the TR-20 model, enter the data, and make preliminary simulations.

Step 2—Examine rainfall and streamflow records to ensure there are periods of record where both rainfall and streamflow are measured.

Step 3—For storm events with both measured rainfall and streamflow, develop a cumulative rainfall distribution to be entered into the TR-20 data file.

Step 4—Runoff volume may be determined for the streamflow hydrograph and is recommended to be the first parameter calibrated. The runoff curve number for the watershed (or its subwatersheds if so divided) may be adjusted such that the storm event rainfall produces the storm event runoff.

Step 5—If the hydrograph at the gage has significant baseflow, the value may be entered in the appropriate location in the TR-20 input file.

Step 6—Timing of the peak discharge at the gage is dependent primarily upon the time of concentration and stream cross-section rating tables. The Manning *n* used for overland flow, concentrated flow, and channel flow is not known precisely. Using gage data may help in refining these estimates. If the times to peak of the measured and computed hydrographs are not similar, the timing factors of the watershed should be adjusted to bring the times to peak into closer agreement.

Step 7—After the runoff volume and timing have been adjusted, the DH may be calibrated by entering a DH in TR-20 with various peak rate factors. The object is to match the peak discharge and shape of the measured hydrograph as closely as possible.

NRCS document also listed some of PRF and corresponding Gamma parameter α found by their study (first two-columns in Table 3). Table 3 also shows computation of $\phi(\alpha)$ by Equation (20) and then computed Gamma parameter α by using Aron and White's equation in Table 2, which is exactly the same as the value in the column one provided by NRCS. NRCS document also shows the equation below

$$PRF = \frac{645.33}{sum\ of\ all\ DUH\ coordinates \times non\,\mathrm{dim}\,ensional\ time\ step} \tag{24}$$

This is the same as Equations (18) and (20) above, where *SF* as the numerical integration of dimensionless UH was used. Equation (24) is valid to present numerical integration under DH if equal time ratio interval is used (traditional NRCS DH in Table 1 has unequal time ratio interval).

It means that through our independent literature review and research development, the research team has achieved the same understanding in the document by NRCS, but NRCS document did not propose any methods/equations to compute Gamma parameter α from PRF. To use equations given in Table 2 are easy and feasible for engineers and designs to compute Gamma parameter *n* or α from PRF. Singh's paper (2000) did criticize NRCS and Snyder SUH, and demonstrated his method, but did not propose what and how NRCS DH should be changed. Singh (2000) proposed his equation to compute *n* from β_o, which is not accurate for low β_o values (see Fig. 5). Singh (2000) also used a complex equation to compute Gamma UH ordinates but mathematically the complex equation is same as simple equation (19) used by Aron and White (1982), by the current study, and by new NRCS document. NRCS document (2003) also did not give any information how PRF can be selected or estimated for different watersheds and regions for engineers to use. The current study outlined procedures to develop regional synthetic unit hydrographs and developed regression equations for T_p and Q_p for Texas watersheds and for engineers and designers to use. Meadows and Ramsey (1991) and Solanki and Suau (unknown year from Internet) had developed regression equations for PRF and for watersheds in South Carolina and Florida regions. They basically corrected PRF to the watershed area and the percent of imperviousness of the watershed (most likely their study and watersheds are for urban areas). Dependence of PRF on watershed parameters for Texas will be further studied later.

Summary and Conclusions

Widely used NRCS dimensionless hydrograph has been carefully reevaluated by using several examples to develop synthetic unit hydrographs. It was found that it is necessary to change the shape of dimensionless hydrograph if different peak rate factors (PRF) are used. Gamma function as dimensional unit hydrograph is proposed for regional unit hydrograph for Texas watersheds. The relationship between PRF and Gamma parameter α has been clearly identified. Unit hydrographs for 90 Texas watersheds have been developed from 1600 recorded rainfall-runoff data pairs by linear programming, then two-parameter Gamma unit hydrographs have been developed. Regional regression equations have been developed for basin-mean time to peak T_p and peak discharge Q_p of Gamma unit hydrographs with respect to watershed parameters: drainage area, main channel slope and length. A procedure has been established for development of synthetic unit hydrographs for Texas watersheds. The study found that mean peak rate factor for Texas watersheds is 370, which is lower than standard NRCS PRF of 484.

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