A New Three Dimensional Finite Difference Model of Ground Water Flow and Land Subsidence in the Houston Area

ROLANDO BRAVO, JERRY R. ROGERS &
THEODORE G. CLEVELAND
Department of Civil and Environmental Engineering, University of Houston, Houston, Texas 77204, USA

ABSTRACT This paper presents a methodology to analyze the subsidence problem in the Houston area using a modified version of the Three Dimensional Finite Difference Ground Water Flow Model developed by the U.S. Geological Survey (USGS). The program simulates the hydrological conditions of the Chicot and Evangeline Aquifers, which underlie Houston, and couples the ground water storage changes in the compressible beds with the aquifer system compaction. The subsidence analysis uses a methodology that is independent of the time interval used in solving the ground water flow governing equation. The regional model is calibrated using actual data from extensometers and piezometers operated by the U.S. Geological Survey in many places throughout Houston. The model uses flux boundary conditions that were estimated using a radial flow analog and Darcy's law. Some head data were generated using the regional variable theory called kriging to supply head estimates in areas where data were unavailable. A one year simulation was made, and a rough estimate of prediction error indicates that the model performs well for locations where data were available.

INTRODUCTION

In the Houston area, land subsidence has occurred for many years. One recognized cause of subsidence in Houston is withdrawal of ground water for municipal, industrial, and agricultural water supply (Williams and Ranzau, 1987). This withdrawal has lowered the static head distribution in the aquifers beneath Houston, and the lowered heads have in turn caused critical subsidence in certain areas. Some land surface has subsided nearly 3 meters since 1906. The U.S. Geological Survey (since the mid 1950's), the Houston-Galveston Coastal Subsidence District (since its creation in 1975), and the City of Houston are all interested in subsidence in the area because parts of the Houston-Galveston area are subject to flood damage which can be intensified by subsidence.

Bravo, et al., (1991) developed a new methodology to predict the compaction and rebound of the soil column given hydraulic heads in the soil. This paper presents a flow and subsidence model to determine the head distributions in the underlying aquifers and to predict regional subsidence.

BACKGROUND

The Houston-Galveston region includes all of Harris and Galveston Counties, and parts of six adjoining counties. Houston is the fifth most populous city in the United States, and third largest in port tonnage. The downtown is 15 meters above sea level; the Johnson Space Center to the south is 5.2 meters above sea level. The region is subject to hurricanes
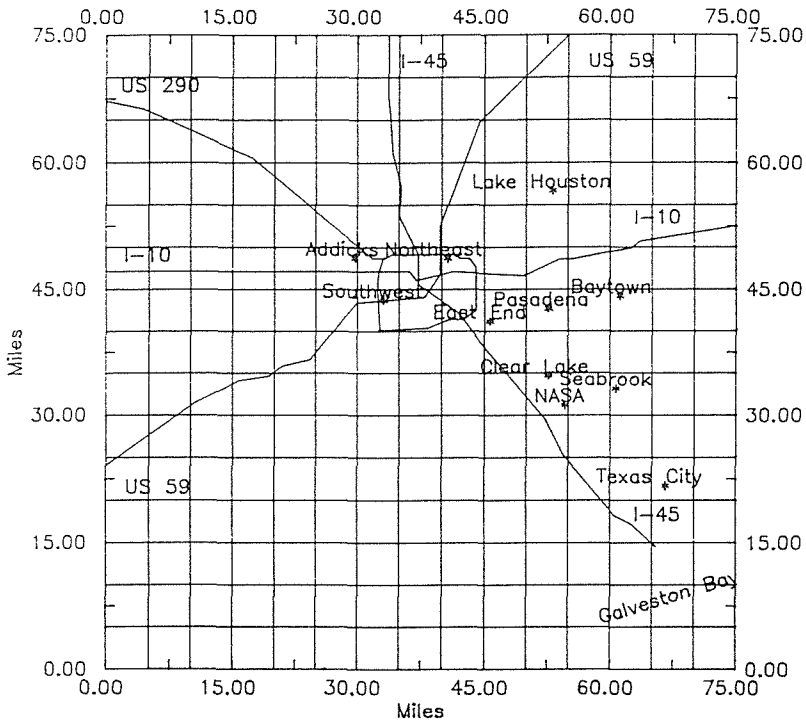
FIG. 1 Location of borehole extensometers of the USGS (Open File Report 89-057). Boundary and grid of the model.

and storms that historically have caused storm surges of 4 to 6 meters and daily rainfall depths up to 107 centimeters. Sections of the region are prone to storm surge flooding and land subsidence compounds the flooding. This problem is the motivation behind the studies of subsidence and ground water flow by Bravo (1990) and earlier researchers.

The major water bearing units in the Houston-Galveston area are the Chicot and Evangeline aquifers. The Chicot aquifer overlies the Evangeline aquifer that overlies the Burkeville confining layer. The relationship of the Chicot aquifer, the Evangeline aquifer, and the Burkeville layer is shown in Figure 2. The Chicot and Evangeline aquifers consist of unconsolidated and discontinuous layers of sand and clay that dip toward the Gulf of Mexico. A detailed description of the subsurface geology is given by Ryder (1988).

PREVIOUS MODELS

The first hydrological model of subsurface flow in the Houston area was the electrical analog model of Wood and Gabrysch (1965). Their model covered about 129.50 km^2 and was used to predict water levels under various conditions of pumping. Its weaknesses were that the aquifers were simulated independently of each other, the western area pumping schedules could not be simulated, the aquifer designation was somewhat fuzzy, and the transmissivities of the aquifers and vertical leakage between the aquifers were not well modeled. In spite of these problems, prior to the 1965 model, predicting aquifer response to various pumping schedules would have been very laborious.

Another early ground water flow model was the electrical analog model by Jorgensen (1975). The model covered about 235.70 km^2 and simulated two layers with vertical leakage. The area modeled was made larger than the Wood and Gabrysch model to
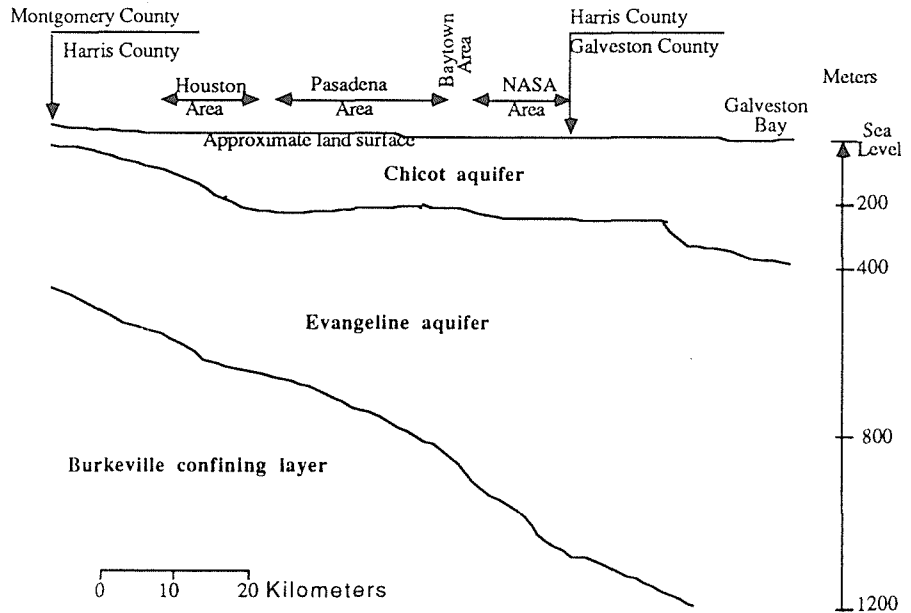
FIG. 2   Hydrologic profile from the Houston area (from Gabrysch, 1976).

minimize boundary effects. Aquifer transmissivities and storage coefficients were estimated from data for many aquifer tests in the Houston area. The model incorporated clay compaction but was not designed to predict subsidence. Jorgensen incorporated more advanced hydrologic concepts but noted that the observed cones of depression in the 1970's extended beyond the model's fixed head boundary. He stated that the model is sensitive to boundary conditions for all simulations beyond 1970.

Meyer and Carr (1979) developed and calibrated a five layer flow and consolidation model of the Chicot and Evangeline aquifers. Their model covered 699.30 km^2 and extended far beyond the Houston area. The model used the USGS-2D Trescott code to solve the flow equations (Trescott et al., 1975). The model had a fine mesh grid in the Houston area and a coarse mesh at its extremities. The arbitrary fixed head boundaries were extended to areas of minimal pumping to reduce boundary effects and eliminate the necessity of imposing flux boundary conditions.

CONCEPTUAL HYDROLOGICAL MODEL

This work describes a methodology to incorporate flux boundary conditions, uses regional variable theory to estimate initial conditions for locations where there are no data, and uses a consolidation theory that is relatively independent of the simulation time steps.

The subsurface lithology of the Houston area is composed of sand and clay layers of varying thickness. Bravo (1990) studied sonic, spontaneous-potential, and conductivity logs for five of the eleven borings shown in Figure 1 (Baytown, Clear Lake, Johnson Space Center, Southwest and Addicks). The logs were manually interpreted to generate geologic profiles of the subsurface at the five sites (Keys, 1971). Interpreted geologic profiles for the Baytown, Clear Lake and Johnson Space Center boring are show in Bravo et al., (1991).

The representation of the subsurface geology was further simplified by concentrating the sand and clay layers in a manner consistent with the stratigraphy in the East-West direction and developing the eight layer conceptual model shown in Figure 3. The North-
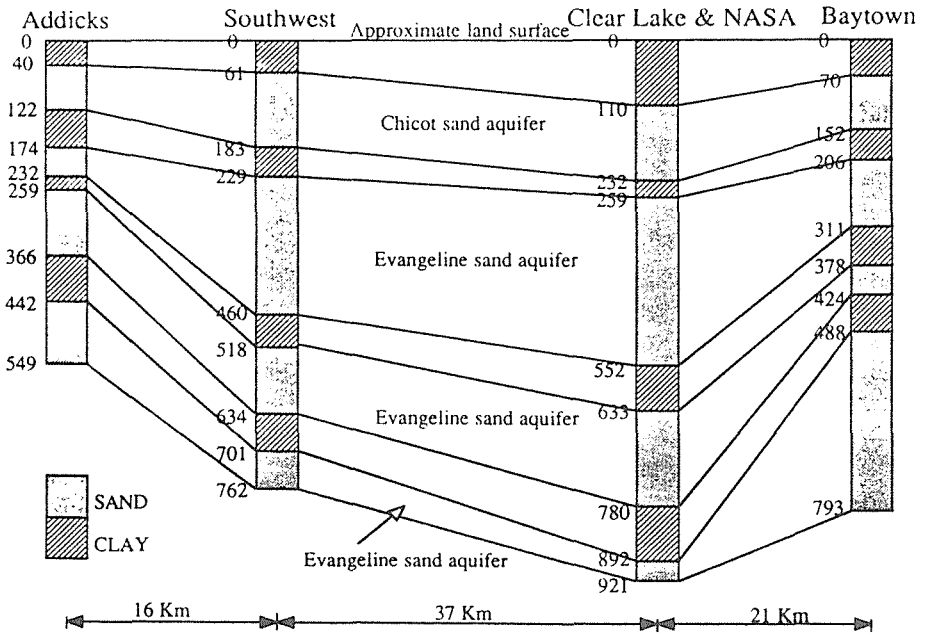
FIG. 3 Conceptual model of the ground water hydrology of the Houston area. The numbers indicate meters below land surface.

South subsurface geology was modeled using the conceptual model and adjusting the thickness of each layer so that the overall aquifer thickness follows the transect shown in Figure 2. The East-West subsurface model was extrapolated horizontally beyond the limits shown in Figure. 3 because there was no further stratigraphic information.

CONCEPTUAL AQUIFER FLOW MODEL

The Chicot Aquifer was modeled as an isotropic aquifer with the potential for either confined or unconfined horizontal flow. The Evangeline Aquifer sand layers were modeled as confined leaky isotropic aquifers. The intervening clay layers were modeled as semi-pervious formations. The effects of delayed storage in the clay layers were modeled as a source term in the flow model but were computed in the consolidation model.

The conceptual hydrogeologic model and flow model are described mathematically by (Bear and Verruijt, 1987)

$$\nabla \cdot (T_i \nabla \phi_i) + \frac{\phi_i - \phi_{i-1}}{\sigma_i} + R_i - P_i = S \frac{\partial \phi_i}{\partial t} + S_{sk} \frac{\partial \phi_i}{\partial t} \qquad (1)$$

subject to the boundary conditions

$$\phi_i(x,y,t) \quad = \quad \text{known on } d\Gamma_1 \text{ (Dirichlet condition)} \qquad (2)$$

$$T_i \frac{\partial \phi_i}{\partial x} \frac{\partial x}{\partial n} \quad = \quad \text{known on } d\Gamma_2 \text{ (Neuman condition)} \qquad (3)$$

and the initial conditions

$$\phi_i(x,y,0) \quad = \quad \text{known on } \Gamma \qquad\qquad (4)$$

where $\phi_i(x,y,t)$ is the piezometric head in sand layer i,

      $T_i(x,y)$ is the transmissivity in sand layer i,

      $\sigma_i(x,y)$ is the conductance of the clay layer between sand layer i-1 and i,

      $R_i(x,y)$ is the recharge in sand layer i,

      $P_i(x,y)$ is the pumping from layer i,

      n   is the outward unit normal vector along the boundary of the flow domain,

      $d\Gamma$ is the boundary of the flow domain ($d\Gamma_1 \cup d\Gamma_2 = d\Gamma$),

      Γ is the flow domain,

      S   is the storage coefficient of the aquifer,

      S_{sk} is the specific storage of the semi-pervious clay layers.

    Unconfined flow in the Chicot Aquifer, when it occurs, was modeled using the Dupuit assumptions and a backwards time linearization. The transmissivity was calculated as the product of saturated thickness and hydraulic conductivity.

    A prescribed piezometric head boundary condition (Dirichlet) was applied along the edge of the model that intersects Galveston Bay, while a prescribed flux boundary condition (Neuman) was applied along the rest of the boundary. The previous regional models of ground water flow in the Houston area used prescribed head everywhere along the boundary. The present work used a flux boundary condition because there were not sufficient data to determine a prescribed head boundary condition for the area studied.

    Piezometric contour maps from 1980 to 1989 were observed to have the same appearance as contour maps that would be expected for radial steady flow to a well. This fact suggested that one test the relationship between radial distances from a hypothetical origin and the piezometric head. In most directions the relationship between the piezometric head and the logarithm of the radial distance was found to be linear and the slopes of the regression lines were almost the same for the ten years studied (Bravo, 1990; Bravo et al., 1990). These slopes can be used to estimate the hydraulic gradient at the boundary.

    The extent of the region studied was chosen to cover the withdrawal areas (pumping areas) for the same decade. The boundary is shown in Figure 2. The radial flow analog and Darcy's law were used to estimate the flux into the domain of interest. The pumping rates for the ten years studied varied from 440 million m^3 year^{-1} to 741 million m^3 year^{-1}; yet the values for the fluxes were relatively constant. Because of this behavior, it was assumed that the fluxes remain constant for prediction horizons of several years. A ground water budget that assumes the clay layers contribute an amount of water equal to 25% of pumping (the proportion concluded by Meyer and Carr, 1979) was satisfied using this flux boundary condition, further strengthening confidence in the flux boundary methodology.

INITIAL CONDITIONS

Initial piezometric heads for all the cells in the model were unavailable. A regional variable theory (kriging) was used to estimate the initial piezometric heads in the cells for which there was no data (Marsily, 1986). A circular search pattern for kriging the data assumed the variation of the heads in the North-South direction were statistically independent for the variation of the heads in the East-West direction (Davis, 1986). This assumption was consistent with the methodology used to determine the flux boundary conditions.

AQUIFER HYDRAULIC CHARACTERICTICS

The transmissivities and storage coefficients for the sand layers were taken from previous studies (Jorgensen, 1975; Meyer and Carr, 1979). The vertical hydraulic conductivities and storage coefficients of the semi-pervious layers were determined independently using the methods developed by Bravo (1990).

CONCEPTUAL SUBSIDENCE MODEL

The principle of effective stress was used to model the relationship of soil compaction and ground water piezometric head (Bear and Verruijt, 1987). In an unconfined aquifer the change in effective stress was expressed as

$$\Delta\sigma' = -\gamma_w (1 - n_e + \theta_w) \Delta h \qquad (5)$$

where $\Delta\sigma'$ is the change in effective stress,

$\quad \gamma_w$ is the specific weight of water,

$\quad n_e$ is the effective porosity of the porous medium,

$\quad \theta_w$ is the moisture content above the water table as a fraction of total volume,

$\quad \Delta h$ is the change in the water table elevation.
In a confined aquifer the change in effective stress was given by

$$\Delta\sigma' = -\gamma_w \Delta\phi \qquad (6)$$

where $\Delta\phi$ is the change in the piezometric head.
    Studies of the change in effective stress and the elastic compaction or expansion of soil indicate that the change in the thickness of an aquifer is proportional to the change in the effective stress. This relationship is expressed as

$$\Delta b = \frac{\Delta\sigma'}{\gamma_w} S_{ske} b_o \qquad (7)$$

where Δb is the change in aquifer thickness,

$\quad S_{ske}$ is the skeletal component of elastic storage,

$\quad b_o$ is the initial thickness of the aquifer.
    The elastic storage coefficient S_{ke} is defined as the product of S_{ske} and b_o. Studies have also shown that when compressible fine grained soils are subjected to stresses greater than a maximum value, the compaction is permanent. This kind of compaction is called inelastic compaction.
The compaction per unit increase in effective stress in the inelastic range is considerably greater than in the elastic range. When the increased effective stress is reduced below the inelastic range the soil resumes its elastic characteristics (with a new initial thickness) unless the effective stress increases beyond the new maximum elastic range. A relation between the inelastic compaction and effective stress similar to the relation for the elastic range was used as a first approximation.
    The specific storage coefficient S_{sk}' of the semi-pervious clay layers in Equation 1 is

determined by both the elastic and inelastic behavior of the soil. The coefficient takes the value of the elastic specific storage whenever the piezometric head is greater than the preconsolidation head. The coefficient takes the value of the inelastic specific storage coefficient when the piezometric head is less than the preconsolidation head.

An implicit approach was used to rewrite the right most term of Equation 1 for modeling the loss or gain of water in the clay layers (Leake and Prudic,1988). This approach allowed the use of much larger time steps in the simulation model than in previous studies which used explicit formulations of the elastic and inelastic storage coefficients (Meyer and Carr, 1979). The implicit formulation is written as

$$\frac{S_{sk}'}{\Delta t} = \frac{S_{sk}^m}{\Delta t}\left[\phi^m - \Phi^{m-1}\right] + \frac{S_{ske}}{\Delta t}\left[\Phi^{m-1} - \phi^{m-1}\right] \tag{8}$$

subject to

$$S_{sk}^m = \left\{ \begin{matrix} S_{ske}, & \phi^m > \Phi^{m-1} \\ S_{skv}, & \phi^m < \Phi^{m-1} \end{matrix} \right.$$

where ϕ^m is the piezometric head at time step m,

Φ^{m-1} is the preconsolidation piezometric head at time step m-1.

This formulation was used in the flow model and once heads were computed for the entire aquifer system, the consolidation was computed using Equations 5 through 7. The values of S_{ske} and S_{skv} were obtained using the methodology described by Bravo (1990).

FLOW AND SUBSIDENCE MODEL SOLUTION

The flow model described by Equations 1 through 3, and Equation 8 was solved using the Modular Three-Dimensional Finite Difference Ground Water Flow Model (MODFLOW) developed by the U.S. Geological Survey (McDonald and Harbaugh, 1988). Transient flow for the geometry defined by the boundary shown in Fig. 4 was modeled using injection wells to simulate the fluxes along the boundary. The subsidence model described by Equations 5 through 7 was solved using the code developed by Leake and Prudic (1988). Their solution code was attached to the MODFLOW code for this research. The subsidence module solved Equations 5 through 7 using the flow solution from Equations 1 through 3 and Equation 8. Because the module also tracked the preconsolidation heads it was an intimate part of the flow model.

RESULTS

The flow model was operated for a simulation period of one year, using initial data from 1983. Figure 4 shows the observed 1984 head distribution in the Chicot Aquifer. Contours are head in feet. The vertical and horizontal scales are in miles. Figure 5 is the simulated head distribution (missing data are estimated by kriging). Figures 6 and 7 are the observed and simulated head distributions in 1984 for the Evangeline Aquifer system. To measure the performance of the model the relative prediction error for the Chicot and Evangeline Aquifers were calculated. The formula used was
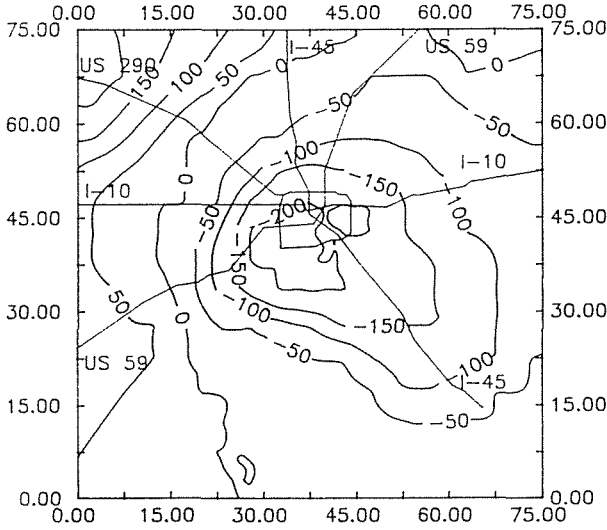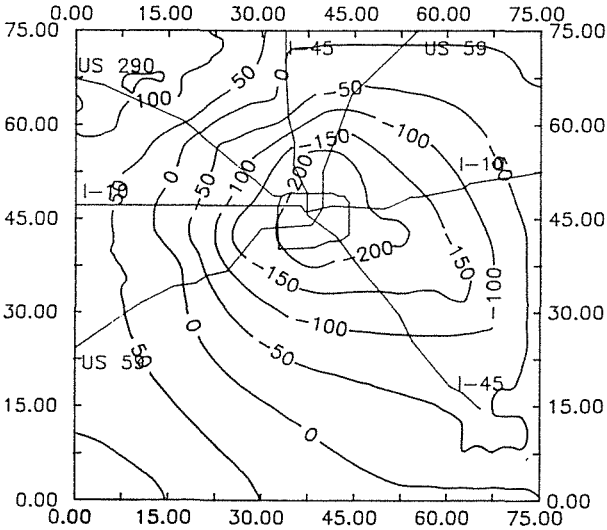
FIG. 4 Chicot aquifer 1984 heads (observed).

FIG. 5 Chicot aquifer 1984 heads (simulated).

$$RPE(x,y) = \frac{\phi_{(x,y)}^{predict} - \phi_{(x,y)}^{actual}}{\phi_{(x,y)}^{actual}} \qquad (9)$$

where RPE(x,y) is the relative prediction error of the flow model.

Figure 8 shows a map of RPE for the Chicot Aquifer. The model performed well in predicting piezometric heads in the Chicot Aquifer at locations where actual data were available. Figure 9 shows a map of RPE for the Evangeline Aquifer. Again the model performs well for those locations where there were data. What is remarkable is that no
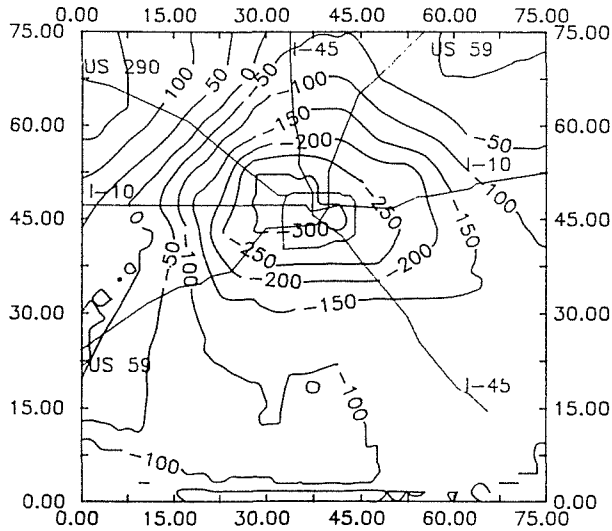
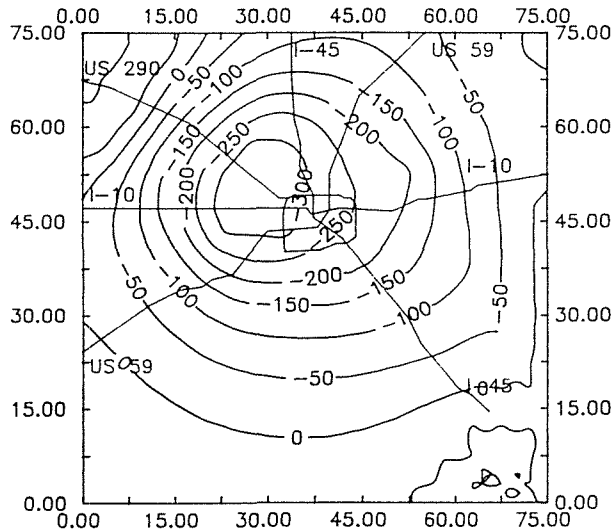**FIG. 6**   Evangeline aquifer 1984 heads (observed).

FIG. 7   Evangeline aquifer 1984 heads (simulated).

parameter identification (inverse estimation) procedures beyond determining the boundary conditions were used; yet the model performed adequately.

Subsidence was calculated but without an initial condition. Figure 10 presents a subsidence prediction for 1984 assuming there is zero subsidence in 1983. This can be interpreted as a map of change in subsidence, much like a ground water drawdown map.

CONCLUSIONS

The University of Houston Civil and Environmental Engineering group has developed a new ground water flow and land subsidence model of the Houston area. Based on the

1983 to 1984 simulation the flow and subsidence model appeared to perform well in areas where data were available; the change in subsidence was consistent with predicted head changes.

The model used regional variable theory for estimating initial conditions at locations where there were no data, and a radial flow analog to estimate flux boundary conditions in the Houston area. The techniques used may be applicable to similar regions; the flux boundary condition eliminates the need to model areas that are greater than the given area of interest.
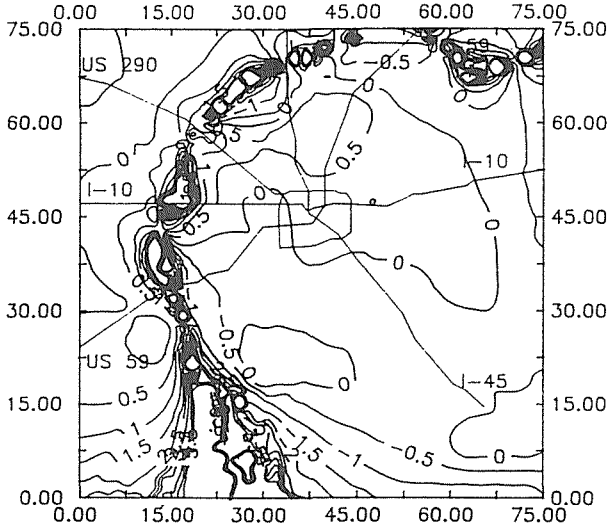

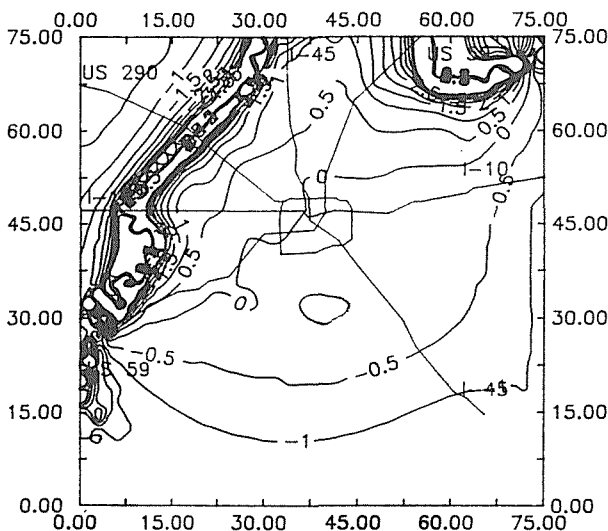**FIG. 8** Relative prediction error for the 1984 Chicot aquifer heads.

FIG. 9 Relative prediction error for the 1984 Evangeline aquifer heads.
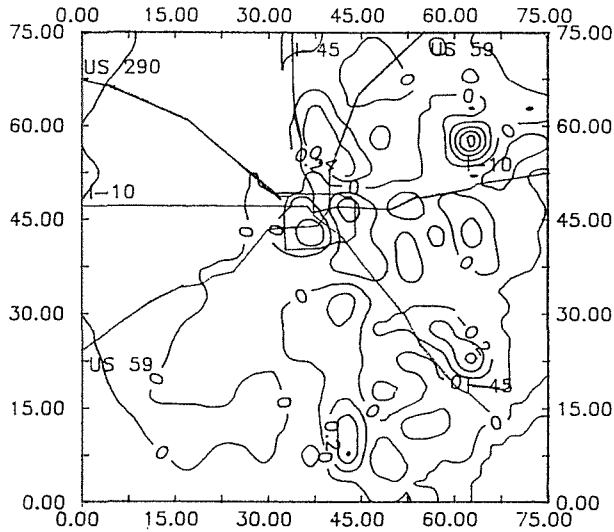
FIG. 10   Subsidence prediction for 1984.

Further research includes a study to determine the sensitivity of the model to changes in aquifer parameters, a study of the influence of storage in the clay layers when the vertical flow assumption is relaxed, and a study of the influence of the search pattern in the kriging algorithm when the assumption of statistical independence of the variation of head with direction is relaxed.

REFERENCES

Bear, J. & Verruijt, A. (1987) Modeling Ground water Flow and Pollution. D. Reidel, Boston.
Bravo, R. (1990) A new Houston ground water flow and subsidence model utilizing three dimensional finite differences. Ph.D. Dissertation, University of Houston, Houston, Texas.
Bravo, R., Rogers, J. R. & Cleveland, T.G. (1990) Prediction of water heads in the Houston area, presented at American Water Resources Association, Texas Section, Austin, Texas, November, 1990.
Bravo, R., Rogers, J.R. & Cleveland, T.G. (1991) Analysis of ground water level fluctuations and borehole extensometer data from the Baytown area, Houston, Texas. Proceedings of the Fourth International Symposium on Land Subsidence, Houston, Texas.
Bravo, R., Rogers, J. R. & Cleveland, T. G. (1991) On the determination of the compressible soil properties required to model subsidence in the area of Houston, Texas. Proceedings of the Fourth International Symposium on Land Subsidence, Houston, Texas.
Davis, J.C. (1986) Statistics and Data Analysis in Geology. Wiley, New York.
Jorgensen, Donald G. (1975) Analog model studies of ground water hydrology in the

Houston district, Texas. U.S. Geological Survey, report 190.

Keys, W.S. & MacCary, L.M. (1971) Applications of borehole geophysics to water resources investigations. U.S. Geological Survey Techniques of Water Resources Investigations Book 2-E1.

Leake, S.A. & Prudic, D.E. (1988) Documentation of a computer program to simulate aquifer-system compaction using the Modular Finite-Difference Ground-Water Flow Model. U.S. Geol. Survey, Open File Report 88-482.

Marsily, Ghilsan de (1986) Quantitative Hydrogeology. Academic Press, New York.

McDonald, M.G. & Harbaugh, A.W. (1988) A modular three-dimensional finite-difference ground-water flow model. U.S. Geological Survey, Techniques of Water Resources Investigations, Book 6-A1.

Meyer, W.R., and Carr, J.E. (1979) A digital model for simulation of ground water hydrology in the Houston area, Texas. Texas Dept. Water Resources LP-103.

Ryder, Paul D. (1988) Hydrogeology and predevelopment flow in the Texas Gulf Coast aquifer system, U.S. Geological Survey water resources investigation report 624-B.

Trescott, P.C., Pinder, G.F. & Larson, S.P. (1975) Finite difference model for aquifer simulation in two dimensions with results of numerical experiments. U.S. Geological Survey, Techniques of Water Resources Investigations, Book 7-C1.

Wood, L.A. & Gabrysch, R.K. (1965) Analog model study of ground water in the Houston district, Texas. Texas Water Commission Bulletin 6508.

Williams III, James F., & Ranzau, Jr., C.E. (1987) Ground water withdrawals and changes in ground water levels, ground water quality, and land surface subsidence in the Houston District, Texas, 1980-1984. U.S. Geological Survey, Water-Resources Investigation Report 87-4153.